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$\pi\pi$-SCATTERING LENGTHS a_0 AND a_2 FROM THE ANALYSIS OF τ-DECAYS

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The analysis of data given by Ford et al./1/ is presented whithin the framework of the theory of three particle production near the threshold/2/. The square of the τ-decay matrix element in this theory with the accuracy to the squared terms by pion relative momenta is given by the expression/2-4/(in the assumption that **rule** $\Delta T = 1/2$ **fulfils** in the τ-decay)

$$|M|^2 = 1 + \frac{1}{18}(a_2 + 5a_0)^2 (K_{13}K_{23} + \frac{1}{2}K_{12}^2 - \frac{3}{2}K_0^2) + \frac{1}{3}a_2(a_2 + 5a_0)[K_{12}(K_{13} + K_{23}) + \\ + 2I(K_{12}) - \frac{1}{2}K_{12}^2 - \frac{3}{2}K_0^2] + \frac{1}{9}(a_0^2 + 25a_2a_0 + 11a_0^2)[I(K_{13}) + I(K_{23})] - C(K_{12}^2 - K_0^2), \quad (1)$$

where K_{12}, K_{13}, K_{23} are the pion relative momenta (index 3 refers to the "odd" pion in the τ- decay); K_0^2 is the pion relative momenta squared in the centre of the Dalitz plot (here and in the following $\hbar = c = m_\pi = 1$); I(K) are the functions of the relative momenta which contain the singularities; a_0 and a_2 are S-wave $\pi\pi$ scattering lengths in the isotope states, T=0 and T=2, respectively; C is the slope parameter in the distribution by K_{12} ($a \approx 0.317C$, where a is the slope parameter in the distribution by the Dalitz-Fabri Y variable). Expression (1) is written in such a form that $|M|^2$ equals the unity and his derivative equals (-C) in the centre of the Dalitz plot.

The distribution of the probability density on the Dalitz plot was fitted by the least squares method (by the a_0 , a_2 , C parameters). The theoretical values of the probability density $\rho_i^t = \frac{1}{\Delta S_i}\int |M|^2 dS_i$ (where ΔS_i is the area of the i-th bin on the

Dalitz plot, the number of bins being 153) were calculated by the Monte-Carlo method using the random number generators on the BESM-6 and CDC-1604A computers. A total of $\sim 6\cdot 10^6$ points have been generated. The accuracy of the integral calculation has been verified. Beginning with $> 2\cdot 10^6$ of the generated points the integral values and fitting results did not change. The minimized functional is:

$$F = \sum_{i=1}^{N} \left[\frac{\rho_i^e - \rho_i^t(a_0, a_1, C)}{\Delta\rho_i^e} \right]^2 , \qquad (2)$$

where N is the number of bins on the Dalitz plot; ρ_i^e and $\Delta\rho_i^e$ (i =1,..., N) are the experimental values of the probability density and their errors in the i-th bin, respectively. Ford et al. give only the statistical errors which are on the average 1-2%. In this case the systematical errors could be of importance. The minimization of expression (2) has been made by the squared functional linearization method[5] with the help of the standard subroutine FUMILI.

The fitted results are listed in Table I. The low confidence level (0.2%) is probably due to the fact that the statistical errors were taken into consideration only. We have tried to reduce the influence of the systematical error contribution by excluding 8 points from the different regions of the Dalitz plot. These points fall out from the regular course of the probability density and give the unusual large (by 6-10 units) contribution into χ^2. After fitting 145 bins (instead of 153) we obtain χ^2 =142 with 142 freedom degrees. This denotes the squared approximation to be in a good agreement with the experiment. In this case the parameter values change slightly.

It is belived that the inclusion of systematical errors will not lead to the considerable change of the parameters. However, its errors are slightly increased.

