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# SCALING IN THE MEAN AND ASSOCIATIVE MULTIPLICITIES FOR THE INCLUSIVE REACTIONS $\bar{p} + p \to K^0_s + x$ AND $\bar{p} + p \to \Lambda + X$ AT 22.4 GeV/c

Dubna - Alma-Ata - Helsinki - Moscow -
Prague - Tbilisi Collaboration

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# SCALING IN THE MEAN AND ASSOCIATIVE MULTIPLICITIES FOR THE INCLUSIVE REACTIONS $\bar{p} + p \to K^{0}_{s} + x$ AND $\bar{p} + p \to \Lambda + X$ AT 22.4 GeV/c

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*Submitted to* ЯФ

## 1. SCALING IN THE MEAN

The hypothesis of scaling in the mean was introduced in paper [1]. From it a scaling equation follows for semi-inclusive one-dimensional distributions of particle c

$$\frac{\langle p_i\rangle_n}{\sigma_n^c}\,\frac{d\sigma_n^c}{dp_i} = \phi_i\left(\frac{p_i}{\langle p_i\rangle_n}\right) \qquad (1)$$

Here $i = T, L$ for transverse ($p_T$) and c.m. longitudinal ($p_L^*$) momenta of particle c, $\langle p_i\rangle_n$ are the mean values of these momenta, and $\sigma_n^c$ is the inclusive cross section of particle c produced in association with n charged particles. The functions $\phi_i$ depend only on the scaling variable $p_i/\langle p_i\rangle_n$, but neither on primary energy nor on charged multiplicity. It has been also assumed that the functions $\phi_i$ are independent of the initial state.

Equation (1) has been tested for $\pi^{\mp}$-meson production in different reactions [1,2a] and for neutral strange particle production in pp-interactions at 19 GeV/c [2b] and 300 GeV/c [2c]. It has been found that scaling in the mean is valid for mesons, but for $\Lambda$-particles only the variable $p_T/\langle p_T\rangle_n$ scales.

In this paper the semi-inclusive processes

$$\bar{p} + p \to K_s^0 + n + \text{neutrals} \qquad (2)$$

$$\bar{p} + p \to \Lambda + n + \text{neutrals} \qquad (3)$$

(n is the number of charged particles) are studied using equation (1).

The experimental data have been obtained in exposures of the hydrogen bubble chamber Ludmila to a RF separated beam of 22.4 GeV/c antiprotons at the Serpukhov accelerator. The amount of interactions is 1123 with $K_s^0$-mesons and 454 with $\Lambda$-hyperons. The scanning, measuring and fitting procedures of the interactions with neutral strange particles have been described elsewhere [3].

The transverse and longitudinal distributions (equation (1)) are presented for $K_s^0$-mesons in Figs.1 and 2 and for $\Lambda$-hyperons in Figs.3 and 4.* The data with $n \geq 6$ have been combined in

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*For clarity our data with $n \geq 6$ and for all events have been shifted along the abscissa axis from the real positions (data at n = 2,4). The data for all events include zero-prong interactions as well.

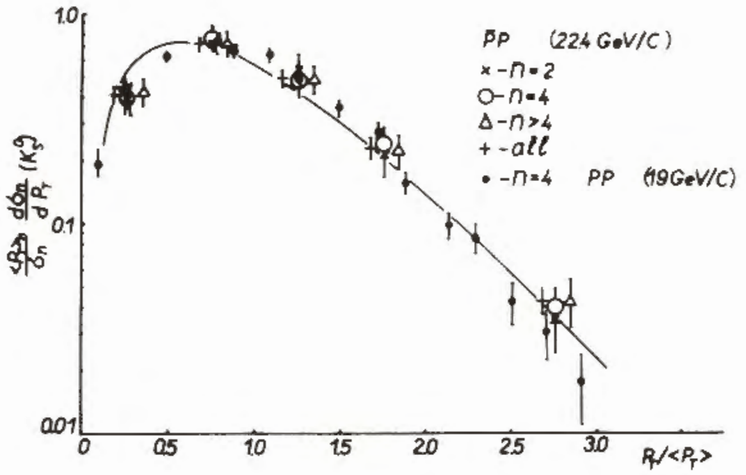

Fig.1. Plot of $(\langle p_T\rangle_n/\sigma_n)\cdot(d\sigma_n/dp_T)$ versus $p_T/\langle p_T\rangle_n$ for $K^0_s$-mesons. The solid line is the result of fitting the $\pi^-$-production in pp-interactions in an energy interval of 13-300 GeV/c.

