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PROBABILITY OF FORMATION OF Σ HYPERONIC ATOMS IN VARIOUS ELEMENTS

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1. Introduction

The study of Σ hyperonic atoms should provide information on the Σ-nucleon interactions and on the Σ hyperon itself. Since Σ hyperon has spin 1/2, the energy levels of Σ hyperonic atoms should show fine structure and thus allow the magnetic moment of the Σ hyperon to be measured. The fine structure splitting increases strongly with the atomic number, so the effect should be studied in the heaviest hyperonic atoms.

In the last two years some evidence for Σ hyperonic atoms has been obtained /1,2,3/ and the intensities of their X-ray lines have been measured for some light elements /3/. However, there is at present no indication of any X-ray line from Σ hyperonic atoms with atomic number higher than 30 /4/.

In this paper we have calculated the intensity of a "beam" of Σ hyperons, stopping in a given target element, relative to the number of the K^- mesons interacting at rest with nuclei of this target. The calculations have been performed for several elements with largely different atomic numbers, namely for C, Al, Fe, Ag, Au and U, in order to obtain information about the Z dependence of the probability of the formation of Σ hyperonic atoms.

Section 2 is concerned with the production of Σ^- hyperons within nuclei. We discuss their production rates, the localization of the production process and the Σ^- hyperon energy spectra. In sections 3 and 4 we calculate the probability that a Σ^- hyperon produced in the nuclear capture of a stopping K^- meson will escape from the parent nucleus. The latter is taken to be like a semi-transparent medium with an absorption coefficient connected with the complex Σ hyperon nuclear potential.

The tramsmission probability $P(r,\Theta,T_\Sigma^{ex})$ of a Σ^- hyperon produced at a given distance r from the centre of the nucleus, moving in a given direction described by an angle Θ and emerging with a kinetic energy T_Σ^{ex}, is calculated in the WKB approximation. Then we calculate an average value of $P(r,\Theta,T_\Sigma^{ex})$ over the angle Θ, the distance r and the energy T_Σ^{ex}. This procedure is repeated for the six discussed nuclei.

The results are compared with the frequency of emission of Σ^- hyperons in nuclear emulsion /5/ as well as with the values quoted by Backenstoss et al. /3,6/ who have measured the intensities of the atomic X-ray lines in several targets. In section 5 we correct the emission rates for the decay in flight and in section 6 we give the probability that a Σ^- hyperon emitted from a K^- meson nuclear capture comes to rest to form a Σ hyperonic atom. Conclusions of this work are presented in section 7.

2. Production of Σ^- Hyperons Inside Nuclei

The K^- meson interactions with nuclei may be of a single nucleon or a multinucleon type. Below we discuss the production rates and energy spectra of the Σ^- hyperons produced in both interaction modes.

2.1 Production Rate

The basic interaction mode between a K^- meson and a nucleus is that of a single-nucleon pionic type :

$$K^- + N \to Y + \pi ,$$

where Y stands for a Σ or a Λ hyperon.

On the assumption that the K-N interaction is charge independent, the rates of the particular interaction channels may be expressed in terms of the following four parameters[7]: $|T_0|^2$, $|T_1|^2$, λ and $|T_{1\Lambda}|^2$ where T_0 and T_1 are the amplitudes for the Σ-π pair production with total isospin 0 and 1, respectively; $T_{1\Lambda}$ is the amplitude for the Λ-π pair production and $\lambda = \frac{1}{\sqrt{6}} Re(T_1^* T_0)$;

$$R(K^- p \to \Sigma^+ \pi^-) \sim \frac{1}{4} |T_1|^2 + \frac{1}{6} |T_0|^2 + \lambda , \qquad (1a)$$

$$R(K^- p \to \Sigma^- \pi^+) \sim \frac{1}{4} |T_1|^2 + \frac{1}{6} |T_0|^2 - \lambda , \qquad (1b)$$

$$R(K^- p \to \Sigma^0 \pi^0) \sim \frac{1}{6} |T_0|^2 , \qquad (1c)$$

$$R(K^- p \to \Lambda^0 \pi^0) \sim \frac{1}{2} |T_{1\Lambda}|^2 , \qquad (1d)$$

$$R(K^- n \to \Sigma^0 \pi^-) \sim \frac{1}{2} |T_{1\Lambda}|^2 , \qquad (1e)$$

$$R(K^- n \to \Sigma^- \pi^0) \sim \frac{1}{2} |T_1|^2 , \qquad (1f)$$

$$R(K^- n \to \Lambda^0 \pi^-) \sim |T_{1\Lambda}|^2 . \qquad (1g)$$

In order to find the branching ratios for reactions (1a) through (1g), three relations between these parameters are needed. The amplitude T_0 is strongly dependent on energy owing to the existence of the Y_0^* resonance, whereas the amplitudes T_1 and $T_{1\Lambda}$ vary with energy only slowly.

