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ON A SCALING LAW FOR THE PROTON FORM FACTORS

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On the basis of the analysis of the data on $e-p$ scattering for $q^2 < 2\,(\mathrm{GeV/c})^2$ (q^2 is the squared momentum transfer) the authors of refs. /1-3/ come to the conclusion that in the region of q^2 from 1 to 2 $(\mathrm{GeV/c})^2$ there occurs a significant deviation from the so-called scaling law

$$G_M(q^2) = \mu\, G_E(q^2)\,. \tag{1}$$

Here $G_E(q^2)$ and $G_M(q^2)$ are the charge and magnetic form factors of a proton and μ is the total magnetic moment of the proton. On the other hand, the data for q^2 from 1 to 3.75 $(\mathrm{GeV/c})^2$ obtained in /4/ are, within errors, compatible with eq. (1).

The present note is devoted to the check of the scaling relation (1) on the basis of the fitting of all the available data on the $e-p$ cross sections. A separate analysis of the data of ref. /1/ has also been made. We have used the method given in refs. /5,6/. This method differs from the generally accepted one (construction of the Rosenbluth plot for a fixed q^2). The proton electromagnetic form factors are directly extracted from the data on the differential $e-p$ cross sections. To this end, a certain functional dependence of the form factors upon q^2 is assumed and the values of the appropriate parameters are found by minimizing the functional χ^2. When analyzing experimental data of different groups normalizing factors, which take into account possible normalization errors, are introduced.

We write

$$G_M(q^2) = S(q^2)\,\mu\, G_E(q^2)\,. \tag{2}$$

For the form factor $G_E(q^2)$ we take the expression

$$G_E(q^2) = \frac{a_1}{1 + a_2 q^2} + \frac{1 - a_1}{1 + a_3 q^2} , \tag{3}$$

which, as is shown in ref. /6/, fits satistactory all the available experimental $e-p$ data. As to the function $S(q^2)$ we will make different assumptions. From the normalization condition $G_E(0) = 1$, $G_M(0) = \mu$ it follows that

$$S(0) = 1 . \tag{4}$$

Next, as is known, at the threshold of the processes $e + \bar{e} \rightleftarrows p + \bar{p}$ for $q^2 = -4M_p^2$ (M_p is the proton mass) there holds the equality

$$G_M(-4M_p^2) = G_E(-4M_p^2). \tag{5}$$

If the form factors G_M and G_E for $q^2 = -4M_p^2$ are not equal to zero (the first experimental data on the process $e + \bar{e} \to p + \bar{p}$, give evidence in favour of this /7/) then from (2) and (4) we get

$$S(-4M_p^2) = \frac{1}{\mu} . \tag{6}$$

Assume first that the function $S(q^2)$ is a ratio of the polynomials of the same degree, i.e., that the form factor $G_E(q^2)$ behaves like $G_M(q^2)$ at $q^2 \to \infty$. By restricting ourselves to the polynomials of the first degree, we have

$$S(q^2) = \frac{a + b\tau}{1 + c\tau} , \tag{7}$$

where

$$\tau = \frac{q^2}{4M_p^2} .$$

From the relations (4) and (6) we find that the function $S(q^2)$ is characterized by one parameter and has the following form:

$$S(q^2) = \frac{1 + [1 - \frac{1}{\mu}(1-c)]\tau}{1 + c\tau} . \qquad (8)$$

As a result of the fitting of all the existing $e-p$ data x/ we have found that

$$c = 1.05 \pm 0.09 . \qquad (9)$$

For the parameters a_1, a_2, a_3 the following values

$$a_1 = -0.48 \pm 0.08$$
$$a_2 = (0.69 \pm 0.05)\ (\mathrm{GeV}/c)^{-2} \qquad (10)$$
$$a_3 = (2.18 \pm 0.08)\ (\mathrm{GeV}/c)^{-2}$$