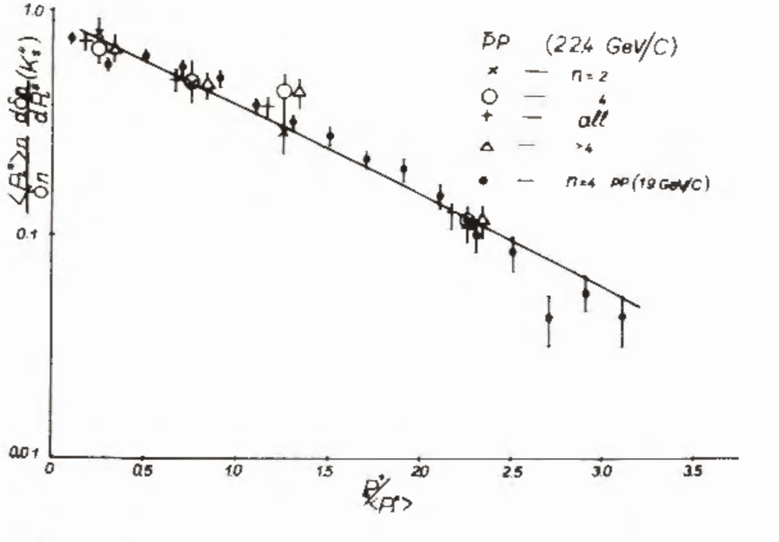

Fig.2. Plot of $(\langle p^*_L\rangle_n/\sigma_n)(d\sigma_n/dp^*_L)$ verses $p^*_L/\langle p^*_L\rangle_n$ for $K^0_s$-mesons. The solid line is the result of fitting the $\pi^-$-production in pp-interactions in an energy interval of 13-300 GeV/c.

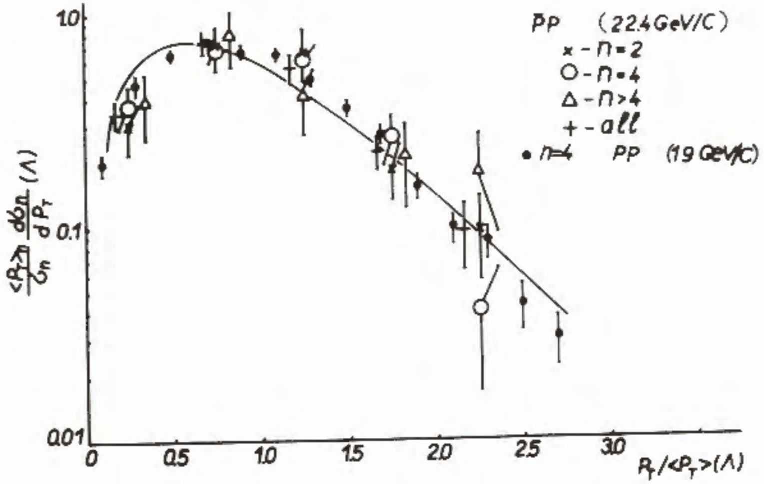

Fig.3. Plot of $(\langle p_T\rangle_n/\sigma_n)(d\sigma_n/dp^*_T)$ versus $p_T/\langle p_T\rangle_n$ for Λ-particles. The solid line is the same as that in fig.1.