The kaon absorption process in nuclear surface and the role of the Y_0^* resonance has been examined by several authors[8,9,10]. Bardeen, Torigoe[8] and Bloom et al.[9] have calculated the ratios :

$$E_1 \equiv R(K^- p \to \Sigma^+ \pi^-) / R(K^- p \to \Sigma^- \pi^+) \qquad (2)$$

and

$$E_2 \equiv [R(K^- p \to \Sigma^+ \pi^-) + R(K^- p \to \Sigma^- \pi^+)] / R(K^- n \to \Sigma^- \pi^0) \qquad (3)$$

taking the values of the parameters of the Y_0^* resonance obtained by Martin and Sakitt[11] or Kim[12] for the K meson-nucleon interaction. Wycech[10] has paid attention to the behaviour ofthe Y_0^* resonance in a nuclear medium. The author has taken into account the exclusion principle effective in the intermediate states of K-N scattering, and the action of the external field produced by the residual nucleus. He has calculated the width and position of the Y_0^* resonance as a function of the nuclear density at the point where the resonance is produced and found that the width of the resonance is enlarged and the effective position is shifted to the higher values of energy. Using the corrected values of the parameters of the Y_0^* resonance, Wycech has calculated the ratios E_1 and E_2 as functions of the nuclear density. Taking the values of E_1 and E_2 at the point where the nuclear density ρ is equal to 6% of the central value (see subsection 2.2), we obtain : $|T_0|^2 = 18\,|T_1|^2$ and $\lambda = 1.08\ /T/^2$. We also consider the ratio :

$$|T_1|^2/|T_{1\Lambda}|^2 \equiv (1-\varepsilon)/\varepsilon . \qquad (4)$$

We assume that the nonresonant amplitudes T_1 and $T_{1\Lambda}$ do not change for the K-N interaction in a nucleus in comparison with the free ones. We have calculated the free ratio, $|T_1|^2/|T_{1\Lambda}|^2$, from the K^--p scattering parameters of Martin and Sakitt and obtained $|T_1|^2/|T_{1\Lambda}|^2 = 2.0$ ($\varepsilon = 0.33$).

In this case the branching ratios for the Σ hyperon production per 1 pionic K^- meson nuclear capture are :

$$R\,(K^-p \to \Sigma^-\pi^+) = 0.193$$
$$R\,(K^-n \to \Sigma^-\pi^0) = 0.044 \qquad (5)$$

and we see that the majority of the Σ^- hyperons originate from K^- meson interactions with protons.

Multinucleon non-pionic K^- meson nuclear interactions of the type $K^- + N + N \to Y + N$ also occur. The Σ^- hyperons may be produced in the following non-pionic K^- meson capture reactions :

$$K^- + n + n \to \Sigma^- + n, \qquad (6a)$$

$$K^- + n + p \to \Sigma^- + p . \qquad (6b)$$

The first of these reactions is rather infrequent[13,5] whereas the rate of the second one is about 19%[13] of all the non-pionic interactions. The results of the analysis of the non-pionic capture process were presented at the International Conference on Hypernuclear Physics at Argonne for the K^- meson interactions in helium, neon and heavy liquid bubble chambers as well as in nuclear emulsion[14]. The probability of the multinucleon K^- meson capture from these data seems to be independent of the nuclear size and is approximately equal to 20%. Taking the value of 20% for the multinucleon K^- meson capture yield in all the target nuclei discussed in our paper, we have obtained the following branching ratios for the Σ^- hyperon production per one K^- meson captured at rest :

reaction :	branching ratio per 1 K^-(%) :	
$K^- p \to \Sigma^- \pi^+$	15.4	(a)
$K^- n \to \Sigma^- \pi^0$	3.5	(b)
$K^- pn \to \Sigma^- p$	3.8	(c)

2.2 Localization of the Production Process

We assume that the K^- mesons are absorbed from the states of maximum possible angular momentum for a given principle quantum number. We take the probability of the K^- meson nuclear capture from different states for each nucleus from the paper of Ericson and Scheck /15/. For a state with a given principle quantum number, n , the distribution of the capture probability as a function of the distance from the centre of the nucleus is proportional to the overlap of the nuclear density, ρ , and the radial kaonic wave function squared :

$$g(r)dr \sim \rho(r) R^2_{n,n-1} r^2 dr. \qquad (7)$$

For the nuclear matter distribution we take the formula of Saxon-Woods. The values of the radius at half density, c , and the skin thickness, t , for different nuclei, except silver , have been taken from ref. /16/. For silver, in the absence of experimental values , we have assumed $c = 1.1 A^{1/3}$ fm and $t = 0.55$ fm, i.e. the values which apply well over the range of medium and heavy elements. The distribution g/r/ calculated in such a way has a maximum beyond the nuclear surface. The average value of the density, ρ , calculated with the distribution g/r/ is about 6% of the density at the centre of the nucleus. The corresponding distance from the centre of the nucleus will be denoted r_{av} .

