объединенный
институт
ядерных
исследований
дубна

E1-80-754

J.Bartke

MULTIPLICITIES OF SECONDARY PIONS IN RELATIVISTIC NUCLEUS-NUCLEUS COLLISIONS AND THE DISTRIBUTION OF THE NUMBER OF INTERACTING NUCLEONS OF THE INCIDENT NUCLEUS

Submitted to International Conference on Nuclear Physics, Berkeley, USA, August 24-30, 1980.

1980

1. INTRODUCTION

Pion production in collisions of relativistic nuclei with nuclear targets has been extensively studied in the last years, and developments in this field could be traced in numerous reviews [1-12]. This report will be devoted to a more detailed discussion of multiplicities of secondary pions produced in relativistic nucleus-nucleus collisions and the distribution of the number of interacting nucleons of the incident nucleus. I will mainly discuss the data from Dubna, where nuclear beams with the highest energies (up to 4.5 GeV/c momentum or 3.7 GeV kinetic energy per nucleon) are available. Three types of track detectors are being used in Dubna for these studies: a 2m propane bubble chamber with thin tantalum target plates inside the chamber volume, a 2m neon-filled streamer chamber "SKM-200" with various internal foil targets (Li, C, Al, Cu, Zr, Pb), and nuclear emulsions. The propane bubble chamber and the emulsions were exposed to the beams of protons, deuterons, helium-4 and carbon-12 nuclei, while the streamer chamber used helium, carbon, oxygen and neon beams.

2. PION MULTIPLICITY DISTRIBUTION IN THE INCLUSIVE SAMPLE

The track detectors with 4π geometry permit one to obtain data on secondary particle multiplicities with relative ease. In the bubble and streamer chambers the external magnetic field allows for the distinction between negatively and positively charged particles and, as it has already become usual, negative secondaries are chosen as best representing pion production processes; the contamination with non-identified electrons and strange particles is expected to be very low here.

As is shown in the studies of high-energy hadron-hadron collisions, already the lowest moments of the pion multiplicity distribution, the average value $\langle n_- \rangle$ and the dispersion $D_- = \sqrt{\langle n_-^2 \rangle - \langle n_- \rangle^2}$, contain interesting information which, when compared to the predictions of various theoretical models, may shed some light on the pion production mechanism.

Table 1. Average multiplicities $\langle n_- \rangle$ and dispersions D_- of the multiplicity distribution of negative secondaries for interactions of p, d, He and C nuclei with carbon and tantalum targets at p/A = 4.2 GeV/c

A_i	A_t	$\langle n_- \rangle$	D_-^2	D_-	N_{tot}
p	C	0.33±0.015	0.28±0.01	0.53±0.01	1620
	Ta	0.45±0.02	0.39±0.02	0.62±0.01	1132
d	C	0.60±0.03	0.48±0.04	0.69±0.03	699
	Ta	0.86±0.03	0.77±0.04	0.88±0.02	1441
He	C	1.02±0.03	0.96±0.04	0.98±0.02	1333
	Ta	1.42±0.06	1.50±0.10	1.23±0.04	1244
C	C	1.50±0.05	1.65±0.08	1.28±0.04	1195
	Ta	3.2 ±0.1	8.4 ±0.4	2.90±0.08	1445

Table 1 shows average multiplicities $\langle n_- \rangle$ and dispersions D_- of the π^- multiplicity distributions for interactions of protons, deuterons, helium-4 and carbon-12 nuclei incident upon carbon and tantalum targets at p/A = 4.2 GeV/c (bubble chamber data). These data differ slightly from those published by us previously[13-16] and are based on the statistics increased to 8037 events in propane and 5262 events in tantalum. The characteristics of interactions with carbon nuclei were obtained from the data on propane by the subtraction procedure described earlier[14-16].

