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E1-80-243

EXPERIMENTS WITH BEAMS OF HIGH ENERGY NUCLEI AND THE PROBLEM OF RELATIVIZATION OF NUCLEAR WAVE FUNCTIONS

Dubna - Košice - Moscow - Strasbourg - Tbilisi - Warsaw Collaboration

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1. INTRODUCTION

In studies of interactions of high energy nuclei (see, e.g., reviews [1-5] and references therein) a problem arises to describe adequately moving relativistic composite systems. A direct way to solve this problem consists in solving the corresponding field theoretical equations (the Bethe-Salpeter equation, quasipotential equations [6-8]) for two- and many-body relativistic composite systems. Since such a program is unlikely to be solved at present, one can choose a heuristic way to find relativistic nuclear wave functions. In such an approach one has to try to guess such relativistic wave functions which reproduce rather well experimental regularities and possess the correct nonrelativistic limit, i.e., in the nonrelativistic limit they must turn into the well-known nonrelativistic nuclear wave functions.

For the deuteron such an attempt has been undertaken in Refs. [9-12]. Experience, accumulated when developing many-body relativistic dynamics [13-15] in terms of the "light front" variables [16], has been used. A relativistic analogue

$$\Phi(x,\vec{p}_\perp) \sim [(\vec{p}_\perp^{\,2}+m^2)/x(1-x) - \alpha_R]^{-1}[(\vec{p}_\perp^{\,2}+m_N^2)/x(1-x) - \beta_R]^{-1} \qquad (1.1)$$

of the Hülthen wave function has been obtained. Note that relativization of other, more refined, deuteron wave functions meets no principle difficulties. The wave function (1.1) is written in an arbitrary reference frame at any momentum of the deuteron as a whole and at any intrinsic momenta of its constituent nucleons. (Details of the corresponding definitions see in Refs. [9-11,15]). In the nucleus rest frame when the momenta $\vec{p}$ of its intrinsic motion obey the condition $|\vec{p}|/m_N \ll 1$, the wave function (1.1) turns to the well-known nonrelativistic Hülthen wave function

$$\Phi(\vec{p}) \sim (\vec{p}^{\,2} + \alpha_{NR}^2)^{-1} (\vec{p}^{\,2} + \beta_{NR}^2)^{-1} \qquad (1.2)$$

with the following relation between the parameters α_R, β_R and α_{NR}^2, β_{NR}^2:

$$\alpha_R = (m_\alpha / m_N)(2m_N^2 - \alpha_{NR}^2); \quad \beta_R = (m_d / m_N)(2m_N^2 - \beta_{NR}^2). \qquad (1.3)$$

The numerical values of the parameters α_R and β_R which have been obtained by fitting the experimental data [12], satisfy the conditions (1.3) with a good accuracy.

Some other attempts of deuteron wave function relativization can be found in Refs. [17-20].

An attempt to find relativistic analogues (in the spirit of Refs. [9-15]) of the wave functions of more complicated nuclei seems to be reasonable. Comparison of the corresponding results with experimental data allows one to answer the question whether this scale-invariant relativization is universal for all light nuclei or not. In the present paper such an attempt is undertaken for ^4He, ^3He and ^3H nuclei.

Recall for completeness some moments of the description of relativistic composite systems in terms of "light front" variables. The nucleus consisting of A particles with total four-momentum P_A is described by means of the relativistic wave function $\Phi_{P_A}^{(A)}(\{x_i^{(A)}; \vec{p}_{i,\perp}\})$ [21] in which the "longitudinal motion" of constituents is parametrized in terms of scale-invariant variables

$$x_i^{(A)} = (p_{i,0} + p_{i,3})/(P_{A,0} + P_{A,3}), \qquad (1.4)$$

where $p_{i,\mu}$ ($\mu = 0,1,2,3$, is the Lorentz index) and $P_{A,\mu}$ are the individual four-momentum of the i-th particle in the composite system and the total four-momentum of the composite system as a whole, respectively. Here and later on the parameters (momenta, masses, etc.) of composite systems are denoted by capital letters, whereas the individual parameters of nucleons are denoted by small letters. The brackets in the argument of the wave function $\Phi_{P_A}^{(A)}$ denote a set of corresponding variables $x_i^{(A)}$, $\vec{p}_{i,\perp}$ which obey the following conditions

$$\sum_{i=1}^{A} x_i^{(A)} = 1; \quad 0 < x_i^{(A)} < 1; \quad \sum_{i=1}^{A} \vec{p}_{i,\perp} = \vec{P}_{A,\perp}. \qquad (1.5)$$