2.3. Internal Energy Spectrum

In order to calculate the energy distribution of the produced

Σ^- hyperons we adopt the Capps model [17]. We use the impulse approximation and treat the outgoing particles as classical particles moving in potential wells. Also, we assume for simplicity that the capture of the K^- mesons is fully localized at the average distance from the centre of the nucleus, r_{av} (see subsection 2.2). The conservation of energy in the reaction (a) is then expressed by the relation

$$m_p + m_K - (E_B + E_{ex}) = m_\Sigma + m_\pi + T_\Sigma^{in} + T_\pi^{in} + V_\Sigma(r_{av}) + V_\pi(r_{av}), \quad (8)$$

where m_p, m_K, m_Σ and m_π are the masses of a proton, a K^- meson, a Σ hyperon and a π meson, respectively; $E_B + E_{ex} = 20$ MeV is the sum of the binding energy of the proton and the excitation energy of the residual nucleus; T_Σ^{in} and T_π^{in} are the internal kinetic energies of the produced Σ hyperon and pion and $V_\Sigma(r_{av})$ and $V_\pi(r_{av})$ are the real parts of the nuclear potentials of the Σ hyperon and pion at the capture point. For the i-th particle we use a potential proportional to the nuclear density, $V_i = -V_i^0 \rho(r)/\rho(0)$. We take $V_\Sigma^0 = -35$ MeV [10] and $V_\pi^0 = -50$ MeV [10]. The analogous equation for the two-nucleon reaction (c) is :

$$m_K + m_p + m_n - 2(E_B + E_{ex}) = m_\Sigma + m_p + T_\Sigma^{in} + T_p^{in} + V_\Sigma(r_{av}) + V_p(r_{av}), \quad (9)$$

where we take $V_p^0 = -60$ MeV [10].

Since the outgoing particles in reactions (a), (b) and (c) are produced inside the nucleus, their measured external energies, T^{ex}, differ from the internal kinetic energies, T^{in}, by the depth of the potential wells. For the i-th particle, the relation between the external and internal energies is :

$$T_i^{ex} = T_i^{in} + V_c^i(r_{av}) + V_i(r_{av}), \quad (10)$$

where V_t^i is the Coulomb potential. This equation, as well as eqs (8) and (9), is only approximate since we neglect the absorption of the final particles.

We first calculate the internal kinetic energy spectra for the Σ^- particles, denoted as $\Sigma^{(1N)}$, originating from the single nucleon reactions. We assume that the Σ^- hyperons are emitted isotropically in the K-N centre of the mass system. For a given value of the total momentum of the kaon and nucleon, p_{KN}, the distribution of the internal kinetic energy of the Σ^- hyperons is constant between the minimum and maximum values of T_Σ^{in}, which correspond to the motion of the Σ^- hyperon in directions parallel and antiparallel to $\vec{p}_{KN}$, respectively. The final internal kinetic energy distribution is obtained by averaging over theinitial momentum, p_{KN}. An empirical form of the momentum distribution has been used, viz.the Gaussian function :

$$f(p_{KN})\,d p_{KN} = \frac{4}{\pi^{1/2} p_0^2}\, p_{KN}^2 \exp\left(-p_{KN}^2/p_0^2\right) d p_{KN} \qquad (11)$$

has been fitted to the experimental distribution of the momentum of the Σ-π pairs from reactions (1a) and (1b) observed in nuclear emulsion by Lovell and Schorochoff /18/. The values of p_0= 130 MeV/c and p_0 = 150 MeV/c have been obtained for the Σ-π pairs produced in the light and heavy emulsion nuclei, respectively.

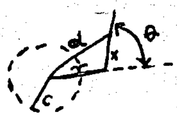
The corresponding internal energy distributions of the Σ hyperons are shown in Fig. 1a. These two distributions do not differ markedly, so we shall not make a serious error if we put now p_0 = 130 MeV/c for the light nuclei C, Al and Fe and p_0 = 150 MeV/c for Ag, Au and U. We have also made a rough evaluation of the internal kinetic energy distribution of the Σ^-hyperons, denoted as $\Sigma^{(2N)}$, produced in the non-pionic reaction (c). We have assumed that the motion of the two nucleons is uncorrelated and that each of their momenta is given by the distribution discussed above. The resulting internal energy spectra for the Σ^-hyperons from reaction (c) are shown in Fig. 1b.

3. Absorption of Σ^-Hyperons within the Parent Nuclei

The fact that some of the created Σ^-particles do not escape from the parent nuclei is connected with the inelastic interactions with the nucleons inside the nucleus and with the capture of a part of the Σ^- hyperons by the potential well. We evaluate the transmission coefficient of the Σ^- hyperons through a nucleus in the WKB approximation. For a Σ^-particle created at a distance r, a direction θ and emerging from the nucleus with a kinetic energy T_Σ^{ex}, the transmission coefficient is given by :

$$P(r, \Theta, T_\Sigma^{ex}) = \exp\left[-2\,\mathrm{Im}\int_0^\infty \sqrt{2m_\Sigma(T_\Sigma^{ex} - V_c(d) - U_\Sigma(d))}\,dx\right]. \qquad (12)$$

In this formula, V_c is the Coulomb potential, $U_\Sigma = V_\Sigma + iW_\Sigma$ is the Σ^-hyperon nuclear optical potential, the parameter x is the distance passed by a Σ^-hyperon and $d = \sqrt{r^2 + x^2 + 2rx\cos\Theta}$ (see the figure below).