The dependence of the average multiplicity $\langle n_- \rangle$ on the mass number A_i of the projectile nucleus for carbon and tantalum targets is shown in fig. 1. This dependence can be described by the power function

$$\langle n_- \rangle = k\, A_i^{\alpha} \,, \qquad (1)$$

where both k and α depend on the mass number A_t of the target nucleus. For carbon target $k = 0.42\pm0.03$, $\alpha = 0.55\pm0.04$, whereas for tantalum target $k = 0.54\pm0.03$, $\alpha = 0.70\pm0.03$[14-16].

Fig. 2 shows D_- plotted versus $\langle n_- \rangle$ for the data of Table 1 together with the streamer chamber data[17]. The trend of the p-p data, $D_- = a\langle n_- \rangle + b$ [18], is designated as a straight line in this figure. It is seen that the multiplicity distributions for nucleus-nucleus collisions for heavier projectiles, such as carbon,

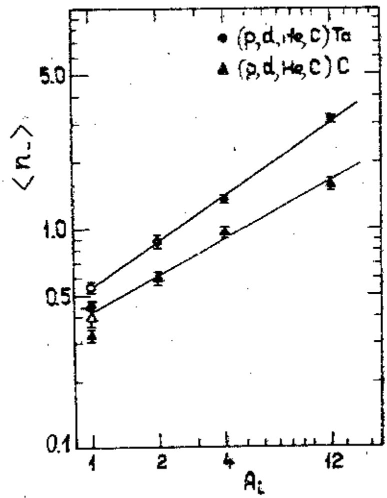
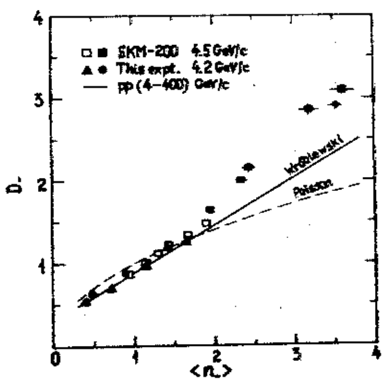


Fig. 1. Dependence of the average multiplicity $\langle n_- \rangle$ of negative secondaries on the mass number A_i of the projectile nucleus in collisions of p, d, He and C nuclei with C and Ta targets at p/A = 4.2 GeV/c.



become wider as compared to nucleon-nucleon collisions with the same average multiplicity. As noted by us earlier/15,16/, the explanation of this effect can be obtained on the basis of the independent collision model. Assuming that the nucleons of the incident nucleus interact independently in the target, one obtains the following expressions for the average multiplicity $\langle n_- \rangle_{A_i A_t}$ and the dispersion $D_{-A_i A_t}$ in nucleus-nucleus (A_i+A_t) collisions in terms of the corresponding quantities characterizing nucleon-nucleus $(N+A_t)$ collisions, $\langle n_- \rangle_{NA_t}$ and D_{-NA_t} :

$$\langle n_- \rangle_{A_i A_t} = \langle \nu_i \rangle \langle n_- \rangle_{NA_t} \qquad (2)$$

$$D^2_{-A_i A_t} = \langle \nu_i \rangle D^2_{-NA_t} + \langle n_- \rangle^2_{NA_t} D^2_\nu \, . \qquad (3)$$

Here $\langle \nu_i \rangle$ is the average number of nucleons of the projectile nucleus which interacted in the target and D_ν is the dispersion of the ν_i distribution, P_ν (ν = 1, 2,... A_i). (For simplicity we omit index "i" in expressions in which ν_i itself is an index).

The data of the Dubna 2m Propane Bubble Chamber Collaboration, in which the non-interacting charged fragments of the projectile

Fig. 2. Dispersion D_- of the multiplicity distribution of negative secondaries in nucleus-nucleus collisions plotted versus average multiplicity $\langle n_- \rangle$.