The superscription of variables $x_i^{(A)}$ means that this variable is defined in the system of particles the number of which is equal to this index. The transformation properties of these functions, when proceeding from an arbitrary reference frame to the reference frame where total transverse momentum $\vec{P}_{A,\perp}$ of the composite system vanishes, and their normalization condition are known [14].

In the next section we shall see how one can get information on the wave functions $\Phi^{(A)}_{P_A}(\{x_i^{(A)}; \vec{P}_{i,\perp}\})$ from experimental observations of the interaction of relativistic nuclei.

2. STUDY OF SPECTATOR-FRAGMENT DISTRIBUTIONS AS A TOOL TO GET INFORMATION ON RELATIVISTIC WAVE FUNCTIONS

Let us consider the process of knocking out one nucleon from the relativistic nucleus A on the hydrogen target. Assuming that the remaining $((A-1)$ nucleons do not interact with the target and still exist in the form of fragment-nucleus, one can calculate the distribution of these fragments theoretically. In the laboratory frame (incoming nucleus moves along the z-axis, target proton is at rest) this distribution looks as follows

$$E^{SP}\frac{d\sigma}{d\vec{P}^{SP}} \sim \frac{\lambda^{1/2}(s_{NN}, m_N^2, m_N^2)}{\lambda^{1/2}(s, M_A^2, m_N^2)}\sigma^{el}_{NN}(s_{NN})\left|\frac{I(X^{SP}, \vec{P}_\perp^{SP})}{1-\alpha X^{SP}}\right|^2, \qquad (2.1)$$

where

$$\lambda(x,y,z) = (x-y-z)^2 - 4yz$$

$$\alpha = 1 + m_N/(E_A + P_{A,3}); \quad X^{SP} = (E^{SP} + P_3^{SP})/(m_N + E_A + P_{A,3}). \qquad (2.2)$$

Energy-momentum conservation leads to the following relation between the Mandelstam variables

$$s_{NN} = s(1-X^{SP}) + M_{SP}^2 - (\vec{P}_\perp^{SP\,2} + M_{SP}^2)/X^{SP}. \qquad (2.3)$$

In formulas (2.1)-(2.2) $P_{A,3}, E_A$ and P_3^{SP}, E^{SP} are the z-components of the momenta and energies of the incident nucleus A and of the spectator-fragment $(A-1)$, respectively, $\sigma^{el}_{NN}(s_{NN})$ is the total elastic cross section of the interaction of the active nucleon from nucleus A with target, M_A is the mass of the incident nucleus, M_{SP} is the mass of the spectator-fragment, m_N is the nucleon mass, $I(X^{SP}, \vec{P}_\perp^{SP})$ is the overlap integral of the relativistic wave functions of the incident nucleus and of the fragment one:

$$I(X^{SP}, \vec{P}_\perp^{SP}) = \int_0^1 \prod_{i=1}^{A-1}(dx_i^{(A-1)\prime}/x_i^{(A-1)\prime})\,\delta(1-\sum_{i=1}^{A-1}x^{(A-1)\prime}) \times$$

$$\times \int \prod_{i=1}^{A-1} d\vec{p}'_{i,\perp}\, \delta^{(2)}(\vec{P}_\perp^{SP} - \sum^{A-1} \vec{p}'_{i,\perp}) \times$$

$$\times\, \Phi_f^{+(A-1)}([x_i^{(A-1)\prime}; \vec{p}'_{i,\perp} - x_i^{(A-1)\prime} \vec{P}_\perp^{SP}])\, \Phi_i^{(A)}([x_i^{(A)}; \vec{p}_{i,\perp}]). \qquad (2.4)$$

The overlap integral is a direct analogue of the corresponding notion which appears in the nonrelativistic theory of nuclear reactions (see, e.g., [22] and references therein).

