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A COMPILATION OF DATA ON THE REAL PARTS OF THE $\pi^{\pm}$ p FORWARD SCATTERING AMPLITUDES

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1. Introduction

Recently we have used a new method to analyze the experimental data on forward elastic $\pi^{\pm}p$ scattering in a model-independent way [1] . This and some other methods of analysing $\pi^{\pm}p$ scattering on the basis of dispersion relations require experimental information on the real parts of the forward scattering amplitudes. The individual data points for the values of these real parts generally have rather large errors, so that reliable information can be extracted from these data only by analyzing simultaneously a large number of independent experimental measurements. It is therefore of great importance in such analyses to utilize a set of data which is as complete as possible.

Experimental data on the real parts of the forward $\pi^{\pm}p$ scattering amplitudes come from many diverse sources. Although a fairly complete list of references to the data is available[2], no full compilation of their numerical values has been published. Compilation [2] includes a summary of information on the forward elastic differential cross section and total cross section for $\pi^{\pm}p$ scattering, from which the values of the real parts of the $\pi^{\pm}p$ forward scattering amplitude can be constructed. After examining the original papers it turned out that the fits to the angular distributions obtained in most of them used, as constraints, either (a) the optical limit in the forward direction (assuming the absence of a real part) or (b) a value of the real part calculated from dispersion relation. Unfortunately the authors of the compilation [2] do not mention these facts and do not distinguish between the theoretical and experimental values of the forward differential cross sections they quote. There, of course, is no objection to the application of procedure (b) to many purposes. In fact, it amounts to the use of additional valuable information. However, values of the real parts of the amplitudes obtained in this way cannot be regarded as purely experimental and should not be used as input data in analyses of the dispersion relations. Data based on such fits are not included in our compilation. Instead, in many cases we reanalysed the original data by fitting to the angular distributions without using the constraints either (a) or (b) and obtained in this way the experimental value of the forward differential cross section.

We present this compilation here, similar to our compilation [3] of the real parts of the $K^{\pm}p$ forward scattering amplitudes, in a form in which data may be used directly in dispersion relation calculations.

2. Determination of the Real Parts from Differential and Total Cross Section Data

The most straightforward method of determining the magnitude of the real part of the forward scattering amplitude in terms of experimental data is provided by the relation

$$(\mathrm{Re}\,F)^2 = \left(\frac{d\sigma}{d\Omega}\right)_0 - \left(\frac{q\,\sigma_{tot}}{4\pi}\right)^2, \qquad (1)$$

where $(d\sigma/d\Omega)_0$ is the elastic differential cross section in the forward direction, σ_{tot} is the total cross section, q is the c.m. momentum, and F is the c.m. forward scattering amplitude.

Values of $(d\sigma/d\Omega)_0$ are obtained by extrapolation of the differential cross sections to the forward direction. Two main methods are used in practice for this extrapolation. Firstly, at energies at which the number of contributing partial waves is not too great (typically, below a few GeV), the differential cross section as a function of the c.m. scattering angle ϑ is often represented in the form of a truncated series of Legendre polynomials

$$\frac{d\sigma(\vartheta)}{d\Omega} = C\sum_{l=0}^{N} A_l P_l(\cos\vartheta) \qquad (2)$$

or simply in the form of a truncated power series

$$\frac{d\sigma(\vartheta)}{d\Omega} = C\sum_{n=0}^{N} a_n \cos^n\vartheta\ , \qquad (3)$$

where C is some fixed normalization constant, usually chosen to be $C=1/q$. The coefficients A_l or a_n are determined by a least squares fit of the parametrization (2) or (3) to experimental values of $d\sigma(\vartheta)/d\Omega$.