nucleus were identified among the secondary particles, make possible a numerical check of the above formulae, as both $\langle \nu_i \rangle$ and D_ν can be obtained from the fragmentation data in a model-independent way[16,19]. The tracks of the non-interacting fragments of the projectile were identified on photographs as those emitted at small angles ($\leqslant 4°$ for exposures at p/A = 4.2 GeV/c) and having high momenta ($\geqslant$3 GeV/c), and their charge was determined using ionization, δ-ray density, and analysis of secondary interactions[13,20]. The number of interacting protons of the projectile nucleus is $n_p = Z_i - Q_s$, where Q_s is the total charge of non-interacting fragments. As the inelastic cross sections of protons and neutrons are the same within experimental errors, and for projectiles with equal numbers of protons and neutrons, the average number of interacting nucleons is

$$\langle \nu_i \rangle = 2 \langle n_p \rangle = 2\,(Z_i - \langle Q_s \rangle)\;. \qquad (4)$$

The values of $\langle \nu_i \rangle$ thus obtained for Ta target are listed in Table 2. It is interesting to note[19] that the values of $\langle \nu_i \rangle$ obtained from the projectile fragmentation data are in good agreement with those obtained from the formula derived in [21] under the assumption of independent nucleon-nucleon collision mechanism ("incoherent composition of collisions of individual nucleons")

$$\langle \nu_i \rangle = \frac{A_i \cdot \sigma_{NA_t}}{\sigma_{A_i A_t}}\;, \qquad (5)$$

where σ_{NA_t} and $\sigma_{A_i A_t}$ are the nucleon-nucleus and the nucleus-nucleus inelastic cross sections[13-16].

It can be shown that the dispersion of the number of interacting nucleons, D_ν , can be also obtained from the projectile fragmentation data. The relevant formulae can be found in[16,19]. Our experimental values of D_ν^2 for Ta target are also listed in Table 2.

Table 2. Average numbers $\langle \nu_i \rangle$ and dispersions D_ν of the number of interacting nucleons of d, He and C nuclei incident upon tantalum target at p/A = 4.2 GeV/c.

A_i	A_t	$\langle \nu_i \rangle$	D_ν^2
d	Ta	1.60±0.04	0.24±0.02
He	Ta	2.86±0.10	1.64±0.09
C	Ta	6.60±0.30	16.8 ±1.0

The results of checking the formulae (2) and (3) are given in Table 3. They have been calculated assuming the following values of the average pion multiplicity and dispersion for N-Ta interactions /19/:

Table 3. Average multiplicities $\langle n_- \rangle$ and dispersions D_- of the multiplicity distributions of negative secondaries for interactions of d, He and C nuclei incident upon tantalum target at p/A = 4.2 GeV/c calculated from formulae (2) and (3) using numerical values for $\langle \nu_i \rangle$ and D_ν^2 given in Table 2.

A_i	A_t	$\langle n_- \rangle$	D_-^2
d	Ta	0.88 ± 0.05	0.81 ± 0.07
He	Ta	1.57 ± 0.08	1.81 ± 0.15
C	Ta	3.63 ± 0.17	8.1 ± 0.7

$$\langle n_- \rangle_{NTa} = 0.55 \pm 0.03$$
$$D^2_{-NTa} = 0.46 \pm 0.03 . \qquad (6)$$

The agreement with the experimental values of Table 1 is good. A relative broadening of the pion multiplicity distribution observed for heavier projectiles is accounted for by the second term in the formula (3). This term is small for light projectiles, such as d or He, and for C it becomes comparable to the first term.

3. PION MULTIPLICITY DISTRIBUTIONS FOR EVENTS WITH FIXED NUMBER OF INTERACTING PROTONS

For events with a fixed number of interacting protons of the incident nucleus, n_p, one might expect that the pion multiplicity distribution is narrower than for the inclusive sample. The reason is that, fixing the number of interacting protons, one allows only the number of interacting neutrons to fluctuate between 0 and N_i, and one can expect also these fluctuations to be damped because of similar space density distributions of protons and neutrons, particularly in light nuclei. This decreases D_ν^2 and thus the overall dispersion $D^2_{-A_iA_t}$. The relevant He + emulsion data /22/ and C + Ta data /9/ seem to follow this prediction. The values of $D_-^2/\langle n_- \rangle$ for

C + Ta collisions at p/A = 4.2 GeV/c with the fixed number of interacting protons are given in Fig. 3 showing a significant narrowing of the pion multiplicity distribution as compared to that for the inclusive sample.