The arguments of the wave function $\Phi_i^{(A)}$ of the incident nucleus are related to the integration variables and observable quantities X^{SP} and $\vec{P}_\perp^{SP}$ in the following way:

$$x_i^{(A)} = \alpha X^{SP} x_i^{(A-1)\prime}; \quad \vec{p}_{i,\perp} = \vec{p}'_{i,\perp}; \quad i = 1,2,\dots,A-1.$$

$$x_A^{(A)} = 1 - \alpha X^{SP}; \qquad \vec{p}_{A,\perp} = -\vec{P}_\perp^{SP}. \qquad (2.5)$$

Thus, observation of the spectator-fragment allows information on the character of the "longitudinal" and transversal motions of nucleons in the incident nucleus to be obtained. In the case $A = 2$ the overlap integral in formula (2.1) is replaced by the deuteron relativistic wave function [9-12].

In formula (2.4) the wave functions $\Phi_i^{(A)}$ and $\Phi_f^{(A-1)}$ of the incoming and outgoing nuclei are defined in the reference frame where their total transverse momenta vanish. They are related to the wave functions with arbitrary total four-momentum P as follows:

$$\Phi_P^{(A)}([x_i^{(A)}; \vec{p}_{i,\perp}]) = \Phi_{P_\perp=0}^{(A)}([x_i^{(A)}; \vec{p}_{i,\perp} - x_i^{(A)} \vec{P}_\perp]). \qquad (2.6)$$

Note that the X^{SP} variable is defined in an arbitrary Lorentz frame, where nucleus A and proton collide along the z-axis as follows:

$$X^{SP} = (E^{SP} + P_3^{SP})/[(E_N + P_{N,3}) + (E_A + P_{A,3})]. \qquad (2.7)$$

Here P_3's and E's in (2.7) are the longitudinal momenta and the energies of corresponding particles. As is easily seen, X^{SP} is a Lorentz-invariant and scale-invariant variable. In the proton rest frame it turns to the form given by Eq. (2.2).

When comparing the theoretical results with the experimental data, we use the following simplest parametrization for the wave function

$$\Phi^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])=C_A \exp\{-a_A^R \sum_{i=1}^{A}[(\vec{p}_{i,\perp}^{\,2}+m_i^2)/x_i^{(A)}]\} \tag{2.8}$$

and similarly for $\Phi^{(A-1)}$. In formula (2.8) a_A^R is an adjustable parameter, C_A is a normalization factor. This parametrization seems to be meaningful for light nuclei ($A \leq 4$). If the scale-invariant parametrization of "longitudinal motion" in the wave functions $\Phi^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])$ is really valid, the parameters a_A^R should be the same at various energies of incoming nuclei.

Since we do not distinguish between protons and neutrons, the wave function $\Phi^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])$ is a symmetric function of its arguments $x_i^{(A)}, \vec{p}_{i,\perp}$. Solving the conditional extremum problem under the conditions (1.5), we obtain that the wave function (2.8) obeys a maximum at zero transverse momenta of constituent nucleons and at the parameters $x_i^{(A)}$ equal to

$$\tilde{x}_i^{(A)}=m_i/(\sum_{i=1}^{A} m_i)=1/A. \tag{2.9}$$

Taking into account the relation of the variables $x_i^{(A)}$ to X^{SP} (see formulas (2.5)), we obtain that in the X^{SP} distributions of the spectator fragments one should expect a maximum at

$$\tilde{X}^{SP}=(A-1)/A[1+m_N/(E_A+P_{A,3})]. \tag{2.10}$$

In the case of the incoming ^4He nuclei with $P_A = 8.56$ GeV/c, which are the subject of our analysis in the next section:

$$\tilde{X}^{SP} \simeq 0.713.$$

Note that such properties as scale invariance of wave functions and the maximum position in the X^{SP} distribution do not depend on the concrete form of (2.8) of the wave functions $\Phi^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])$ and remain valid at their arbitrary parametrization.