are found. The quality of fitting of the experimental data in the considered case ($\chi^2 = 396$ for $\bar{\chi}^2 = 313$) is practically the same as in the case of parametrization of the form factors G_M and G_E by the sum of two poles with independent parameters /6/. We note that, within errors, the values (10) coincide with those obtained in the latter case. From (8) and (9) we obtain that the quantity $\frac{\mu G_E}{G_M} S^{-1}$ for q^2 equal to 1,5,10, and 25 $(\mathrm{GeV}/c)^2$ is 1.01 ± 0.01, 1.02 ± 0.04, 1.02 ± 0.05 and 1.03 ± 0.06, respectively. Thus, the quantity $\frac{\mu G_E}{G_M}$ found from the fitting of all the available $e-p$ data under the assumption that $G_E(q^2)$ and $G_M(q^2)$ are given by expressions (3) and (8) does not deviate within errors from unity over the whole experimentally studied interval of q^2.

We stress that this conclusion is obtained under the condition that the form factors obey the constraint (5). Using the relations

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x/ For references see paper /6/. Instead of the former Bonn data we have used the recent ones /1/.

(2), (3), and (8), we have performed an analysis of the experimental data for $q^2 \leq 2\ (\mathrm{GeV}/c)^2$ obtained in ref. /1/ , in which the deviation from eq. (1) has been reported. The parameter c is found to be $c = 0.85 \pm 0.19$ ($\chi^2 = 26$ for $\bar{\chi}^2 = 49$). The quantity $\frac{\mu G_E}{G_M}$ for q^2 equal to 0.5, 1, and 2 $(\mathrm{GeV}/c)^2$ is 0.99 ± 0.02, 0.98 ± 0.03 and 0.96 ± 0.05, respectively.

Next, in order to obtain better fitting of all the experimental $e-p$ data we consider the different behaviour of the form factors $G_E(q^2)$ and $G_M(q^2)$ at $q^2 \to \infty$. Assuming that $S(q^2)$ is the ratio of the polynomial of the second degree in q^2 to that of the first degree, by means of (4) and (6), we get

$$S(q^2) = \frac{1 + [1 + e - \frac{1}{\mu}(1-d)]\tau + e\tau^2}{1 + d\tau}. \tag{11}$$

It is obvious that for $e = 0$ the expression (11) turns into (8). As a result of fitting the parameter e is found, within errors, to be zero ($e = -0.02 \pm 0.14$). The introduction of it does not improve the quality of the description of the experimental data ($\chi^2 = 396$ for $\bar{\chi}^2 = 312$).

Finally we have fitted the data for the case when the function $S(q^2)$ is represented by a ratio of the first degree polynomials and the constraint (5) is not imposed. The two parameters characterizing $S(q^2)$ turn out in this case to be strongly correlated and are determined with large errors. The quality of fitting remains at the same level.

In conclusion we note that we have also made an analysis of the data of ref. /1/ for the interval $q^2 \leq 2\ (\mathrm{GeV}/c)^2$, taking for $S(q^2)$ as in paper /1/ , the expression

$$S(q^2) = \frac{1}{1 + \beta q^2}. \tag{12}$$

At $\chi^2 = 29$ and $\bar{\chi}^2 = 49$ it is found that $\beta = -(0.026 \pm 0.028)(\mathrm{GeV}/c)^{-2}$

By fitting the values of the quantity $\frac{\mu G_E}{G_M}$ obtained from the Rosenbluth plot in ref. /1/ the parameter β is found to be $\beta = -(0.059 \pm 0.020)\ (\mathrm{GeV}/c)^{-2}$. In ref. /4/ , it is obtained that $\beta = -(0.051 \pm 0.030)\ (\mathrm{GeV}/c)^{-2}$.

Thus the analysis of all the available $e-p$ data shows that over the whole studied region of the momentum transfer the ratio $\frac{\mu G_E}{G_M}$ does not deviate, within errors, from unity. It is interesting to note that this is in agreement with the constraint (5).

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