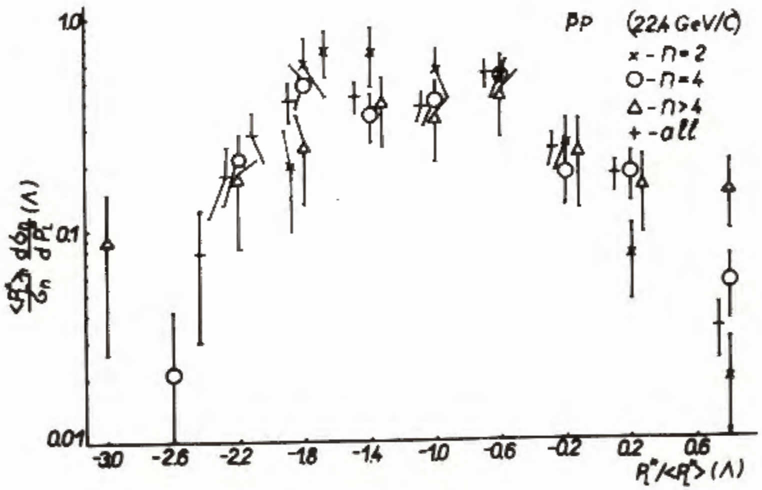

Fig.4. Plot of $(\langle p^*_L\rangle_n/\sigma_n)\cdot(d\sigma_n/dp^*_L)$ versus $p^*_L/\langle p^*_L\rangle_n$ for Λ-particles.

Table

Average values of $p_T$ and $p_L^*$ (in the c.m.s.) for $K_s^\circ$ and $\Lambda$ - particles.

| Particle | Multiplicity | Number of events | $\langle p_T \rangle_n$ GeV/c | $\langle p_L^* \rangle_n$ GeV/c |
|---|---|---|---|---|
| $K_s^\circ$ | 2 | 265 | 0.42 ± 0.02 | 0.65 ± 0.04 |
| | 4 | 375 | 0.43 ± 0.01 | 0.57 ± 0.03 |
| | ≥ 6 | 435 | 0.43 ± 0.01 | 0.44 ± 0.02 |
| | all | 1123 | 0.423± 0.008 | 0.56 ± 0.02 |
| $\Lambda$ | 2 | 166 | 0.47 ± 0.02 | -1.58 ± 0.08 |
| | 4 | 185 | 0.49 ± 0.02 | -1.09 ± 0.07 |
| | ≥ 6 | 81 | 0.47 ± 0.03 | -0.69 ± 0.08 |
| | all | 454 | 0.48 ± 0.01 | -1.26 ± 0.05 |

order to improve statistics. The points from the pp-experiment at 19 GeV/c are given for comparison.

For $K_s^\circ$-mesons the hypothesis of scaling in the mean is valid within errors for both transverse and longitudinal variables. For $\Lambda$-hyperons the scaling in the mean holds for the transverse variable, but for the longitudinal variable the shape of the distribution depends on multiplicity. For low multiplicities it is steeper. A similar effect has been also observed in the pp-experiment at 19 GeV/c.

The curves in Figs.1,2 and 3 describe the scaling in the mean for $\pi^-$-production in pp-interactions in an energy interval of 13-300 GeV/c /1/. One can see that the data points follow these curves closely. A similar feature has been observed for $\pi^\circ$-mesons in $\pi^- p$-interactions at 5 GeV/c /2a/. Thus, one can see that scaling in the mean distributions for both $p_T$ and $p_L^*$ - variables are very similar for different types of mesons. In addition, for the $p_T$-variable these distributions are very alike for mesons and $\Lambda$-hyperons.

The mean values of the transverse and longitudinal momentum for $K_s^\circ$ and $\Lambda$ are presented in the Table. As is seen, the absolute values of $\langle p_L^* \rangle_n$ for $\Lambda$-hyperons decrease with increasing n, i.e., the leading behaviour* of $\Lambda$'s is more pronounced for small multiplicities. So the kinematical limits for the variables $p_L^*/\langle p_L^* \rangle_n$ vary strongly for different n** and scaling in the mean is violated at large negative values of $p_L^*/\langle p_L^* \rangle_n$ (for $p_L^*/\langle p_L^* \rangle_n \lesssim -1.8$ in Fig.4). Thus, the violation of the law of scaling in the mean for $\Lambda$'s in this kinematical region is connected with the leading effect.