The Nuclear potential for the hyperon has been obtained by Wycech[10] using the data on the Σ-N interactions presented by Alexander at the International Conference on Hypernuclear Physics at Argonne [19]. The real part of the potential was introduced in section 2.3; the imaginary part is :

$$W_\Sigma \equiv W_{\Sigma p} + W_{\Sigma n} =$$
$$= \left\{-70(\rho_p/\rho_p(0))\left[1-0.6(\rho_p/\rho_p(0))\right] - 12(\rho_n/\rho_n(0))\left[1-0.8(\rho_n/\rho_n(0))\right]\right\} \mathrm{MeV}.. \qquad (13)$$

We use this form of the potential assuming that $\rho_n = \rho_p$. Also, we allow for the space correlations between the nucleons in a nucleus by multiplying the short-range nuclear potential by the two-nucleon correlation function, c^2. The following form of the function c^2 has been used :

$$c^2(x,d) = \left[1 - \exp(-x/a)\right]\left[1 - D^2(x,d)\,\delta_{\sigma\sigma'}\,\delta_{\tau\tau'}\right]. \qquad (14)$$

The first factor corresponds to the hard core nucleon-nucleon repulsion; the parameter a is taken to be equal to 1.1 fm. The second factor is related to the Pauli principle which means in this case that the distance between two protons (or neutrons) in the same spin state cannot be arbitrarily small. The statistical correlation function has been calculated by Brueckner[20] for the case of non-interacting nucleons :

$$D = \frac{3}{(k_F x_{12})^2}\left[\frac{\sin(k_F x_{12})}{k_F x_{12}} - \cos(k_F x_{12})\right], \qquad (15)$$

where x_{12} is the distance between two nucleons and $\hbar k_F$ is the Fermi momentum of nucleons, related to the nuclear density by :

$$\hbar k_F = \hbar\,(3\pi^2)^{1/3}\,\rho^{1/3} \qquad (16)$$

We have approximated the formula of Brueckner by the simple expression for D^2 :

$$D^2 = \exp\left[-x/x_{corr}\right], \tag{17}$$

where $x_{corr} = 0.85\,(\rho_0/\rho)^{1/3}$ fm. In eq. (14) the indexes σ, τ and σ', τ' indicate the spin and isospin of the two nucleons. Let us assume that the kaon has been captured on the first nucleon and the second is its neighbour. Then, after averaging over spins and the isospin τ', the imaginary part of the nuclear potential, W_Σ, is replaced by the following effective forms for the Σ^--hyperons origina - ting from protons (τ = 1) and neutrons (τ = -1), respectively :

$$\tau = 1 \qquad W_\Sigma \longrightarrow \left[W_{\Sigma n} + W_{\Sigma p}\left(1-\tfrac{1}{2}D^2\right)\right]\left[1-\exp(-x/a)\right],$$

$$\tau = -1 \qquad W_\Sigma \longrightarrow \left[W_{\Sigma p} + W_{\Sigma n}\left(1-\tfrac{1}{2}D^2\right)\right]\left[1-\exp(-x/a)\right]. \tag{18}$$

The corresponding transmission probability for a Σ^- hyperon emerging with a kinetic energy T_Σ^{ex} is :

$$P_\tau\,(T_\Sigma^{ex}) = \frac{1}{4\pi}\int g(r)\,P(r,\theta,T_\Sigma^{ex})\,dr\,d\Omega \,. \tag{19}$$

The transmission probabilities of Σ^- hyperons originating from protons and neutrons as functions of energy for the six discussed nuclei are shown in Fig. 2. For comparison, the curves obtained without nuclear correlations are drawn. We see that for the Σ^- hyperons emitted with an energy of T_Σ^{ex} = 20 MeV, the values of the transmission coefficient obtained when the nucleon correlations are taken into account decrease from about 60% for the light nuclei to about 40% for uranium. Also, we see that the local "dilution" of the nuclear matter around the Σ^- hyperon production point increases the Σ^- transmission coefficient by about 20%.

Finally we have averaged the transmission coefficient over energy. Having in mind the internal kinetic energy spectra of the Σ^- hyperon and eq. (10) which relates the internal and external kinetic energies, we see that many of the Σ^- hyperons have an energy insufficient to leave the nucleus.(The corresponding T_Σ^{ex} turns out to be negative). The rate of such hyperons increases with increasing atomic number since the Coulomb potential in heavy nuclei is higher. The value of $V_{\Sigma}(r_{av}) + V_c(r_{av})$ for the heaviest of the discussed nuclei, uranium, is - 27 MeV. Hence, more than 70% of the Σ^- hyperons produced in the interactions of K^- mesons with single nucleons in uranium is captured in the potential well. The obtained probability that a Σ^- hyperon will escape the parent nucleus is shown in Table I for the $\Sigma^{(1N)}$ and $\Sigma^{(2N)}$ hyperons separately.