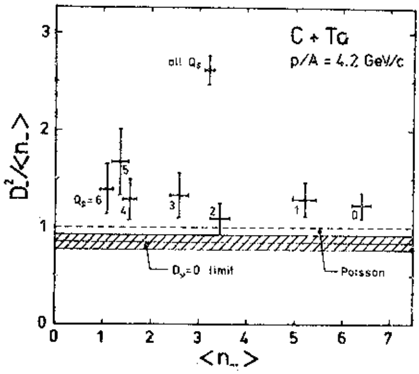


Fig. 3. Values of the ratio $D^2_-/\langle n_-\rangle$ plotted versus $\langle n_-\rangle$ for C + Ta collisions at p/A = 4.2 GeV/c with the fixed number of interacting protons. Q_s denotes the total charge of non-interacting fragments of the projectile nucleus.

A similar picture was observed in the recent paper of the Berkeley-Darmstadt group[23] where the sample of inelastic Ar + KCl collisions at E/A = 1.8 GeV was subdivided into groups according to the total charge of participating nucleons, and these subgroups were shown to have $D^2_-/\langle n_-\rangle \cong 1$.

Collision events which exhibit the absence of charged projectile fragments ($Q_s = 0$ or $n_p = Z_i$) are usually considered as "central". Pion multiplicity distributions for such events were shown to be compatible with the Poisson shape[24,23].

For a subgroup of nucleus-nucleus collisions in which the fluctuations of the number of interacting nucleons were fully eliminated, the independent collision model predicts (see formulae (2) and (3)):

$$\frac{D^2_{-A_iA_t}}{\langle n_-\rangle_{A_iA_t}} = \frac{D^2_{-NA_t}}{\langle n_-\rangle_{NA_t}} \qquad (7)$$

or, using the values (6),

$$D^2_-/\langle n_-\rangle = 0.84 \pm 0.08 \qquad (8)$$

for C + Ta interactions at p/A = 4.2 GeV/c. This value is marked in Fig. 3. as a shaded band. The recent results from the Dubna streamer chamber, obtained at a similar momentum of p/A = 4.5 GeV/c and with an additional "veto" counter of neutral fragments of the projectile, indeed show that in the case of such a strong selection the pion multiplicity distributions exhibit further narrowing, the value of the ratio $D_-^2/\langle n_- \rangle$ falling below unity[25] in accordance with the above prediction. This would mean that the Poisson shape of the pion multiplicity distributions for "central" collisions, observed in refs.[23,24] and interpreted in favour of the hypothesis of Gyulassy and Kauffmann [26], might have been accidental and due to particular selection criteria.

4. DISTRIBUTION OF THE NUMBER OF INTERACTING NUCLEONS OF THE INCIDENT NUCLEUS

It would be interesting to find the shape of the P_ν distribution in nucleus-nucleus collisions. In refs.[9,16] it has been noticed that the experimental values of the dispersion D_ν are significantly greater than it would have been expected for the flat P_ν distribution (P_ν = const) which, together with almost the same average value $\langle \nu_i \rangle$, suggests that small and large values of ν_i would be enhanced.

It can be shown that higher moments of the P_ν distribution could be obtained from the projectile fragmentation data in the model-independent way[19]. In the case of an incident nucleus with charge Z, the moments of the P_ν distribution up to the Z-th order can be calculated. This would give more detailed information on the shape of the P_ν distribution, but, providing only Z constraints, does not allow one to obtain P_ν for $\nu = 1, 2, \dots A$.