It has been proposed /4/ that the scaling in the mean is a consequence of: a) Feynman scaling /5/, b) KNO-scaling /6/ and c) factorization of the cross section over longitudinal and transverce momenta

$$\frac{d^2\sigma_n}{dp_L^*\,dp_T} = f_1\left(p_L^*, \frac{n}{\langle n \rangle}\right) \cdot f_2(p_T), \qquad (4)$$

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* The production of $\Lambda$'s mainly in the proton fragmentation region.

** The $p_L^*/\langle p_L^* \rangle_n$ kinematical limits are -1.94 for n = 2, -2.74 for n = 4, -4.22 for n ≥ 6 and -2.43 for all events.

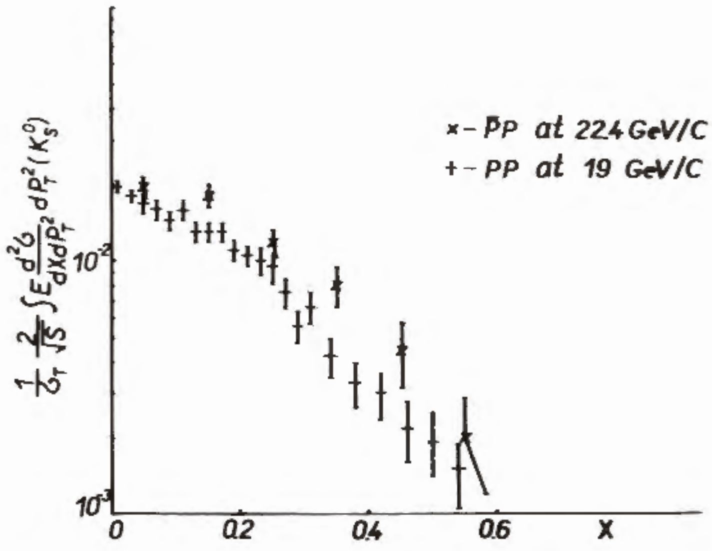

Fig.5. Distribution of the invariant function $\frac{1}{\sigma_{tot}}\frac{2}{\sqrt{s}}\int E^*\frac{d^2\sigma}{dx\,dp_T^2}dp_T^2$ versus x for $K^\circ_s$ -mesons.

where $n/\langle n\rangle$ is the KNO-variable /6/. In other words, it has been supposed that the hypothesis of scaling in the mean contains no new information. In order to check this assumption, we have tested the conditions a) and c) for $K^\circ_s$ inclusive production. Figure 5 shows the invariant normalized inclusive x-distributions ( x is the Feynman variable) for $K^\circ_s$ production in our experiment and in pp-interactions at 19 GeV/c /7/. One can see that there is a significant difference between the presented distributions in the intermediate region of x, i.e., Feynman scaling does not hold for different initial states.

In fig.6 (a,b,c,d) we show the $p^*_L$-distributions of $K^\circ_s$ in different intervals of $p_T$ for all events and separately for multiplicities 2, 4 and 6. It is seen that in all considered cases the shapes of the distributions change with $p_T$, i.e., the factorization condition (4) is not fulfilled. The same result has been found for pp-interactions at 19 GeV/c /2b/.

