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e - d SCATTERING AND NEUTRON FORM FACTORS

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In this work the statistical analysis of all available data on e-d elastic cross section

$$e + d \to e + d$$

was carried out for the purpose to estimate the neutron electromagnetic form factors.

We used the method of data analysis described in detail in [1]. This method permits the form factors to be obtained directly from the differential cross section data. In this case a certain functional dependence of the form factor is assumed, and the appropriate parameters are found by minimizing the functional χ^2

$$\chi^2 = \sum_k \sum_i \frac{1}{(\Delta_i^k)^2} \left(\frac{d\sigma^k}{d\Omega_i} - N_k \frac{d\sigma^k}{d\Omega_i}(th) \right)^2 \qquad (1)$$

where $\frac{d\sigma^k}{d\Omega_i}$ and Δ_i^k are the differential cross sections and the corresponding errors for the i-th point of the k-th experiment, $\frac{d\sigma^k}{d\Omega_i}(th)$ is the computed differential cross section of e-d scattering. When analysing experimental data of different groups the normalization factors N_k are introduced as variating parameters which take into account the possible normalization errors.

We present here the results of the analysis of experimental data obtained in [2-9].

As is known [10], the cross section for elastic scattering in the one photon approximation has the following form:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \cdot \left[A(q^2) + B(q^2) \cdot tg^2 \theta/2 \right] \qquad (2)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{\alpha}{2E_0}\right)^2 \frac{\cos^2\theta/2}{\sin^4\theta/2} \cdot \frac{1}{1 + \frac{2E_0}{M}\sin^2\theta/2} \qquad (3)$$

$$A(q^2) = G_C^2 + \frac{8}{9}\eta^2 G_Q^2 + \frac{2}{3}\eta G_M^2 \qquad (4)$$

$$B(q^2) = \frac{4}{3}\eta(1+\eta) G_M^2, \quad \eta = \frac{q^2}{4M_d^2}$$

In the impulse approximation the charge form factor G_c, the magnetic one G_m, and the quadrupole form factor G_Q are bound to the nucleon electromagnetic form factors by the following relations:

$$G_C = (G_{EP} + G_{EN}) \cdot C_E$$

$$G_Q = (G_{EP} + G_{EN}) \cdot C_Q \qquad (5)$$

$$G_M = \frac{M_d}{M_P}\left[(G_{MN} + G_{MP}) \cdot C_S + \frac{1}{2}(G_{EP} + G_{EN}) \cdot C_L \right],$$

where M_D and M_p are the deuteron and proton masses

$$
\begin{aligned}
C_E &= \int \{u^2(r) + w^2(r)\} j_0\left(\frac{qr}{2}\right) dr \\
C_Q &= \frac{3}{\sqrt{2}\,\eta} \int W(r)\left\{u(r) - \frac{w(r)}{\sqrt{8}}\right\} j_2\left(\frac{qr}{2}\right) dr \\
C_L &= \frac{3}{2} \int w^2(r)\left\{ j_0\left(\frac{qr}{2}\right) + j_2\left(\frac{qr}{2}\right)\right\} dr \\
C_S &= \int \left\{u^2(r) - \frac{w^2(r)}{2}\right\} j_0\left(\frac{qr}{2}\right) dr + \\
&+ \frac{1}{\sqrt{2}} \int W(r)\left\{U(r) + \frac{W(r)}{\sqrt{2}}\right\} j_2\left(\frac{qr}{2}\right) dr
\end{aligned}
\qquad (6)
$$

In the relations (6) $U(r)$ and $W(r)$ are the deuteron wave functions (for S and D states); $j_0\left(\frac{qr}{2}\right)$ and $j_2\left(\frac{qr}{2}\right)$ are the Bessel spherical functions.

In our calculations we have used the wave functions from [II], the parameters of which were obtained by fitting the values of the numerical wave functions [I2] determined with the Hamada-Johnston potential. It should be noted, as was shown in [I3], this potential allows one to get a good agreement between the values of $G_{EN}(0) = \frac{dG_{EN}}{dq^2}\Big|_{q^2=0}$ extracted from n-e scattering data [I4] and e-d elastic scattering calculations in a range of small square transferred momenta

$$
\begin{aligned}
U(r) &= N\left(e^{-\alpha r} + \sum_{j=1}^{n} c_j e^{-\epsilon_j r}\right) \\
W(r) &= \rho N\left(\alpha r h_2(i\alpha r) + \sum_{j=1}^{n} c_j' \epsilon_j' r h_2(i\epsilon_j' r)\right)
\end{aligned}
\qquad (7)
$$

where

$$
\begin{aligned}
x h_2(ix) &= e^{-x}\left[1 + 3/x + \frac{3}{x^2}\right] \\
\alpha &= 0.2338\,F^{-1},\ N = 0.8896\,F^{-1/2},\ \rho = 0.0269
\end{aligned}
\qquad (8)
$$

To avoid wave function singularity at r=0 the following conditions are necessary */

$$
\sum_{j=1}^{5} c_j' = -1\ ;\quad \sum_{j=1}^{5} c_j' / \epsilon_j'^2 = -1\ ,\quad \sum_{j=1}^{4} c_j = -1 \qquad (9)
$$

More precise parameter values satisfying to the conditions (9) are given in Table I.