T a b l e I

nucleus	Transm. coeff. of Σ^- per 1 K^-	Transm. coeff. of Σ^- per 1 K^-
C	0.50	0.71
Al	0.50	0.71
Fe	0.49	0.66
Ag	0.32	0.61
Au	0.21	0.52
U	0.10	0.45

These numbers may be compared with the experimental ones of Koch et al. /13/ and the European K^- Collaboration /5/ who have evaluated the absorption of the Σ^- hyperons in the parent nuclei for nuclear emulsion. The result obtained by Koch et al. is 0.49 ± 0.14. The values obtained by the European K^- Collaboration for all the charge states of the Σ hyperon are between 0.55 and 0.45, but the authors say that the value 0.45 is preferable for the Σ^- hyperons, as it makes some allowance for the trapping of the Σ^- hyperons by the Coulomb field. Treating our results for carbon and silver as representative of the light and heavy nuclei of the nuclear emulsion, respectively, and combining them with the weights 0.37 and 0.63 (the K^- meson absorption probabilities on the light and heavy nuclei in nuclear emulsion /21/) we have found that the transmission probability of the Σ^- hyperon through the nuclei of the nuclear emulsion is 0.39. This result is in agreement with both discussed experimental values.

4. External Energy Spectra and the Total Number of the Emitted Σ^- Hyperons

The external kinetic energy spectra of the Σ^- hyperons have been obtained by shifting the corresponding internal kinetic energy spectra (see subsection 2.3) to the lower energies according to eq. (10) and multiplying the result by the transmission probability $P_{+1}(T_{\Sigma}^{ex})$ x). In Figs 3a and 3b the calculated external energy spectra of the Σ^- hyperons resulting from reaction (a) in carbon and silver, respectively, are compared with the experimental energy distributions of ref.[18]. The agreement between the experimental distributions and our calculation is satisfactory since there is a bias in the low energy region caused by the fact that the short range Σ^- hyperons cannot be distinguished from the non-pionic decays of hyperfragments.

In order to find the total rate of the Σ^- hyperons emitted per one stopping K^- meson, we have multiplied the transmission coefficient of the Σ^- hyperons originating from the single nucleon and multinucleon reactions by the corresponding production rates. The emission probabilities for different nuclei are shown in Fig. 5a. Combining, as before, the results for carbon and silver with the weights 0.37 and 0.63, we have found that the emission probability of the Σ^- hyperon in nuclear emulsion per one stopping K^- meson is 9.7%. This result may be compared with the corresponding experimental value (8.1 ± 1.3)% obtained by the European K^- Collaboration [5].

Backenstoss et al.[3,6] have measured the relative intensities of the X-ray lines of the S, Cl and Zn Σ hyperonic and kaonic atoms. The authors have presented [6] the values corrected for different population of the upper level of the lines being compared, for the nuclear absorption of the Σ^- hyperons from the upper level and for the decay in flight. They have obtained the following results : (9.3 ± 1.7)% for the line 6 → 5 in sulphur, (9.3 ± 2.4)% and (13.3 ± 2.5)%, respectively, for the lines 7 → 6 and 6 → 5 in chlorine, (7.3 ± 1.8)% and (5.6 ± 1.4)%, respectively, for the lines 9 → 8 and 8 → 7 in zinc. These results may be compared with our Σ^- emission probability for aluminium (Z = 13) and iron (Z = 26), which are equal to 12.1% and 11.8% , respectively.

We see that our results are consistent with the experimental ones. A small difference may be caused by the fact that, owing to the Fermi motion, the "hole" connected with nucleon correlations would

x) The transmission probability for τ = +1 has been used because the majority of the Σ^- hyperons originates from protons.

be partially "filled up" during the passage of the Σ^- hyperon through the nucleus. Consequently, the correct values of the transmission probability should be somewhere between the full and dotted-dashed lines in Fig. 2. The other doubtful point in our calculations is the assumption that K^- mesons are captured from the states of maximum angular momentum (circular orbits). The transmission coefficient is strongly dependent on the localization of the production process, Therefore, the capture from other states (elliptical orbits) which takes place in the more dense parts of the nucleus, would reduce the transmission coefficients considerably.

5. Decay in Flight

A Σ^- hyperon emitted from a nucleus passes through the target losing its energy by collisions with atoms until it slows down sufficiently to be captured by the Coulomb field of a nucleus.

Since the time necessary to stop a Σ^- hyperon (the moderation time) is of the same order as its lifetime, τ_Σ , a significant part of the Σ^- hyperons may decay in flight. We have calculated the moderation time, t , as a function of the primary energy of the Σ^- hyperon for the discussed targets. .

