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# DEUTERON FORMFACTOR FROM EXPERIMENTAL DATA ON THE p - d ELASTIC SCATTERING AT SMALL ANGLES

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Ядерный формфактор дейтрона из экспериментальных данных по упругому $p-d$ рассеянию на малые углы

На основе теории Глаубера из экспериментальных данных о дифференциальных сечениях упругого рассеяния протонов на дейтронах, протонов на протонах и нейтронов на протонах в интервале энергий 10-26 Гэв определен структурный (ядерный) формфактор дейтрона.

Из числа рассмотренных волновых функций наилучшее согласие с экспериментом дает волновая функция Брессела и Кермана. Полученные данные согласуются с результатами экспериментов по $e-d$ рассеянию.

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Nikitin V.A., Shafranova M.G., Truong Bien, Zolin L.S. **E1 - 7207**

Deuteron Formfactor from Experimental Data on the $p-d$ Elastic Scattering at Small Angles

The nuclear deuteron formfactor has been determined, using the Glauber model, from the experimental data on elastic scattering differential cross-sections of protons on deuterons, protons on protons and neutrons on protons in the energy range of 10-26 GeV.

Among the wave functions considered the Bressel-Kerman wave function is in the best agreement with experiment.

The comparison of the $p-d$ and $e-d$ data shows that the distributions of nuclear matter and charge in the deuteron are equal within the errors.

**Preprint. Joint Institute for Nuclear Research.**
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## 1. $p-d$ Scattering and Nuclear Deuteron Formfactor

In order to determine the nuclear formfactor of the deuteron in the framework of the Glauber theory, we have analysed the experimental data on the $p-d$ elastic scattering at 11.2; 14.4; 15.9; 20.5 and 26.5 GeV/c /1/ and on the $p-p$ /2, 3/ and $n-p$ elastic scattering in the same energy region. The $p-d$ elastic scattering differential cross section at small momentum transfers $q$ can be expressed through the $p-p$ and $p-n$ (or $n-p$) elastic scattering amplitudes on free nucleons by the following formula /4,5/:

$$\left|\frac{d\sigma}{dt}\right|_{pd} = |S(\frac{t}{4})[f_c(t)+\exp(i\chi_{cp})f_{pp}(t)+\exp(i\chi_{cn})\times f_{np}(t)] + \frac{i\hbar}{\sqrt{\pi}}\exp(i\chi_{cpn})f_{np}(\frac{t}{4})\cdot f_{pp}(\frac{t}{4})\cdot IG|^2, \quad (1)$$

where $t$ is the four momentum transfer squared. At small $q$ $|t| = q^2$; $f_{pp}(t)$ and $f_{np}(t)$ are the $p-p$ and $n-p$ elastic scattering amplitudes; $S(q)$ is the nuclear deuteron form factor, where $S^2(q) = \int_0^\infty (u^2 + w^2) j\,(dr)\,dr$ ($u$ and $u$ are the deuteron wave functions in $S-$ and $D-$ states, respectively, $r$ is the distance between nucleons in the deuteron, $j_0$ is the Bessel function).

$$S^2(q/2) = S_0^2(q/2) + S_2^2(q/2).$$

$S_0(q)$ and $S_2(q)$ are the deuteron form factors in $S-$ and $D$-states, correspondingly.

The following parametrization was used for the $p-N$ elastic scattering amplitude:

$$f(t) = \frac{\sigma}{4\hbar\sqrt{\pi}} (a+i)\, e^{-\frac{1}{2}bt},$$

where $b$ is the slope parameter; $\sigma$ is the total cross section /6/. $a = \frac{Re\, f(0)}{Im\, f(0)}$ is the ratio of the real to the imaginary part of the elastic scattering amplitude.

The Coulomb scattering amplitude takes the form:

$$f_c = \frac{2n\hbar\sqrt{\pi}}{t} e^{-i\eta} \cdot F_p(t); \quad \eta = 2n\ln\frac{1.06\cdot\hbar}{R\sqrt{|t|}},$$

where $n = 1/137.04\,\beta_{lab}$; $\beta_{lab}$ is the incident particle velocity in the lab. system in $c$ units; $F_p(t)$ is the electromagnetic proton formfactor; $R$ is the $N-N$ interaction radius (1 fermi).

$\chi_{cp}$, $\chi_{cn}$, $\chi_{cpn}$ are the averaged values of the Coulômb phase shifts for the scattering on proton, neutron and for the double ($p$ and $n$) scattering.