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*/ The parameters given in [II] satisfy only approximately to these conditions.

T A B L E I

	ϵ_γ	c_γ	ϵ'_γ	c'_γ
1.	5.733 α	- 0.63608	4.833 α	-20.34
2.	12.844 α	- 6.615	10.447 α	-36.60
3.	17.331 α	15.2162	14.506 α	-123.02
4.	19.643 α	-8.9651	16.865354 α	305.12
5.			21.154 α	-126.16

It was denoted in [15] that the statistically satisfactory fit to all the available data on elastic scattering is possible if the following expressions are assumed for the proton form factors:

$$G_{EP} = \frac{A}{1+Bq^2} + \frac{1-A}{1+Dq^2} \qquad (10)$$

$$G_{MP} = \frac{G_{EP}\cdot\mu_p}{1+Eq^2} \qquad (11)$$

where $A=.45$; $B=0.67(\mathrm{GeV}/c)^2$; $D=2.23(\mathrm{GeV}/c)^2$; $E=1.10^{-2}(\mathrm{GeV}/c)^2$

The expressions (10),(11) for the proton form factor were used when analysing data on e-d elastic scattering

For the neutron form factors the following dependences on q^2 are assumed similar to those considered in [13] for the proton form factors*/

$$G_{MN} = \mu_N G_{EP}/(1-aq^2)$$

$$G_{EN} = -b\tau\mu_N G_{EP}/(1+cq^2) \qquad (12)$$

where $\tau = \frac{q^2}{4M_p^2}$,

μ_N is the neutron magnetic moment in nuclear magnetons.

The results of experimental data analysis [2-9] are in the 3-d column of Table II. As is seen from the Table, the fit is satisfactory (C.L.=2.6%) but the parameter a is defined with the error which is more than the parameter value. In the 4-th column of the Table the data analysis results are given for the case when

*/ The analogous parametrization for G_{EN} was assumed in [7] when analysing e-d scattering data in a range of transferred momenta $5F^{-2} - 14F^{-2}$.

a=b=c=0 (the neutron electric form factor equals zero, the magnetic form factor is given by scaling law). As is seen from the Table, the fit to all the data is also satisfactory. In the 5-th column we present e-d data analysis results for the case a=c=0. (The neutron magnetic form factor is given by a precise scaling law, and $G_{EN}=-\tau \cdot G_{MN}=-\mu\tau G_{EP}$, i.e. the Pauli form factor $F_{IN}=0$ [16]) In the 6-th column we have $a = 0, b \neq 0, c \neq 0$. Finally, in the 7-th column the data analysis results are given for the case when the normalization factors are assumed to be equal to unity. The fitting of the experimental data was significantly deteriorated.

As a result of the analysis of all the available data on e-d elastic scattering we can make the following conclusions:

I. It is impossible to obtain the satisfactory fit to all the data without introducing the normalization factors.
2. The neutron magnetic form factor agrees within error limits with scaling law.
3. The systematic errors in the normalization do not allow to draw the definite conclusions on the value of the neutron electric form factor. The satisfactory fit to all the experimental data both for the case $G_{EN}=0$ and $G_{EN}\neq 0$ is possible by changing the norms.

TABLE II

Number of experimental points and reference		χ^2= II3 $\bar{\chi}^2$=86(C.L.=2.6%)	χ^2= I05.4 $\bar{\chi}^2$=89(II.7%)	χ^2= 134. $\bar{\chi}^2$= 88	χ^2= III $\bar{\chi}^2$=87(C.L.=4%)	χ^2= 299 $\bar{\chi}^2$= 94
6 2/	N_1	I.03±0.0I	I.00±0.0I	I.02±0.0I	I.04±0.0I	I
4 3/	N_2	I.I4±0.04	I.I0±0.04	I.I0±0.04	I.II±0.04	I
7 4/	N_3	I.07±0.0I	I.02±0.0I	I.05±0.0I	I.07±0.0I	I
6 5/	N_4	0.98±0.03	0.87±0.03	0.94±0.03	0.96±0.03	I
I4 6/	N_5	I.04±0.03	0.94±0.03	0.0I±0.03	I.04±0.03	I
I0 7/	N_6	I.I0±0.02	0.90±0.02	I.I2±0.02	I.II±0.02	I
37 8/	N_7	I.I9±0.03	0.90±0.02	I.40±0.04	I.I9±0.03	I
I3 9/	N_8	0.96±0.0I	0.82±0.0I	0.94±0.0I	0.96±0.0I	I
	$a\,(\mathrm{GeV}/c)^2$	0.02±0.05	0	0	0	0.I3±0.06
	b	−I.I3±0.I3	0	−0.57±0.02	−I.I5±0.I4	−I.05±0.I4
	$c\,(\mathrm{GeV}/c)^2$	3.I6±0.80	0	0	3.22±0.79	3.I3±0.94

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