In order to normalize the relativistic wave functions correctly, one has to known in general the form of all the interactions inside the relativistic system [14]. Assuming, however, that the total quasi-potential does not depend on the total four-momentum of the composite system, one gets the following normalization condition:

$$\int_0^1 \prod_{i=1}^{A}(dx_i^{(A)}/x_i^{(A)})\,\delta(1-\sum_{i=1}^{A}x_i^{(A)})\int\prod_{i=1}^{A}d\vec{p}_{i,\perp}\,\delta^{(2)}(\vec{P}_{A,\perp}-\sum_{i=1}^{A}\vec{p}_{i,\perp})\times$$

$$\times|\Phi_{P_A}^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])|^2=1. \tag{2.11}$$

Inserting the wave function $\Phi^{(A)}([x_i^{(A)};\vec{p}_{i,\perp}])$ in the form of (2.8) into the normalization condition (2.11), we get the following approximate expression for the normalization factor:

$$C_A \simeq (2a_A^R/\pi)^{3(A-1)/4} (\sum_{i=1}^{A} m_i)^{[3(A-1)+1]/4} (\prod_{i=1}^{A} m_i)^{-1/4} \exp[a_A^R (\sum_{i=1}^{A} m_i)^2]. \quad (2.12)$$

(Details of the calculation see in Appendix A).

As was already mentioned in Introduction, one of the guiding points in the choice of relativistic wave functions is their correct nonrelativistic limit. The nonrelativistic wave function $\Phi_{NR}^{(A)}([\vec{p}_i])$, the relativistic analogue of which is given by Eq. (2.8) is of the Gaussian form

$$\Phi_{NR}^{(A)}([\vec{p}_i]) = (4a_A^{NR}/\pi)^{3(A-1)/4} \exp[-a_A^{NR} \sum_{i=1}^{A} \vec{p}_i^{\,2}] \quad (2.13)$$

and is normalized by the condition

$$\int \prod_{i=1}^{A} d\vec{p}_i \, \delta^{(3)}(\sum_{i=1}^{A} \vec{p}_i) \, |\Phi_{NR}^{(A)}([\vec{p}_i])|^2 = 1. \quad (2.14)$$

The condition of the correct nonrelativistic limit gives the following relation between the parameters of the relativistic and nonrelativistic wave functions

$$a_A^R = (2m_N/M_A)\, a_A^{NR}. \quad (2.15)$$

(Details of the nonrelativistic limit see in Appendix B).

Putting now the wave functions $\Phi^{(A)}$ and $\Phi^{(A-1)}$ into the overlap integral (2.4) and integrating over transverse momenta (the calculation is similar to that of the normalization factor and is given in Appendix A), one has

$$I(X^{SP}, \vec{P}_\perp^{SP}) = C_A C_{A-1} [\pi/(a_{A-1}^R + a_A^R/\alpha X^{SP})]^{A-2} \times$$

$$\times \exp[-a_A^2 m_N^2/(1-\alpha X^{SP})] \exp[-a_A^R \vec{P}_\perp^{SP\,2}/\alpha X^{SP}(1-\alpha X^{SP})] J(X^{SP}), \quad (2.16)$$

where

$$J(X^{SP}) = \int_0^1 \prod_{i=1}^{A-1} dx_i^{(A-1)} \delta(1 - \sum_{i=1}^{A-1} x_i^{(A-1)}) \times$$

$$\times \exp[-(a_{A-1}^R + a_A^R/\alpha X^{SP}) \sum_{i=1}^{A-1} (m_i^2/x_i^{(A-1)})]. \quad (2.17)$$