The P_ν distribution ($\nu = 1, 2, \dots A$) can be obtained if the projectile fragmentation data are supplemented with data on pion multiplicities in groups of events with a given number of interacting protons[27]. The relations between unknown probabilities P_ν ($\nu = 1, 2, \dots A$) and measured probabilities of the interaction of various numbers of protons of the projectile nucleus, W_n ($n=0,1,\dots Z$), provide us with Z+1 linear equations :

$$W_0 = \sum_{\nu=1}^{N} P_\nu \, C_N^\nu / C_A^\nu \quad \text{and} \quad W_n = \sum_{\nu=n}^{N+n} P_\nu \, C_Z^n \, C_N^{\nu-n} / C_A^\nu , \qquad (9)$$

where C are the binomial coefficients[16,19]. Further Z+1 equations of similar structure can be obtained using the experimental values of average pion multiplicities in subsamples of events with various numbers of interacting protons and assuming that pion production occurs in independent interactions of the nucleons of the projectile nucleus. Thus we have altogether 2(Z+1) equations with A unknowns, which constitute an overdetermined system for light projectile nuclei with A=2Z.

In the case of the deuteron incident upon tantalum, one obtains $P_1 = 0.44$, $P_2 = 0.56$. In the case of the ^{4}He nucleus incident upon tantalum, the system of $2(Z+1) = 6$ equations can be solved using standard minimization procedure. One obtains: $P_1 = 0.21$, $P_2 = 0.20$, $P_3 = 0.13$, $P_4 = 0.41$. However, the quality of the fit is poor, and a better solution is obtained dropping one of the six equations, namely that for average multiplicity in the case of two interacting protons of the ^{4}He nucleus. Then we obtain: $P_1 = 0.19$, $P_2 = 0.17$, $P_3 = 0.14$, $P_4 = 0.49$ with χ^2/NDF = 0.79/1. This distribution is shown in fig. 4a.

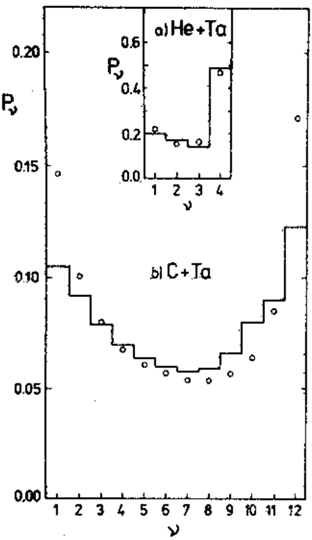


Fig. 4. Probability distributions of the number of nucleons of the projectile nucleus interacting with the target for a) He+Ta and b) C+Ta collisions at p/A = 4.2 GeV/c. Results of the multiple scattering model calculations are denoted by small circles.

In the case of incident carbon nuclei the standard method of solution fails, the solutions being highly unstable often yielding negative values of P_ν. Therefore we tried to obtain a physically sensible solution using the method of regularization described in refs. /28,29/. The basis of this method is the requirement of regularity of the P_ν distribution (minimizing the integral $\int (dz/ds)^2 ds$, where $z(s)$ is the unknown solution), which seems to be justified in our case. The result of this procedure is shown in Fig. 4b. This P_ν distribution reproduces well the experimental distribution of W_n and the average pion multiplicities in subsamples of events with the given number of interacting protons. Details will be given in a separate paper by M.Kowalski and J.Bartke.

The P_ν distributions thus obtained are also in fair agreement with the results of theoretical calculation by Shabelsky and Cheplakov using the multiple scattering (Glauber-type) model. In this calculation the Gaussian density distribution was used for helium nuclei, and the Saxon-Woods density distribution for carbon and heavier nuclei*).

5. CONCLUSIONS

Concluding, one can say that the presented data on multiplicities of secondary pions do not contradict the assumption that the nucleons of the projectile nucleus interact independently in the target, or, at least, that this mechanism of interaction is the dominant one in relativistic nucleus-nucleus collisions. This suggests that in looking for any new effects in these collisions one should study other characteristics such as momentum distributions or correlations, particularly in interactions in which many nucleons are involved (i.e.,central collisions).

The author would like to thank Professor V.G.Grishin and Drs. R.Dymarz, A.P.Gasparian, E.O.Okonov and G.L.Vardenga for helpful discussions as well as Professor Yu.M.Shabelsky and Dr. A.P.Cheplakov for having made available to me the results of their calculations.