Thus, the results presented in figs.5 and 6 do not support the assumption that the scaling in the mean is only the consequence of the above laws a), b), c). Another picture of connection between the scaling laws is considered in papers /8/,

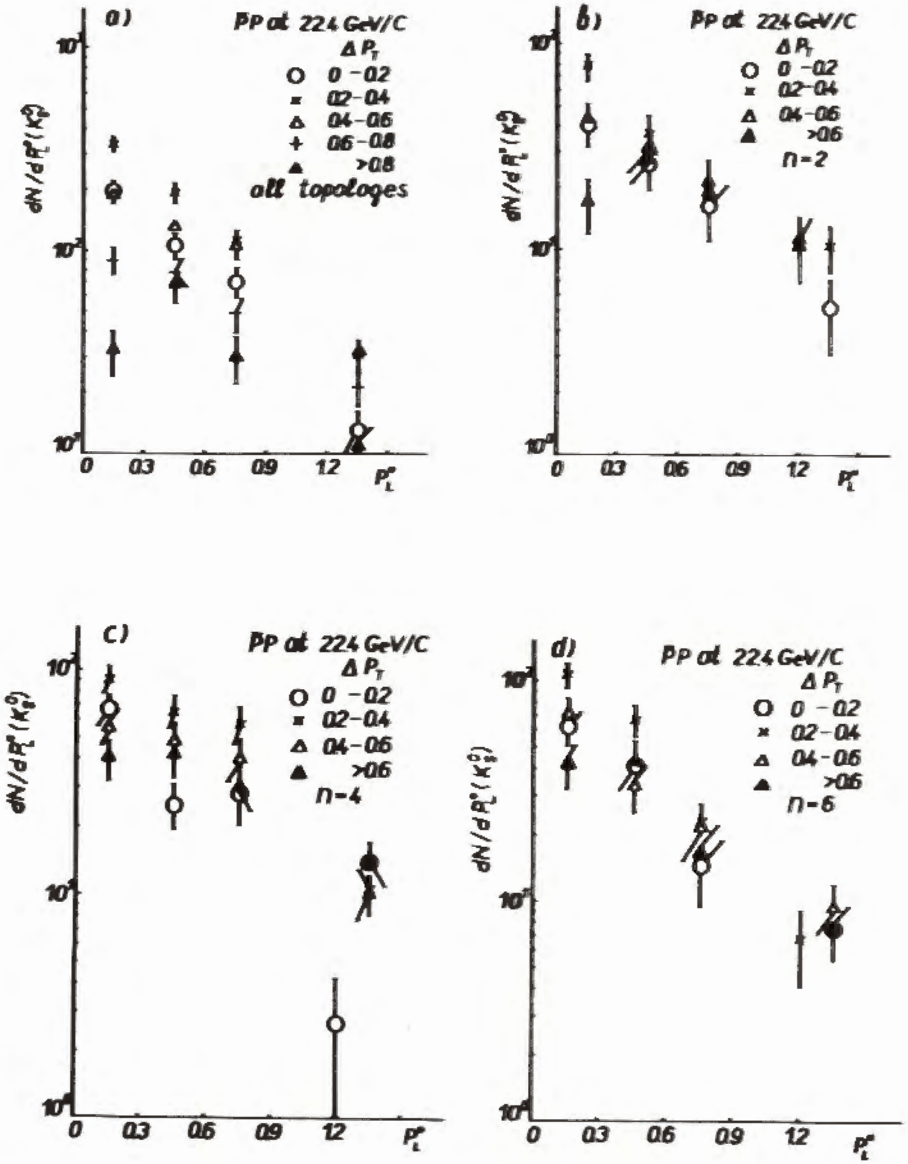

Fig.6. $p^*_L$ -distribution of $K^\circ_s$ for different intervals of $p_T$.

where it has been obtained that at asymptotic energies the validity of KNO-scaling and scaling in the mean leads to the violation of Feynman scaling. This violation in the central region (at $x = 0$) may correspond (with certain additional assumptions) to the rise of the cross section of meson production, which is also experimentally observed /9/ in pp-interactions up to ISR energies.