Thus the $P_{\perp}^{SP}$ distribution is obtained in the analytic form. The integral $J(X^{SP})$ is calculated approximately by means of the multi-dimensional saddle point method. Finally the overlap integral takes the form.

$$I(X^{SP}, \vec{P}_{\perp}^{SP}) \simeq C_A C_{A-1} \left(\prod_{i=1}^{A-1} m_i\right)^{1/2} \left(\sum_{i=1}^{A-1} m_i\right)^{-[3(A-2)+1]/2} \times$$
$$\times [\pi/(a^R_{A-1} + a^R_A/aX^{SP})]^{3(A-2)/2} \exp[-a^R_A m^2_N/(1-aX^{SP})] \times$$
$$\times \exp[-(a^R_{A-1} + a^R_A/aX^{SP}) (\sum_{i=1}^{A-1} m_i)^2] \times \qquad (2.18)$$
$$\times \exp[-a^R_A \vec{P}_{\perp}^{SP^2}/aX^{SP}(1 - aX^{SP})].$$

Formula (2.1) with the overlap integral in the form of (2.18) is ready now for comparison with experimental data.

3. COMPARISON WITH EXPERIMENT

The results obtained have been compared with experimental data on the spectator-fragment (^3H and ^3He) distributions in the reactions

$$^4He + p \to {}^3H + p + p, \qquad (3.1a)$$

$$^4He + p \to {}^3He + p + n. \qquad (3.1b)$$

Details of the experiment and the operational definition of spectators can be found in Ref. /23/ .

The experimental data have obtained from an analysis of two- and three- prong events in ^{4}Hep interactions at $P_{^4He}$ = = 8.56 GeV/c. The ^{4}He nuclei were accelerated at the Dubna synchrophasotron and then transported to the 100-cm hydrogen bubble chamber. Details of the experiment can be found in Ref. /23/.

After scanning, measurements of the selected events and their kinematic identification, the events corresponding to the reactions (3.1a) and (3.1b) were separated. The spectator nuclei ^3H and ^3He were defined as the lowest momentum products of corresponding reactions in the rest frame of ^4He. For comparison with the theoretical calculation 720 events of the reaction (3.1a) and 711 events of the reaction (3.1b) were used.

The total elastic cross section σ_{NN}^{el} in formula (2.1) is approximately constant and is equal to 24 mb in this energy range (see, e.g., ref. /24/).

The $d\sigma/dX^{SP}$ and $d\sigma/dP_\perp^{SP}$ experimental distributions have been analysed in the proton rest frame. They are related to the invariant differential cross section (2.1) as follows:

$$d\sigma/dX^{SP} = \int_0^{P_{\perp,max}^{SP}} \frac{d\sigma}{dX^{SP}dP_\perp^{SP}} dP_\perp^{SP}, \qquad (3.2a)$$

$$d\sigma/dP_\perp^{SP} = \int_{X_{min}^{SP}}^{X_{max}^{SP}} \frac{d\sigma}{dX^{SP}dP_\perp^{SP}} dX^{SP}, \qquad (3.2b)$$

$$d\sigma/dX^{SP}dP_\perp^{SP} = 2\pi(P_\perp^{SP}/X^{SP}) \frac{d\sigma}{d\vec{P}^{SP}/E^{SP}}. \qquad (3.2c)$$

The integration limits X_{min}^{SP} and X_{max}^{SP} are taken from the corresponding X^{SP} distributions. Their numerical values are: $X_{min}^{SP} = 0.635$; $X_{max}^{SP} = 0.805$.