The second method is usually employed for the analysis of angular distribution at higher energies, where the elastic scattering at small angles appears to have a diffractive character. In this case, the measured values of

$$\frac{d\sigma(t)}{dt} = \frac{\pi}{q^2}\frac{d\sigma(\vartheta)}{d\Omega} \qquad (4)$$

in a restricted range of scattering angles near the forward direction are usually fitted to an exponential form such as

$$d\sigma(t)/dt = \exp(a + bt + ct^2)\ . \qquad (5)$$

Here t is the usual momentum-transfer variable given by

$$t = 2q^2(1 - \cos\vartheta) . \tag{6}$$

In most cases, a good fit to the data is found by retaining only the first one or two of the parameters a,b,c in Eq.(5).

The method of determining the real part of the forward scattering amplitude and its sign as well, which is more precise than that provided by Eq.(1) is based on the use of the interference between the nuclear and Coulomb amplitudes. In this case one fits the measured values of $d\sigma(t)/dt$ to the form (for details see e.g. 4,5)

$$\frac{d\sigma^{\pm}(t)}{dt} = \frac{F_c^2}{t^2} \mp (2F_c/|t|\, Im f_{\pm} [\alpha_{\pm} \cos 2\delta \pm \pm \sin 2\delta] + (1+\alpha_{\pm}^2)(Im f_{\pm})^2 , \tag{7}$$

where $f_{\pm}$ - forward $\pi^{\pm}p$ scattering amplitude, $F_c = [(2\sqrt{\pi})e^2/\beta c] \times$ $\times$(form factor), δ - the relative phase introduced between the nuclear and Coulomb amplitudes by the long-range Coulomb force and

$$\alpha_{\pm} \equiv \frac{Re f_{\pm}}{Im f_{\pm}} . \tag{8}$$

It should be stressed that the extrapolation of the differential cross sections to the forward direction, simple in principle, is, in fact, a very sensitive procedure and is based on the rather arbitrary assumption that nothing abnormal will occur near the forward direction. For example in the recent paper [6] it has been shown that in the case of the violation of the Pomeranchuk theorem the sharp forward peak may become significant only in the region $t \lesssim 0.003$ GeV^{-2}, where Coulomb effects are comparable with strong interactions. This possibility may throw doubt on the usual extrapolation procedure by which the phases of forward scattering amplitudes at high energies are obtained.

3. The Data Compilation

We have found from the available literature suitable experimental data on the angular distributions of the elastic differential cross sections at 97 energies for $\pi^- p$ scattering [7-29] and at 49 energies for $\pi^+ p$ scattering [30-43]. For each energy at which a reliable extrapolation to the forward direction was made in the paper originally reporting the experimental data, we used the reported value of $(d\sigma/d\Omega)_0$ or $(d\sigma/dt)$ for our determination of the real part of the forward scattering amplitude. In those cases when we performed the extrapolation to the forward direction we give the minimal number of terms in (3) for the satisfactory fit and the corresponding value of χ^2/(number of degrees of freedom). In all cases we made use of the most up-to-date values of σ_{tot} [44] in calculating the real parts from Eq.(1).

In addition we have found the values of α measured at 22 energies for $\pi^- p$ scattering [4,45-47] and at 8 energies for $\pi^+ p$ scattering [4]. In these cases we evaluated the real parts using the values of σ_{tot} quoted in original papers.

For convenience, we have converted the real parts of the c.m.amplitudes obtained from Eq.(1) to the laboratory frame, since the dispersion relations in which these data are used are normally written for the laboratory amplitudes. In Tables 1 and 3 we present for $\pi^- p$ and $\pi^+ p$ scattering respectively, the magnitudes of these real parts, $|\mathrm{Re}\, f|$, their errors, $\Delta\, \mathrm{Re}\, f$, the magnitudes of the ratio α, $|\alpha|$, their errors, $\Delta\alpha$, using the pion laboratory momentum K as the kinetic variable. The errors $\Delta\, \mathrm{Re}\, f$ and $\Delta\alpha$ take into account the estimated errors on both $(d\sigma/d\Omega)_o$ and σ_{tot} in Eq.(1). In Tables 2 and 4 we present for $\pi^- p$ and $\pi^+ p$ scattering respectively, the measured values of α, their errors, $\Delta\alpha$, the values of the real parts, $\mathrm{Re}\, f$, obtained from (8) and their errors $\Delta\, \mathrm{Re}\, f$. The errors $\Delta\, \mathrm{Re}\, f$ in this case take into account the estimated errors on both α and $\mathrm{Im}\, f$ in Eq.(8).