The moderation time of a particle which range in material is R is given by :

$$t = \int_0^R \frac{dl}{v(l)} = \int_0^R \frac{\sqrt{p(l)^2c^2 + m_\Sigma^2 c^4}}{c^2\, p(l)}\, dl\,, \tag{20}$$

where v(l) and p(l) are the velocity and the momentum of the particle at a distance l from the point of production, respectively. The momentum p(l) is simply related to the distance l :

$$p(l) = a\,(R - l)^b. \tag{21}$$

We have found the parameters a and b using the tabulated range-energy functions for protons /22/ and the fact that for a given velocity the range of a particle is proportional to its mass,

$$\left.\frac{R_\Sigma}{R_p}\right|_v = \left.\frac{E_\Sigma}{E_p}\right|_v = \frac{m_\Sigma}{m_p}\,. \tag{22}$$

In the nonrelativistic approximation the moderation time of eq. (20)

for a Σ^- particle emitted with momentum p is :

$$t = \frac{m_\Sigma}{(1-b)} p^{1/b - 1} . \qquad (23)$$

The probabilities of survival, $\exp(-t/\tau_\Sigma)$ in the different targets are drawn as a function of external kinetic energy, T_Σ^{ex}, in Fig. 4.

We have also tried to evaluate the rate of the Σ^- hyperon inelastic interactions in flight with target nuclei. For the particles of the range R this rate is equal to $\exp(-\sigma R)$ where σ is the Σ^- hyperon-nucleus inelastic cross-section and R is the number of nuclei enclosed in a volume element of dimensions 1 cm^2x R.

The cross-section for the absorption of the Σ^- hyperon in the nucleus is approximately equal to the geometric cross-section and for different target nuclei it varies within the range 0.16 $\div$ 1.5 b. In the interesting energy region (0 $\div$ 180 MeV), the numbers R are enclosed in the range from 10^{22} for the heavy target nuclei to 10^{23} for the light ones. The product σR is then at most of the order of 10^{-2} so the interactions of the Σ^- hyperons in flight with target nuclei may be neglected.

6. Probability of Σ^- Hyperons Stopping in the Target

In order to obtain the rate of Σ^- hyperons stopping in the targets considered in this work, we have multiplied the external energy spectra by the corresponding functions $\exp[-t(T_\Sigma^{ex})/\tau_\Sigma]$ and summed over energy.

The results are shown in Table II. The rates of the Σ^- hyperons stopping in the targets normalized to that in carbon are also shown.

Table II

nucleus	R(Σ^- stop) (%)	$\frac{R(\Sigma^- \text{ stop})}{R(\Sigma^- \text{ stop in carbon})}$
C	7.8	1.0
Al	8.1	1.0
Fe	10.1	1.3
Ag	7.0	0.9
Au	5.1	0.65
U	2.9	0.37

We see that the rate of the stopping Σ^- hyperons per one stopped K^- meson tends to decrease with increasing atomic number, but it can also change significantly from element to element. The important factor in this rate is the probability that the Σ^- hyperon does not decay in flight which, in turn, depends on the density of the medium. For targets which density is large (e.g. iron : 7.96 g / cm^3 compared with aluminium : 2.7 g /cm^3) the moderation time is small and so is the probability of the Σ^- hyperon decay in flight. Therefore, we do not generalize our results for other elements. The rates of the Σ^- hyperons stopping in the same targets are also shown in Fig. 5b separately for the $\Sigma^{(1N)}$ and $\Sigma^{(2N)}$ hyperons and also for the sum ($\Sigma^{(1N)} + \Sigma^{(2N)}$).

7. Conclusions

The probability of Σ^--hyperon emission from nuclei decreases approximately monotonically with atomic number. This is on account both of a decrease in the transmission coefficient of the Σ^- hyperon through the larger nuclei, and of the greater probability of trapping the Σ^- hyperon in their Coulomb field. On the other hand, the atomic capture of the Σ^- hyperon is more probable in heavy elements because of their larger stopping power. Therefore, the Σ hyperonic atoms are expected to be produced in heavy elements. The probability of formation of heavy Σ hyperonic atoms should not be much smaller than that for light Σ atoms, the corresponding ratio being about 50%.