According to Franco /5/, $\chi_{cp}$, $\chi_{cn}$, $\chi_{cpn}$ weakly depend on energy and are equal approximately to 0.06.

The Glauber integral IG is decsribed as:

$$IG = \int_0^\infty S_0(q)\, \frac{Im\, f_{pp}(q)\cdot Im\, f_{np}(q)}{Im\, f_{pp}(0)\cdot Im\, f_{np}(0)}\, q\, dq$$

(if the $D$ -state contribution is neglected). The Glauber integral was calculated for different deuteron wave functions and different slope parameters $b$. It turned out that the IG was weakly dependent on the parameter of the $p-p$ and $p-n$ elastic scattering amplitude and on the version of the deuteron wave function. Its value varies between 0.026 and 0.028 $mb^{-1}$ which agrees with the results obtained by measuring the total $p-d$, $p-p$ and $n-p$ cross sections and the total $\pi-d$ and $\pi-p$ cross sections.

In order to determine the deuteron form factor, we have taken only the experimental data obtained at 10-26 GeV because in this energy range the slope parameter of the $n-p$ elastic scattering is known and within the errors coincides with that of the $p-p$ elastic scattering. As an illustration of this fact, the data on the $p-p$ and $n-p$ elastic scattering slope parameter at $|t|$ = 0.6 - - 0.7 $(GeV/c)^2$ /7-13/ are shown. The total cross sections and, consequently, the optical points at these energies for $n-p$ and $p-p$ interactions are equal within the errors /6,10,13,14/ . To determine the deuteron form factor, the slope parameter of the $n-p$ elastic scattering was taken to be equal to that of the $p-p$ elastic scattering. The ratio of the real to the imaginary part of the $n-p$ elastic scattering amplitude was taken to be equal to the value in the $p-p$ elastic scattering.

Fig. 2 presents the values of $S^2(t/4)$ obtained from the analysis of the $p-d$ elastic scattering data at 10 - 26 GeV for $|t| < 0.2\ (GeV/c)^2$. All these points satisfactorily lie on the curve that can be presented by the following empirical formula:

$$S^2(t/4) = exp(\beta t + \gamma t^2),$$

$$\beta = 25.9 \pm 1.2\,(Gev/c)^{-2},$$

$$\gamma = 60 \pm 5\,(Gev/c)^{-4}.$$

Only the statistical error is shown in the points. The corridor of errors takes into account statistical and systematical errors in the $p-d$ elastic scattering differential cross section and also errors in the slope parameters of the $p-p$ and $n-p$ elastic scattering.

As is seen from the figure, the form factor obtained in this paper satisfactorily agrees with the presented theoretical curves in the region of the largest $t$. In the region of small $t$ an inessential disagreement is observed using the Humbertson wave function, the best agreement in all the interval using the Bressel-Kerman function.

## 2. $p-d$ and $e-d$ Elastic Scattering at Small Angles

Let us compare the obtained data with the results of $e-d$ elastic scattering experiments. Assuming in an impulse approximation that the electron is a structureless particle the $e-d$ elastic scattering differential cross section can be expressed in terms of the following formula:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[A(q^2) + B(q^2)\cdot tg^2\frac{\theta}{2}\right]$$

( $\theta$ is the scattering angle in the lab. coordinate system) or in the form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{mott} \cdot \{G_0^2(q^2) + G_2^2(q^2) + G_{mag}^2(q^2)\times$$

$$\times[1 + 2(1+\eta_d)\,tg^2\frac{\theta}{2}]\} \qquad (2)$$

$G_0(q^2), G_2(q^2)$ and $G_{mag}(q^2)$ are the electrical dipole, electrical quadrupole and magnet dipole form factors of the deuteron; $(d\sigma/d\Omega)_{Mott}$ is the Mott scattering cross section, $\eta_d = \frac{q^2}{4M_d^2}$. $M_d$ is the deuteron mass.

In the small angle scattering region the electrical form factor plays a main role (see fig. 3). On the other hand, as the deuteron is a system with law bound energy, the electromagnetic form factor can be written as /20/:

$$G_0(q^2) = (G_{Ep} + G_{EN})\cdot(1+\eta_d)^{-1}\cdot\int_0^\infty (u^2+w^2)\,j_0\,\frac{(qr)}{2}\,dr, \qquad (3)$$

$G_{EP}$, $G_{EN}$ are the proton and neutron electrical form factors, respectively.