We use the natural units (n.u.), $h=c=m_\pi=1$. If other units are preferred one can make use of the relations 1Ref[fm] = 1.41 Ref[n.u.], 1 Ref[GeV.mb] = 2.79 Ref[n.u.].

In many cases, the central value of $(\mathrm{Re}\, F)^2$ calculated from Eq.(1) turned out to be negative, although the calculated error was large enough to allow also a reasonable range of positive values. In these cases, we set $\mathrm{Re}\, F=0$ with a smaller error. If, however, it was found that no positive values were included within the error limits, we concluded that the data on $(d\sigma/d\Omega)_o$ and σ_{tot} at the energy in question are incompatible (probably because their errors have been underestimated). Since we have no well-defined procedure for enlarging the errors in these cases, we merely indicate these points in Tables 1 and 3 by means of an asterisk and no values for the real parts are given for them.

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Table I. Magnitudes of the real parts of the $\pi^- p$ forward scattering amplitude in the laboratory system and of $\alpha_- \equiv Ref_-/Imf_-$, as a function of the pion laboratory momentum k. The optimal number of parameters in the expansion (3) and the quantity χ^2/(number of degrees of freedom) are given in those cases in which no extrapolation of the differential cross section was made in the original paper reporting the experimental data.

k (GeV/c)	Reference	|Ref| (n.u.)	Δ Ref (n.u.)	|α|	Δα	Number of param.	$\frac{\chi^2}{N}$
0.256	[7]	0.34	0.04	0.74	0.09	4	I.I2
0.336	[8]	0.09	0.07	0.I9	0.I5		
0.342	[9]	0.30	0.04	0.64	0.08		
0.406	[9]	0.29	0.03	0.82	0.I0		
0.427	[I0]	0.I0	0.06	0.30	0.I9	4	0.87
0.490	[9]	0.07	0.I5	0.I8	0.40		
0.490	[II]	0.26	0.04	0.68	0.I2	4	0.79
0.532	[II]	0.I6	0.26	0.38	0.6I	4	0.69
0.532	[I2]	0.22	0.04	0.5I	0.09	4	0.72
0.549	[9]	0.00	0.22	0.00	0.48		
0.557	[I3]	0.00	0.I5	0.00	0.33		
0.573	[II]	0.I8	0.04	0.37	0.08	4	I.3I
0.583	[I4]$^{x)}$	-	-	-	-		

0.6I0	[I3]	0.33	0.2I	0.59	0.38		
0.6I4	[II] x)	-	-	-	-	4	I.48
0.6I4	[I2]	0.I7	0.06	0.30	0.I0	3	I.06
0.658	[I5] x)	-	-	-	-	4	I.29
0.66I	[I3]	0.05	I.4I	0.07	I.56		
0.675	[II] x)	-	-	-	-	5	I.I4
0.675	[I6]	0.30	0.50	0.37	0.62	5	0.5I
0.683	[I7] x)	-	-	-	-		
0.699	[I3]	0.2I	0.27	0.23	0.29		
0.707	[I5] x)	-	-	-	-	4	I.76
0.726	[II]	0.05	0.I9	0.05	0.20	4	I.43
0.726	[I6] x)	-	-	-	-	4	I.62
0.730	[I8]	0.00	0.I3	0.00	0.I4		
0.750	[I3]	0.32	0.I7	0.35	0.I9		
0.773	[I9]	0.00	0.24	0.00	0.26	6	I.28
0.777	[II]	0.28	0.39	0.32	0.44	7	I.I3
0.799	[I3]	0.00	0.26	0.00	0.30		
0.826	[I5] x)	-	-	-	-	5	0.73
0.848	[I6]	0.00	0.I7	0.00	0.I9		
0.855	[I3]	0.00	0.33	0.00	0.37	6	2.25
0.875	[20]	0.00	0.I6	0.00	0.I7	7	I.89
0.904	[22] x)	-	-	-	-		
0.922	[I3]	0.00	0.40	0.00	0.32		
0.925	[20]	0.I2	I.49	0.09	I.I6	8	I.40
0.956	[I3] x)	-	-	-	-		
0.975	[20]	0.70	0.2I	0.43	0.I3	8	0.46
0.979	[I3] x)	-	-	-	-		