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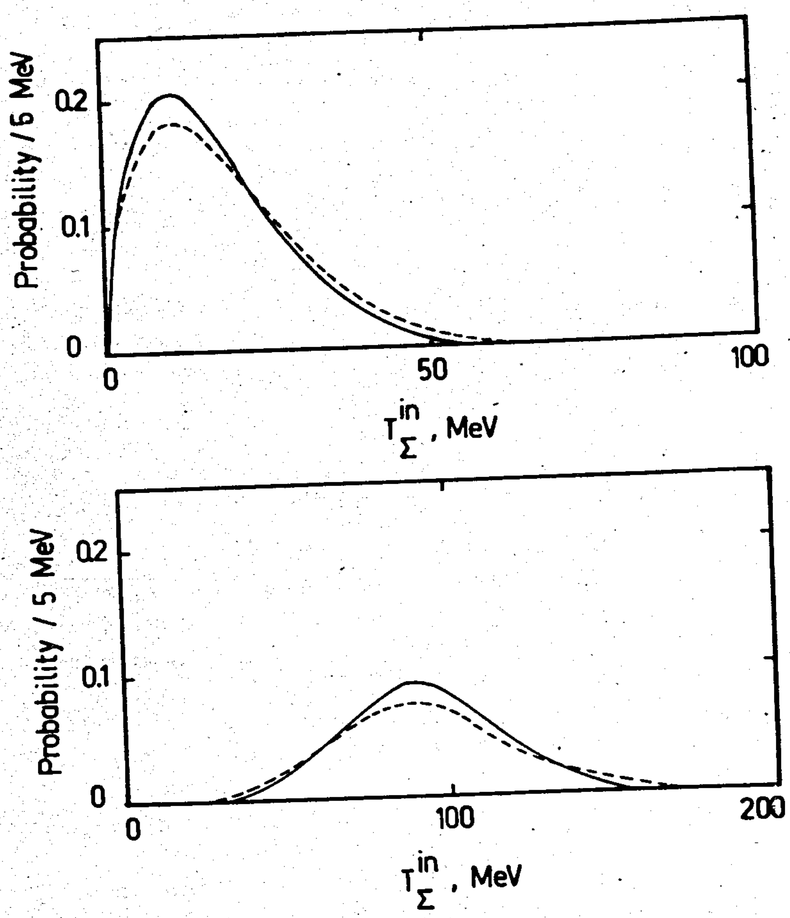

Fig.1.

The kinetic energy distribution of the Σ^- hyperons produced a) in the one-nucleon and b) two-nucleon K^- capture. The solid lines are obtained for p_o = 130 MeV/c, the dashed lines - for p_o = 150 MeV/c.

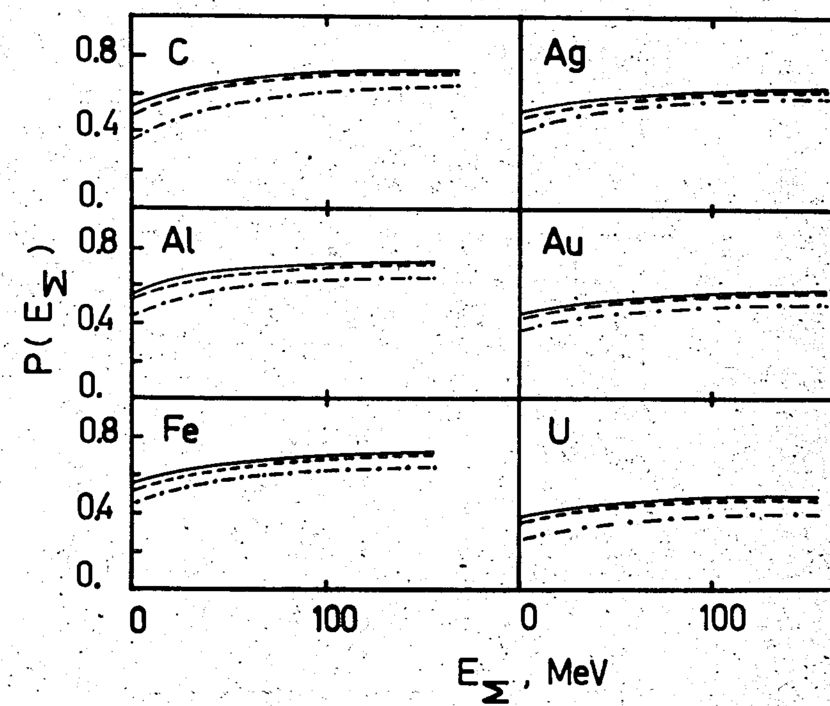

Fig.2.

The transmission probability of Σ^- hyperons originatin from protons (solid line), neutrons (dashed line), eit protons or neutrons without correlations (dashed-dotte line).

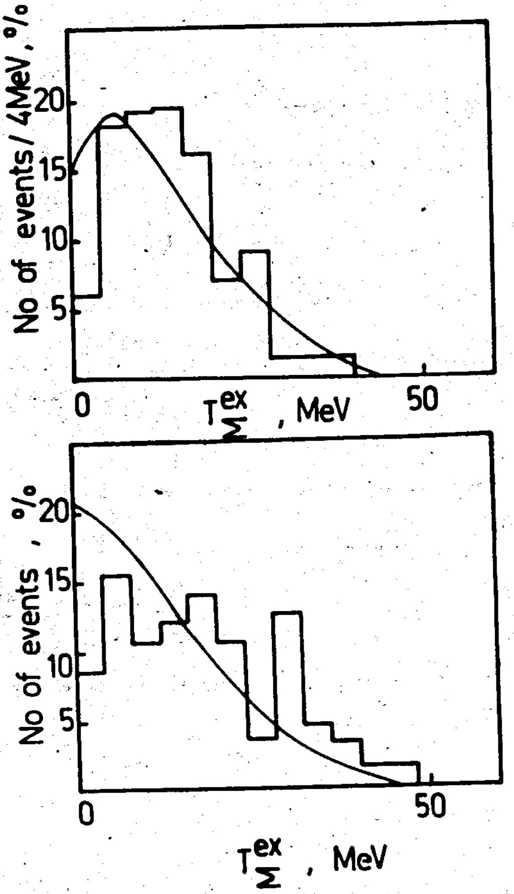

Fig.3.