From the $e-d$ elastic scattering experimental data obtained in the region of small $q$, it is possible to determine $G_0$. Taking the electrical proton form factor from the $e-p$ elastic scattering experiments, one can determine the electrical neutron form factor by formula (3) using different theoretical deuteron wave functions.

The nuclear deuteron form factor $S(q)$ can be obtained from the $p-d$ elastic scattering experimental data using the Glauber model. This has been done in the lst part.

Let us analyse if the $e-d$ scattering agrees with the $p-d$ data. Both the $e-d$ and $e-n$ experimental data show that the electrical neutron form factor is near zero.

Figure 4 presents the known $e-d$ data and the curve with the corridor of errors calculated by the formula:

$$A(q^2) = S^2(q/2)\,G_{EP}^2(q),$$

where $S^2(q/2)$ is the empirical value obtained in this experiment $(S^2(t/4) = exp(\beta t + \gamma t^2)$ $\beta = 25.9 \pm \pm 1.2\ (GeV/c)^{-2}$, $\gamma = 60 \pm 5\ (GeV/c)^{-4}$ ), $G_{EP}$ is the electrical proton form factor determined by the dipole formula $G_{EP} = 1/(1+q^2/a)^2$ ( $a = 0.71\ (GeV/c)^2$ ). The electrical neutron form factor was taken to be equal to zero. 223 experimental points lie in the shaded region.

One can see from the figure that the data on the $e-d$ and $p-d$ elastic scattering are in good agreement. This fact indicates that the distributions of nuclear matter and of charge in the deuteron are equal within the errors.

The experimental data on the nuclear deuteron form factor as well as the theoretical ones can be used to analyse the $e-d$ elastic scattering experimental data for determining the electrical neutron form factor at small $q$. They can be used also for extraction of the $p-n$ data and for studying the interactions of particles with deuterons.

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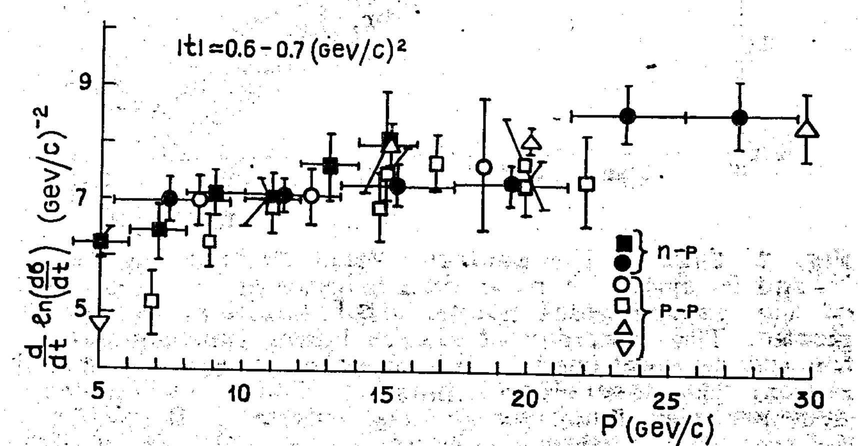


Fig. 1. Slope parameter of the $n-p$ and $p-p$ elastic scattering at $|t|=0.6 - 0.7$ (GeV/c)$^2$:

■ /8/, ● /7/ } $n-p$

○ /9/, □ /10/, Δ /11/, ∇ /12/ } $p-p$

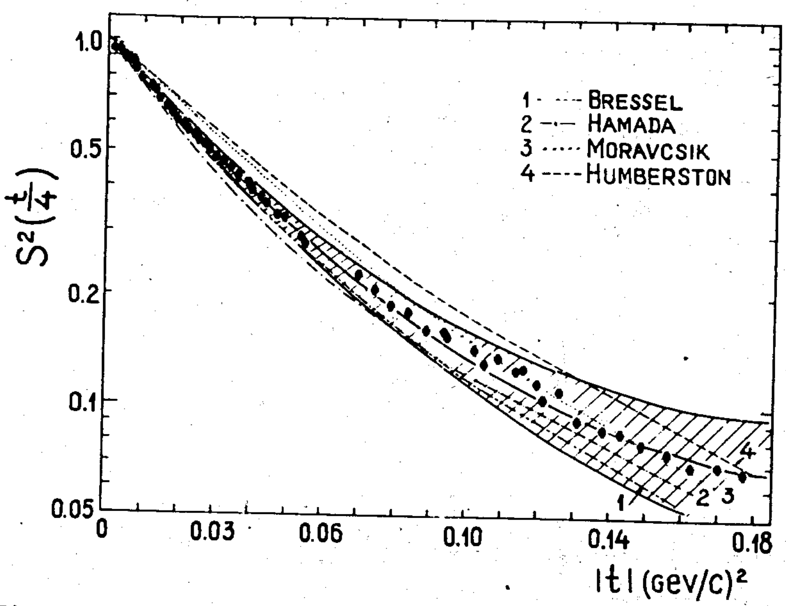


Fig. 2. Sum of the deuteron form factors squared in $S$- and $D$- states. $S\ (t/4)$ as a function of $t$. One part of the experimental points with statistical errors is plotted. The corridor of errors taking into account all the errors mentioned in the paper is shown as a shaded region. The theoretical values of $S^2(t/4)$ for different deuteron wave functions are also shown: 1. Bressel and Kerman /15/ 2. Hamada-Johnston /16/ 3. Moravcsik-3/17/ 4. Humberston /18/



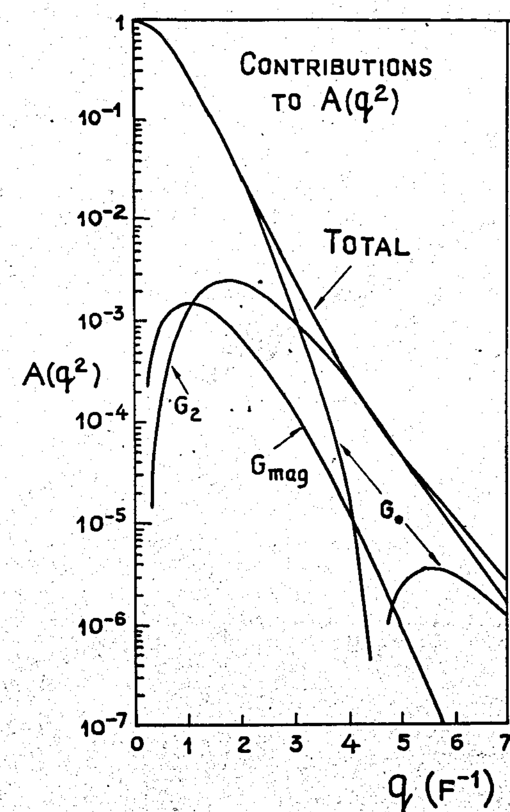


Fig. 3. Electromagnetic deuteron form factor squared $A(q^2)$. The contribution of electrical dipole, electrical quadrupole and magnetic dipole form factors of the deuteron to $A(q^2)$ is shown according to /19/.

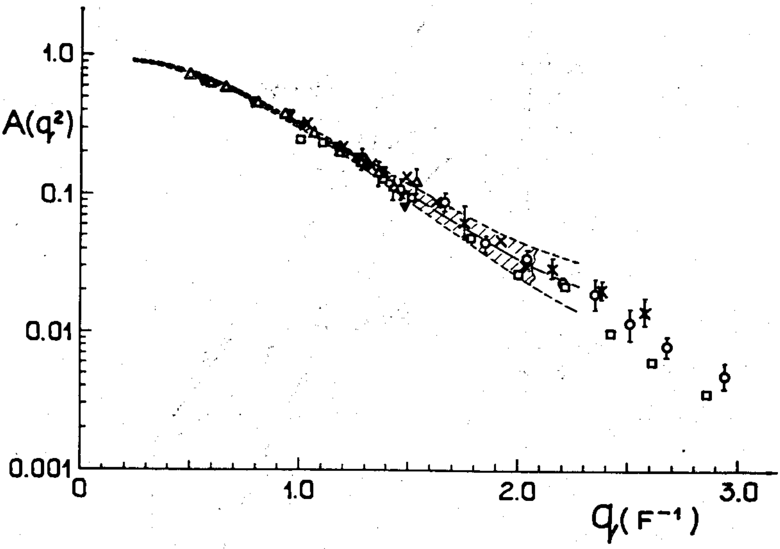


Fig. 4. Experimental value of the deuteron electromagnetic form factor squared $A(q^2)$: 1) from the $e-d$ data;

× - 400 MeV
o - 500 MeV /21/
Λ - 188 MeV
□ - 180 - 500 MeV /22/
∇ - 100 - 250 MeV /23/

2) from the $p-d$ data in the energy range 10 - 26 GeV 223 experimental points lying in the shaded region (this paper) have been obtained. The shaded area shows the corridor of errors. The main part of them is a systematic error.