It should be noted that the deviation from the linearity of the odd meson energy spectrum in the case of the τ-decay has been first found by Ford et al. In our paper this experimental fact is explained by the contribution of terms which contain the $\pi\pi$ scattering lengths, that is by $\pi\pi$ scattering effects (Fig. 1). The fit with the fixed values of $a_0 = a_2 = 0$ gives $\chi^2/\bar{\chi}^2$ =1.37 (the confidence level of $\sim$ 0.1%). Note, that Ford et al., who have taken into account only analytical terms in the square of matrix element, i.e. have used its incomplete expression, described satisfactorily the distribution of the probability density on the Dalitz plot by including in $|M|^2$ the fourth order terms in pions relative momenta.

Since the theory is semiphenomenological, an important problem arises which terms of the expansion in the relative momenta can be neglected (that must be decided from the comparison with the experiment). Besides, it should be noted that the inclusion of the terms, of the following, third order gives a principal possibility to determine the sings of the scattering lengths, because in this case the terms of the third degree by a_0 and a_2 appear in the expression for the square of matrix element /2-4/. We have analysed the data by Ford et al. taking into account the cubic terms. The results of the fit are given in Table I.

Table I.

	Account of quadratic terms		account of cubic terms: fit from 145 bins of the Dalitz plot	
	Fit from 153 bins of Dalitz plot	Fit from 145 bins of Dalitz plot	1 solution	2 solution
a_0	$\pm(0.58\pm0.04)$	$\pm(0.57\pm0.04)$	$-\ 0.48\pm0.03$	0.67 ± 0.04
a_2	$\mp(0.16\pm0.01)$	$\mp(0.16\pm0.01)$	0.14 ± 0.009	-0.18 ± 0.01
c	0.8640 ± 0.0084	0.8670 ± 0.0086	0.8662 ± 0.0086	0.8617 ± 0.0096
(a)	(0.2739 ± 0.0027)	(0.2748 ± 0.0027)	(0.2746 ± 0.0027)	(0.2732 ± 0.0030)
χ^2	201	142	141	157.5
confidence level	0.2%	50%	50%	18%

As is seen from Table I the adding of the cubic terms did not improve χ^2. We have received two solutions with different signs for the scattering lengths. The values of scattering lengths are not different (with the accuracy of ~ 15-20%) from those corresponding to the fit using the quadratic terms. Our analysis has shown that the cubic terms give a small contribution compared with quadratic terms.

The obtained values of the scattering lengths do not agree with the values of $a_0=0.2$ and $a_2=-0.06$ predicted by Weinberg /6/ on the basis of the current algebra. It seems, that the effect of the higher, fourth order, terms would be small and its account in the square of the τ -decay matrix element would not change essentially the results, presented in Table I, as the lower terms already describe well the experimental data. But the direct answer to this question can be obtained after the analysis of experimental data considering the fourth order terms. This analysis should be carried out after accounting all systematical errors.

Note that the value $a_0 = (0.6 \pm 0.25) \lambda_\pi$ obtained from the analysis of 1609 K_{e4} -decays [7] is in a good agreement with the value $a_0 = (0.57 \pm 0.04) \lambda_\pi$ from our paper.

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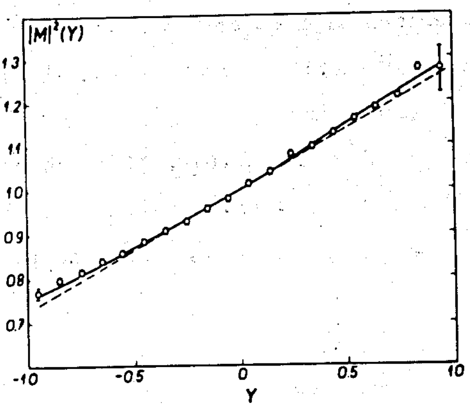

Fig. 1. The square of the matrix element distribution by the Dalitz-Fabri variable Y. The points are the experimental distribution from Ford et al. /1/. The statistical errors are given only. Dotted line is the fit with $|M|^2 = 1 + aY$. Solid line is the fit by formula (1).