0.995	[I3]	0.00	0.I5	0.00	0.09		
I.000	[20]	0.22	0.68	0.I3	0.39	8	0.75
I.003	[I5]	0.66	0.I9	0.38	0.II	7	I.I6
I.004	[I3] x)	-	-	-	-		
I.024	[I3]	0.2I	0.4I	0.I2	0.24		
I.030	[20] x)	-	-	-	-	7	I.72
I.030	[I6]	0.27	0.4I	0.I6	0.25	8	0.60
I.042	[I3]	0.66	0.I5	0.40	0.09		
I.046	[23] x)	-	-	-	-		
I.055	[20]	0.37	0.98	0.23	0.6I	9	I.II
I.068	[I3]	0.63	0.2I	0.4I	0.I3		
I.080	[20]	0.69	0.I7	0.46	0.I2	8	I.09
I.088	[I3]	0.58	0.29	0.40	0.20		
I.I07	[I3]	0.37	0.25	0.27	0.I8		
I.I2I	[I5]	0.77	0.09	0.58	0.07	7	0.49
I.I25	[I3]	0.68	0.09	0.5I	0.06		
I.I5I	[I6]	0.45	0.I5	0.34	0.I2	7	0.60
I.I65	[I3]	0.I4	0.74	0.II	0.57		
I.I67	[I3]	0.39	0.I7	0.30	0.I3		
I.I74	[I3]	0.69	0.I2	0.54	0.09		
I.I80	[20]	0.09	I.08	0.07	0.85	8	2.05
I.2I4	[I3] x)	-	-	-	-		
I.25I	[I3]	0.35	0.43	0.27	0.32		
I.260	[I3]	0.22	0.38	0.I6	0.29		
I.280	[I3]	0.00	0.I8	0.00	0.I4		
I.280	[20]	0.00	0.44	0.00	0.33	8	2.05
I.323	[I3]	0.00	0.23	0.00	0.I7		
I.332	[22]	0.I9	0.83	0.I4	0.60		

I.343	[I3] x)	-	-	-	-		
I.360	[20] x)	-	-	-	-	6	I.54
I.38I	[I3] x)	-	-	-	-		
I.407	[I3] x)	-	-	-	-		
I.440	[20] x)	-	-	-	-	7	I.I0
I.446	[I3] x)	-	-	-	-		
I.470	[I3] x)	-	-	-	-		
I.505	[I3]	0.00	0.23	0.00	0.I5		
I.505	[20] x)	-	-	-	-	8	0.68
I.509	[I3] x)	-	-	-	-		
I.568	[I3] x)	-	-	-	-		
I.579	[20] x)	-	-	-	-	7	I.I3
I.590	[24]	0.00	0.I9	0.00	0.I2		
I.603	[I3] x)	-	-	-	-		
I.700	[26] x)	-	-	-	-	6	I.I9
I.700	[27]	0.00	0.I6	0.00	0.I0	I0	0.75
I.720	[25] x)	-	-	-	-	8	I.I5
I.880	[26]	0.00	0.I2	0.00	0.06	9	I.05
2.000	[28] x)	-	-	-	-	I0	I.I0
2.070	[25] x)	-	-	-	-	I0	0.72
2.269	[25] x)	-	-	-	-	I0	0.95
2.460	[25] x)	-	-	-	-	I0	0.80
2.500	[26] x)	-	-	-	-	9	0.82
3.I50	[23]	0.00	0.47	0.00	0.I6		
4.I30	[23]	0.96	0.47	0.27	0.I3		
4.950	[23]	I.43	0.49	0.35	0.I2		
8.500	[29]	I.0I	0.87	0.I5	0.I3		