The upper limit $P_{\perp,max}^{SP}$ is a kinematical bound obtained from the positivity of the $\lambda(s_{NN}, m_N^2, m_N^2)$ factor in formula (2.1)

Table

Reaction	$a_3^R, (GeV/c)^{-2}$	$a_4^R, (GeV/c)^{-2}$	$\frac{\chi^2}{N_p}(\frac{d\sigma}{dX^{SP}})$	$\frac{\chi^2}{N_p}(\frac{d\sigma}{dP_\perp^{SP}})$
$^4Hep \to {}^3Hpp$	16-fixed	7,415±0,253	29.47/18	27.52/22
$^4Hep \to {}^3Hepn$	16-fixed	5,894±0,211	54.46/18	43.68/22

It is of the form

$$P_{\perp,max}^{SP\,2} = (sX^{SP} - M_{SP}^2)(1 - X^{SP}) - 4m_N^2 X^{SP}. \qquad (3.3)$$

Results of a joint analysis of the X^{SP} and $P_{\perp}^{SP}$ distributions for the ^3H spectator in the reaction (3.1a) and results of an analysis of the same distributions for the ^3He spectator in the reaction (3.1b) are given in the table. When fitting the data, the parameters of the wave functions of ^3H and ^3He nuclei are determined with large error bars. This is the reason why we have fitted the data at a fixed value of the parameter $a_3^R = 16$ $(GeV/c)^{-2}$ which corresponds to $a_3^{NR} \approx 24$ $(GeV/c)^{-2}$ in the nonrelativistic parametrization. It should be noted that even the parameters of the nonrelativistic Gaussian wave functions of ^4He, ^3He and ^3H nuclei are not determined well enough. In the current literature their values vary in rather wide range (see, e.g., ref. /25/). The largest number of data is available on the parameters of the ^4He nucleus. However, numerical values of the parameter a_4^{NR} varies in a range of $a_4^{NR} = 21\text{-}26$ $(GeV/c)^{-2}$. As is seen from the table, the value of the parameter a_4^R, obtained in the fitting procedure, is somewhat smaller than the values of $a_4^R = 10\text{-}13$ $(GeV/c)^{-2}$ predicted by (2.15).

The experimental and theoretical $P_{\perp}^{SP}$ and X^{SP} distributions of the ^3H spectator in the reaction (3.1a) are given in Figs.1a and 1b. Figures 2a and 2b give the same distributions

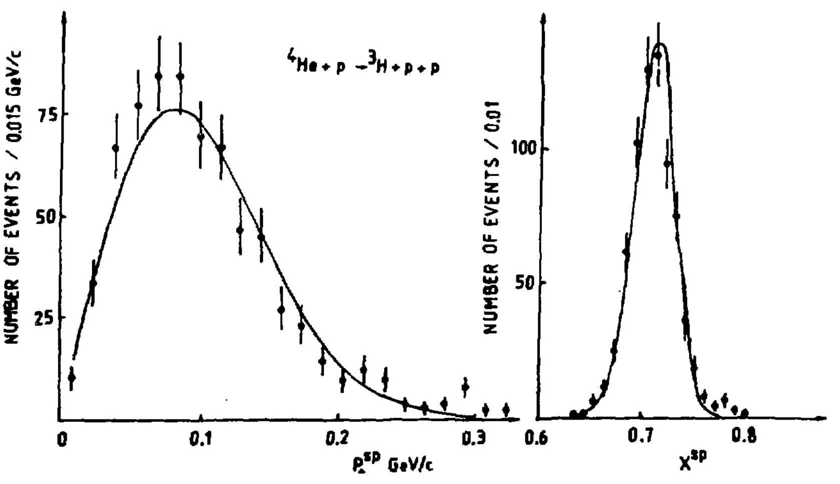

Fig.1. a) $P_{\perp}^{SP}$ distribution of spectator-fragment ^3H in the reaction $^4Hep \to {}^3Hpp$. b) X^{SP} distribution of spectator-fragment ^3H in the reaction $^4Hep \to {}^3Hpp$.