The energy spectrum of the Σ^- hyperons from reaction (2) in a) carbon and b) silver; histograms from Lovell and Schorochoff (ref. 18).

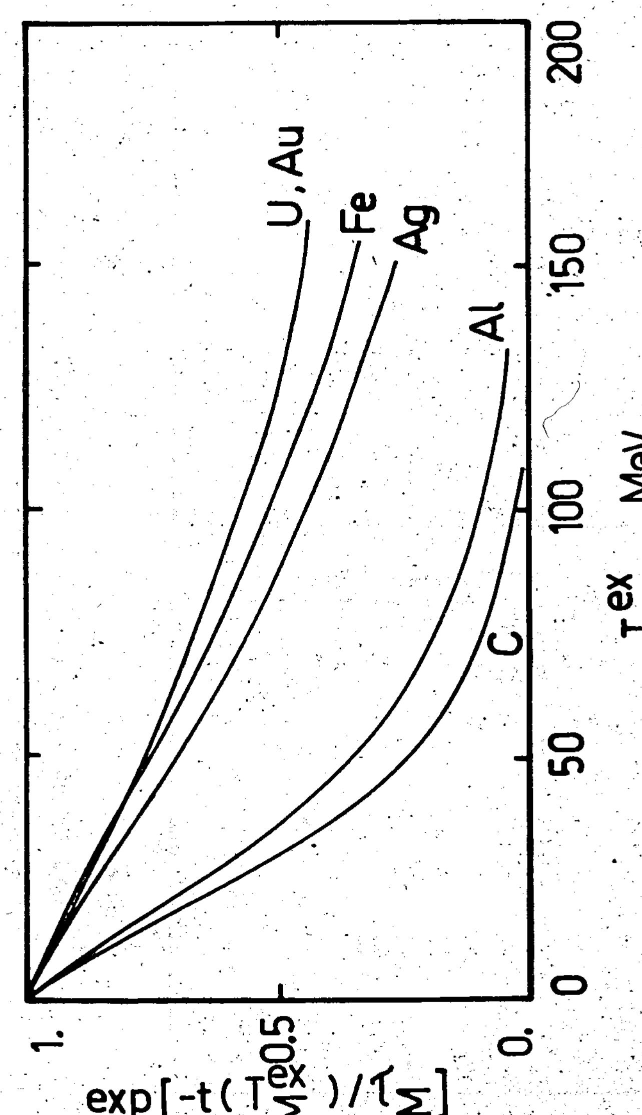

Fig.4. Probability of survival of the Σ^- hyperons as a function of energy in different targets.

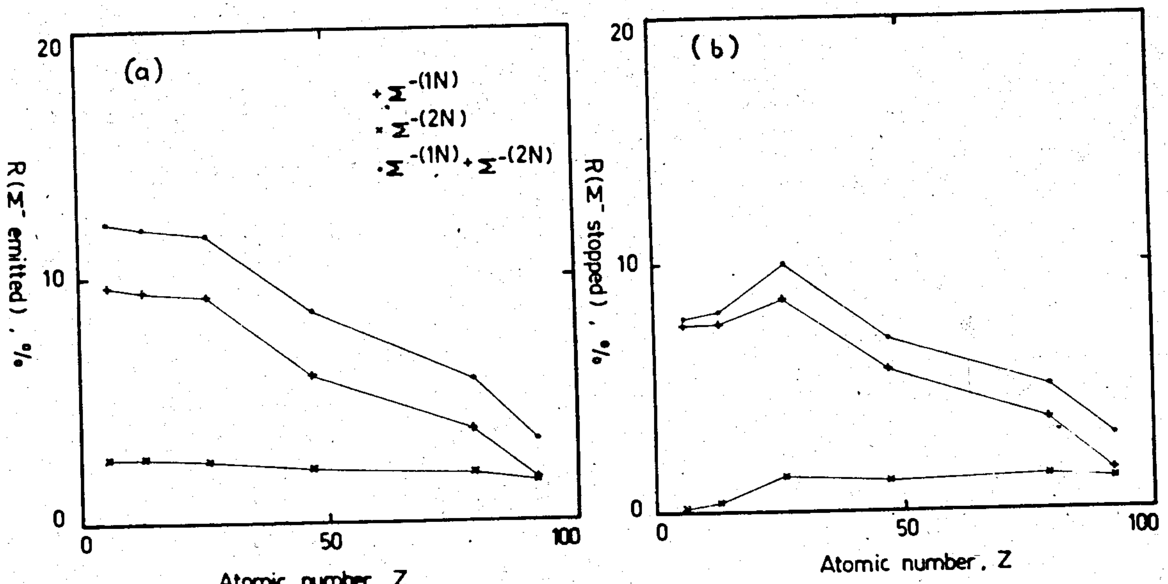

Fig.5.

The rate of the Σ^- hyperons a) emitted, b) stopped per K^- meson stopping.