I2.400	29	0.90	I.79	0.I0	0.I9
I8.400	29	4.63	2.28	0.35	0.I7

x) In these cases, the value of $(ReF)^2$ calculated from Eq.(1) was found to be negative within the errors.

Table 2. The measured values of $\alpha_- \equiv \mathrm{Re}f_-/\mathrm{Im}f_-$ and of the real parts of the $\pi^- p$ forward scattering amplitude in the laboratory system, as a function of the pion laboratory momentum k.

k (GeV/c)	Reference	Ref (n.u.)	Δ Ref (n.u.)	α	$\Delta\alpha$
I.9I	[45]	-0.20	0.08	-0.I0	0.04
2.44	[45]	-0.34	0.I0	-0.I4	0.04
3.06	[46]	-0.47	0.I4	-0.I7	0.05
3.48	[47]	-0.54	0.22	-0.I7	0.07
4.00	[2I]	-I.I5	I.05	-0.33	0.30
4.I7	[45]	-0.54	0.I8	-0.I5	0.05
4.56	[46]	-0.62	0.I6	-0.I6	0.04
4.95	[45]	-0.58	0.I6	-0.I4	0.04
5.65	[45]	-0.55	0.I8	-0.I2	0.04
6.I3	[47]	-0.55	0.I8	-0.22	0.09
7.89	[4]	-0.764	0.I62	-0.I23	0.026
9.84	[4]	-0.967	0.I44	-0.I28	0.0I9
9.89	[4]	-I.I92	0.I82	-0.I57	0.024
II.89	[4]	-I.092	0.I6I	-0.I22	0.0I8
I4.I6	[4]	-I.I84	0.262	-0.II3	0.025
I5.99	[4]	-I.487	0.222	-0.I27	0.0I9
I8.I9	[4]	-I.49I	0.356	-0.II3	0.027
20.I5	[4]	-I.452	0.378	-0.I00	0.026
20.38	[4]	-I.746	0.279	-0.II9	0.0I9
22.I3	[4]	-I.76I	0.428	-0.III	0.027
24.22	[4]	-2.I26	0.50I	-0.I23	0.029
26.23	[4]	-2.593	0.54I	-0.I39	0.029

Table 3. Magnitudes of the real parts of the π^+p forward scattering amplitude in the laboratory system and of $\alpha_+ \equiv \mathrm{Ref}_+/\mathrm{Imf}_+$. The conventions are the same as in Table I.