## 2. MULTIPLICITIES ASSOCIATED WITH STRANGE PARTICLES IN $\bar{p}p \to K^o_s + X$ AND $\bar{p}p \to \Lambda + X$ REACTIONS AT 22.4 GEV/C

Associative multiplicities in inclusive processes have been studied as a function of missing mass $M_x$ for this and other experiments /10/. The following logarithmic dependence has been predicted /11/.

$$n(M_x^2) = A + B \ln (M_x^2) \qquad (5)$$

in the framework of the multiperipheral model for particle production in the fragmentation region. In eq.(5) B is the universal slope parameter. The prediction has been made assuming that a main contribution to these processes comes from the diagrams where system X is produced in the virtual interaction of some effective trajectory with one of the primary particles. The $M_x^2$ value is analogous to the c.m.s. energy squared for real processes.

In Fig.7 the average charged multiplicities $n(M_x^2)$ are presented as a function of $M_x$ (to logarithmic scale) for inclusive production of $K_s^o$ and $\Lambda$. The fit to expression (5) (solid line in Fig.7) yields for the slope parameter of $\Lambda$'s $B = 1.26 \pm 0.14$ ($\chi^2/N_F = 1.5/6$). For $K_s^o$-mesons the fit is not satisfactory ($\chi^2/N_F = 27/7$). For comparison with other results we have also made the same fit for $\Lambda$'s with $\cos\theta < -0.5$* (~ 85% of the sample) where $\theta$ is the polar angle in the c.m.s. We got $B = 1.31 \pm 0.11$, which is almost the same as without the cut. Both values of the parameter B for $\Lambda$-particles agree rather well with the values obtained in other experiments /10/.

Analogously to real processes, one can assume that for system X produced with $\Lambda$-particles the Wroblewski /12/ and KNO /13/ relations have been applied in the following way:

$$D(M_x^2) = a + b < n(M_x^2) > \qquad (6)$$

and

$$\frac{<n(M_x^2)>}{\sigma(M_x^2)} \sigma_n(M_x^2) = \Psi\left(\frac{n(M_x^2)}{<n(M_x^2)>}\right), \qquad (7)$$

where $D(M_x^2)$ is the square root of the dispersion of the associative multiplicity distribution, $\sigma(M_x^2)$ and $\sigma_n(M_x^2)$ are the total and topological cross sections for definite values of $M_x^2$.

The experimental dependence of $D(M_x^2)$ on $<n(M_x^2)>$ is shown in Fig.8 for different $M_x^2$ intervals. After fitting the experimental data by the function (6), we have obtained the parameters $a = 0.28 \pm 0.3$ and $b = 0.40 \pm 0.09$ (straight line in fig.8). In Fig.9 is shown the KNO dependence (7) for three intervals of $M_x^2$. One can see that the KNO distributions for all $M_x^2$ intervals are similar.

An analytical expression for the associative KNO scaling has been obtained in the framework of renormalizable quantum field teeory using the renormalization group method /15/. For system X with mass $M_x$ we get

$$\Psi(Z(M_x^2)) = C \frac{\alpha^\alpha}{\Gamma(\alpha)} Z^{\alpha-1}(M_x^2) e^{-\alpha Z(M_x^2)}, \qquad (8)$$

where $Z(M_x^2) = n(M_x^2)/<n(M_x^2)>$, C is a normalization para-

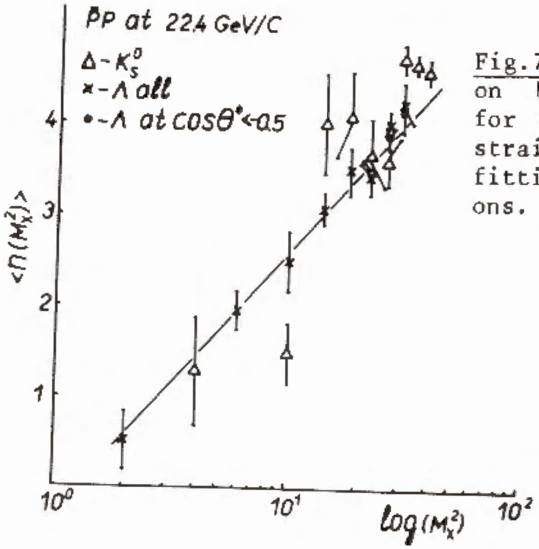

Fig.7. Dependence of $<n(M_x^2)>$ on $M_x^2$ (to logarithmic scale) for $K_s^o$ and $\Lambda$-particles. The straight line is the result of fitting the data for $\Lambda$-hyperons.