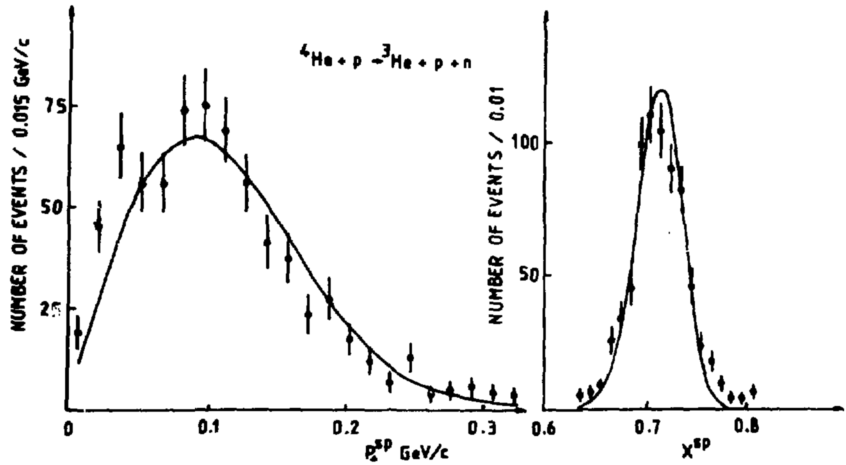

Fig.2. a) $P_\perp^{SP}$ distribution of spectator-fragment ^3He in the reaction $^4Hep \to {}^3Hepn$. b) X^{SP} distribution of spectator-fragment ^3He in the reaction $^4Hep \to {}^3Hepn$.

for the ^3He spectator in the reaction (3.2b). The theoretical curves correspond to the values of the parameters a_3^R and a_4^R presented in the Table. A small difference in the parameter a_4^R in the two considered cases is probably due to a small admixture of the misidentified ^4He from the elastic channel to the ^3He sample.

Note that the maximum position in the X^{SP} distributions coincides with the predicted value (2.1) to a good precision.

4. DISCUSSION

The analysis has been made of the spectator fragments in the knockout reactions with beams of high energy nuclei. Consideration is based on many-body quasi-potential dynamics in terms of "light front" variables. It is evident that the simplest parametrization of the relativistic nuclear wave functions which has been used in this paper gives a good qualitative description of corresponding experimental data. This fact is reflected by the values of χ^2/N_p (N_p is the number of the fitted experimental points) given in the Table. In order to achieve a better quantitative agreement with experiment, one could consider more refined forms of the corresponding wave functions. However, some regularities (such as scale invariance with respect to the $x_i^{(A)}$-variables) do not depend on the particular parametrization of the wave functions. The study of the spectator distributions in a wide range of energies of incident nuclei allows one to check the validity of the scale-invariant parametrization of the "longitudinal motion" in moving relativistic composite systems.

The method developed here can also be used to study other processes involving relativistic nuclei.

The authors express their deep gratitude to N.S.Amaglobely, A.M.Baldin, S.B.Gerasimov, T.I.Kopaleishvili, V.A.Matveev, R.M.Muradyan, Ya.A.Smorodinsky, A.N.Tavkhelidze for helpful discussion of the problems we have concerned here.

APPENDIX A

Inserting the relativistic wave function (2.8) in the reference frame with zero total transverse momentum (see transformation property (2.6)) into the normalization condition (2.11), one gets

$$|C_A|^2 \exp(2a_A^R \vec{P}_{A,\perp}^2)\, I_x * I_\perp = 1, \tag{A.1}$$

where

$$I_x = \int_0^1 \prod_{i=1}^{A} (dx_i^{(A)}/x_i^{(A)})\,\delta(1-\sum_{i=1}^{A} x_i^{(A)}) \exp[-2a_A^R \sum_{i=1}^{A} (m_i^2/x_i^{(A)})], \tag{A.2}$$

$$I_\perp = \int \prod_{i=1}^{A} d\vec{p}_{i,\perp}\, \delta^{(2)}(\vec{P}_{A,\perp} - \sum_{i=1}^{A} \vec{p}_{i,\perp}) \exp[-2a_A^R \sum_{i=1}^{A} (\vec{p}_{i,\perp}^{\,2}/x_i^{(A)})]. \tag{A.3}$$