k (GeV/c)	Reference	|Ref| (n.u.)	ΔRef (n.u.)	|α|	Δα	Number of param.	χ^2/N
0.36I	[30]	I.07	0.0I	0.96	0.0I	4	2.22
0.427	[3I]	I.08	0.0I	I.52	0.0I	4	2.23
0.490	[II]	0.93	0.0I	I.80	0.02	3	0.79
0.532	[II]	0.94	0.05	2.I7	0.I0	5	I.36
0.532	[I2]	0.90	0.02	2.07	0.04	4	0.99
0.573	[II]	0.89	0.04	2.30	0.I0	5	0.9I
0.573	[32]	0.76	0.06	I.96	0.I6		
0.6I4	[II]	0.69	0.08	I.97	0.23	5	2.I0
0.6I4	[I2]	0.76	0.02	2.I7	0.05	5	I.2I
0.658	[33]	0.69	0.06	2.I3	0.I9	5	0.78
0.675	[II]	0.60	0.05	I.90	0.I5	5	I.0I
0.707	[33]	0.58	0.05	I.86	0.I6	5	2.3I
0.726	[34]	0.59	0.04	I.89	0.I2		
0.726	[II]	0.56	0.05	I.79	0.I5	6	I.30
0.726	[35]	0.63	0.II	2.02	0.35	6	I.23
0.777	[II]	0.42	0.03	I.29	0.I0	5	2.I4
0.826	[33]	0.36	0.I3	0.93	0.35	6	0.83
0.875	[20]	0.34	0.05	0.72	0.II	5	0.93
0.9I0	[36]	0.I7	0.22	0.3I	0.4I		
0.925	[20]	0.00	0.20	0.00	0.35	6	I.3I
0.949	[35]	0.25	0.I2	0.40	0.I9	4	0.89
0.950	[20]	0.I3	0.23	0.2I	0.37	6	I.I8
0.975	[20]	0.00	0.25	0.00	0.39	7	I.43

I.000	[20]x)	-	-	-	-	6	I.I0
I.003	[33]	0.00	0.33	0.00	0.47	6	I.58
I.030	[35]	0.25	0.I4	0.34	0.I9	4	I.27
I.030	[20]	0.II	0.6I	0.I5	0.84	7	I.50
I.030	[37]	0.27	0.05	0.37	0.07		
I.080	[20]	0.32	0.33	0.40	0.4I	8	I.35
I.I2I	[33]	0.69	0.I3	0.8I	0.I5	7	0.86
I.I3I	[38]x)	-	-	-	-	4	0.77
I.I80	[20]	0.00	0.46	0.00	0.47	8	I.93
I.I8I	[35]x)	-	-	-	-	5	0.I9
I.232	[39]x)	-	-	-	-	5	4.20
I.280	[20]x)	-	-	-	-	7	I.07
I.360	[20]	0.07	3.93	0.04	2.57	8	I.08
I.444	[33]	0.54	0.29	0.32	0.I7	7	I.06
I.500	[40]x)	-	-	-	-	8	I.26
I.579	[20]	I.07	0.24	0.64	0.I4	8	0.59
I.600	[4I]	0.57	0.27	0.35	0.I6		
I.689	[33]	0.67	0.II	0.42	0.07	7	0.63
I.760	[42]	0.82	0.09	0.52	0.06		
2.000	[28]x)	-	-	-	-	8	I.78
2.000	[40]	0.00	0.59	0.00	0.36	8	0.67
2.080	[42]	0.00	0.30	0.00	0.I7		
2.500	[40]x)	-	-	-	-	9	I.I4
2.920	[43]x)	-	-	-	-		
8.500	[29]	0.00	I.5I	0.00	0.25		
I2.400	[29]	0.00	2.00	0.00	0.23		

x) In these cases, the value of $(ReF)^2$ calculated from Eq.(1) was found to be negative within the errors.

Table 4. The measured values of $\alpha_+ \equiv Ref_+/Imf_+$ and of the real parts of the π^+p forward scattering amplitude in the laboratory system, as a function of the pion laboratory momentum k.

k (GeV/c)	Reference	Ref (n.u.)	Δ Ref (n.u.)	α	$\Delta\alpha$
7.76	[4]	-I.I94	0.I30	-0.2I2	0.023
9.86	[4]	-I.55I	0.I54	-0.22I	0.022
I0.02	[4]	-I.43I	0.I00	-0.20I	0.0I4
II.95	[4]	-I.567	0.II7	-0.I87	0.0I4
I4.00	[4]	-I.84I	0.233	-0.I90	0.024
I6.02	[4]	-I.870	0.242	-0.I70	0.022
I7.96	[4]	-I.753	0.3I9	-0.I43	0.026
20.I9	[4]	-2.470	0.494	-0.I80	0.036