REFERENCES

1. Heckman H. In: Proc. 5th Intern. Conf. on High Energy Physics and Nuclear Structure, Uppsala, 1973. Tibell G. ed., North-Holland (Amsterdam-London) 1974, p. 403.
2. Baldin A.M. In: Proc. 6th Intern.Conf. on High Energy Physics and Nuclear Structure, Santa Fe and Los Alamos, 1975. Nagle D.E., Burman R.L., Storms B.G., Goldhaber A.S. and Hargrave C.K. ed., American Institute of Physics (New York) 1975, p. 621.
3. Schroeder L.S. ibid., p. 642.
4. Schroeder L.S. Acta Phys. Polon.,1977, B8, p. 355.
5. Steiner H. In: Proc. 7-th Intern.Conf. on High Energy Physics and Nuclear Structure, Zürich, 1977. Locher M.P. ed., Birkhäuser Verlag (Basel u. Stuttgart) 1977, p. 261.
6. Feshbach H. ibid., p. 291.

*) Similar calculations for helium nuclei incident upon various targets are described in ref./30/.

7. Baldin A.M. Report to the 7th Intern. Conf. on High Energy Physics and Nuclear Structure, Zürich, 1977; JINR E1-11368, Dubna, 1978.
8. Baldin A.M. In: Proc. 19th Intern. Conf. on High Energy Physics, Tokyo, 1978. Homma S., Kawaguchi K., Miyazawa H. ed., Physical Society of Japan (Tokyo) 1979, p. 455.
9. Bartke J. In: Proc. 8th Intern.Conf. on High Energy Physics and Nuclear Structure, Vancouver, 1979. Measday D.F. and Thomas A.W. ed., North-Holland (Amsterdam-New York-Oxford) 1980, p. 481; Nucl. Physics, 1980, A335, p. 481.
10. Siemens P.J. ibid., p. 491; Nucl. Physics, 1980, A335, p. 491.
11. Nagamiya S. ibid., p. 517; Nucl.Physics, 1980, A335, p. 517.
12. Soloviev M.I. In: Proc. Intern. Conf. on Extreme States in Nuclear Systems, Dresden, 1980.
13. Abdrakhmanov E.O. et al. JINR P1-11517, Dubna, 1978; Yad.Fiz., 1978, 28, p. 1304.
14. Angelov N. et al. JINR P1-12281, Dubna, 1979; Yad. Fiz., 1979, 30, p. 1590.
15. Angelov N. et al. JINR E1-12548, Dubna, 1979.
16. Abdrakhmanov E.O. et al. Z.Physik, 1980, C5, p.1.
17. Aksinenko V. et al. Nucl. Physics, 1979, A324, p. 266.
18. Wróblewski A.K. Acta Phys. Polon., 1973, B4, p. 857.
19. Baatar Ts. et al. JINR P1-80-209, Dubna, 1980.
20. Gasparian A.P., Grigalashvili N.S. JINR 1-11335, Dubna, 1978.
21. Białas A. et al. Nucl. Physics, 1976, B111, p. 461.
22. Shabratova G.S. JINR P1-12869, Dubna, 1979.
23. Sandoval A. et al. GSI Preprint 80-15, Darmstadt, 1980.
24. Bartke J. et al. JINR E1-13038, Dubna, 1980.
25. Anikina M.Kh. JINR E1-80-651, Dubna, 1980.
26. Gyulassy M., Kauffmann S.K. Phys.Rev.Lett., 1978, 40, p. 298.
27. Bartke J. In: Proc. Intern. Conf. on Extreme States in Nuclear Systems, Dresden, 1980, Comment after Soloviev's M.I. talk.
28. Tikhonov A.N. Dokl. Akad. Nauk SSSR, 1963, 151, p. 501.
29. Tikhonov A.N. Dokl. Akad. Nauk SSSR, 1963, 153, p. 49.
30. Khorozov S.A. JINR 2-80-142, Dubna, 1980.

Received by Publishing Department on November 24 1980.