Consider first the integral $I_\perp$. After putting the Fourier-transformation of the δ function into (A.3), $I_\perp$ takes the form of two-dimensional Gaussian integrals. Calculating them, one obtains

$$I_\perp = \{\pi^{A-1}/(\prod_{i=1}^{A} \alpha_i)[\sum_{i=1}^{A}(1/\alpha_i)]\}\exp[-\vec{P}_{A,\perp}^2/\sum_{i=1}^{A}(1/\alpha_i)]. \tag{A.4}$$

Here the following notation is introduced:

$$\alpha_i = 2a_A^R/x_i^{(A)} . \tag{A.5}$$

Inserting (A.5) into (A.1) and taking into account the conditions (1.5), one obtains the normalization condition in the following form

$$|C_A|^2 (\pi/2a_A^R)^{A-1} J_x = 1, \tag{A.6}$$

where

$$J_x = \int_0^1 \prod_{i=1}^{A} dx_i^{(A)} \delta(1-\sum_{i=1}^{A} x_i^{(A)}) \exp[-2a_A^R \sum_{i=1}^{A} (m_i^2/x_i^{(A)})]. \tag{A.7}$$

The integral J_x cannot be calculated exactly in the analytic form. This is the reason why we calculate it approximately using the multi-dimensional saddle point method (see, e.g., /26/ and references therein). Performing all necessary calculations, one has

$$J_x \simeq (\pi/2a_A^R)^{(A-1)/2} (\prod_{i=1}^{A} m_i)^{1/2} (\sum_{i=1}^{A} m_i)^{-[3(A-1)+1]/2} \exp[-2a_A^R (\sum_{i=1}^{A} m_i)^2]. \tag{A.8}$$

Putting (A.8) into (A.6), one obtains just the expression (2.12) for the normalization factor C_A.

The method of calculation of the overlap integral $I(X^{SP}, \vec{P}_\perp^{SP})$ is completely similar to the calculation in the normalization procedure. Therefore we do not give here details of this calculation.

APPENDIX B

Let us expand the combination $(\vec{p}_{i,\perp}^{\,2} + m_i^2)/x_i^{(A)}$ in power series in the parameters $p_{i,3}/m_i$, $p_{i,\perp}/m_i$, $P_{A,3}/M_A$, $P_{A,\perp}/M_A$ which are small in the nonrelativistic limit. Restricting ourselves to the quadratic terms and writting the result in the nucleus rest frame $\vec{P}_A = 0$, we get

$$(\vec{p}_{i,\perp}^{\,2} + m_i^2)/x_i^{(A)} \simeq (m_i - p_{i,3}) M_A + (M_A/2m_i)\vec{p}_i^{\,2}. \tag{B.1}$$

Then the wave function (2.8) takes the following form in the nonrelativistic limit (without taking into account the normalization factor)

$$\Phi^{(A)} \sim \exp\{-a_A^R \sum_{i=1}^{A} [(\vec{p}_{i,\perp}^{\,2} + m_i^2)/x_i^{(A)}]\} \to$$
$$\to \exp[-a_A^R M_A \sum_{i=1}^{A} m_i] \exp[-a_A^R M_A \sum_{i=1}^{A} (\vec{p}_i^{\,2}/2m_i)]. \tag{B.2}$$

From the condition that the limiting expression (B.2) should coincide with the nonrelativistic wave function (2.13), one gets the relation (2.15) between the parameters of the relativistic and nonrelativistic wave functions.

The normalized wave functions are related to each other as follows

$$\Phi_R^{(A)}(\{x_j^{(A)};\vec{p}_{i,\perp}\}) \to m_N^{(A-1)/2} A^{[3(A-1)+1]/4} (m_N/M_A)^{3(A-1)/4} \times$$

$$\times \exp\{2m_N^2 a_A^{NR} A[(m_N/M_A)A-1]\} \Phi_{NR}^{(A)}(\{\vec{p}_i\}). \qquad (B.3)$$

In (B.3) $\Phi_R^{(A)}$ is the wave function (2.8) normalized by (2.11), $\Phi_{NR}^{(A)}$ is the wave function (2.13) normalized by (2.14).

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Received by Publishing Department
on March 26 1980.