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* In Fig.7 these new data are denoted by circles if they do not coincide with those observed for all $\Lambda$'s (crosses in Fig.7).

meter and $\alpha = 1/b^2$ ( b is the same as in eq.(6)). The curve in Fig.9 is the result of fitting our data by expression (8) with the parameter $C = 1.86 \pm 0.11$ ($\chi^2/N_F = 4.2/10$). One can see that the data are quite well described by expression (8).

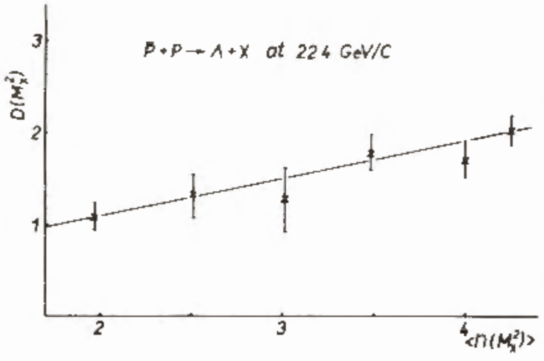

Fig.8. Dependence of $D(M_x^2)$ on $\langle n(M_x^2)\rangle$ for Λ's for the intervals $\Delta M_x^2 = $ (4-8, 8-12, 12-16, 16-24, 24-30, 30-32) $(GeV/c)^2$. The straight line is the fitting result.

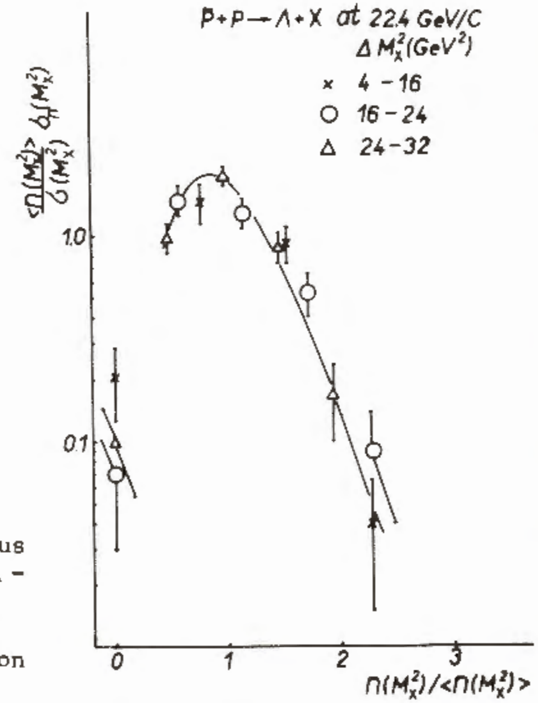

Fig.9. Plot of $(\sigma_n(M_x^2) \times \langle n(M_x^2)\rangle)/\sigma(M_x^2)$ versus $n(M_x^2)/\langle n(M_x^2)\rangle$ for Λ-particles. The curve is the result of fitting by the theoretical expression (8).

## 3. CONCLUSIONS

1. The scaling in the mean is shown to be valid in the variables $p_T/\langle p_T\rangle_n$, $p_L^*/\langle p_L^*\rangle_n$ for inclusive $K_s^0$-production. The scaled distributions coinside with the data obtained in pp- and $\pi^-$p-interactions.
2. For Λ particles the scaling in the mean in the variable $p_L^*/\langle p_L^*\rangle_n$ is violated which can be due to the leading particle behaviour.
3. The dependence of the mean associative multiplicities on $\ln(M_x^2)$ for Λ-production is linear, the slope agreeing rather well with the values obtained in other experiments.
4. The associative KNO-scaling for Λ's is observed in our experiment. The KNO distribution has the shape predicted by renormalizable quantum field theory.

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