
K.D. Tolstov

COMPARISON OF NEW DATA ON P-P ELASTIC SCATTERING<br>AT 15-30 GEV/C<br>WITH CALCULATIONS BASED<br>ON THE MODEL OF THE DISCRETE VALUES $<\mathbf{P}_{1}^{2>1 / 2}$

The experimental data on $p-p$ elastic scattering at $>7 \mathrm{GeV}$ in a wide range of angles - from small ones (when the Coulomb scattering contribution is unessential) to angles of $\sim 70^{\circ}$ (when differential cross sections at a given energy tend to saturation) have been described in refs. $/ 1,2 /$ by the formula:
$\frac{\mathrm{d} \sigma}{\mathrm{d} \cdot|\mathrm{i}|}=\left\{\mathrm{C}_{1} \exp \left(\frac{\mathrm{t}+\frac{\mathrm{t}^{2}}{4 \mathrm{p}^{2}}}{\left\langle\mathrm{p}_{\square}^{2}\right\rangle}\right)+\mathrm{C}_{2} \exp \left(\frac{\mathrm{t}+\frac{\mathrm{t}^{2}}{4 \mathrm{p}^{2}}}{4\left\langle\mathrm{p}_{\square}^{2}\right\rangle}\right)+\mathrm{C}_{3} \exp \left(\frac{\mathrm{t}+\frac{\mathrm{t}^{2}}{4 \mathrm{p}^{2}}}{9\left\langle\mathrm{p}_{\square}^{2}\right\rangle}\right)+\right.$

$$
+2\left(C_{1} C_{2}\right)^{1 / 2} \cos \phi_{2} \exp \left(\frac{t+\frac{t^{2}}{4} p^{2}}{\frac{8}{5}\left\langle p_{1}^{2}\right\rangle}\right)+2\left(C_{1} C_{3}\right)^{1 / 2} \cos \phi_{3} \exp \left(\frac{1+\frac{1^{2}}{4 p^{2}}}{-\frac{9}{5}-\left\langle p_{1}^{2}\right\rangle}\right)+
$$

$$
\left.+2\left(\mathrm{C}_{2} \mathrm{C}_{3}\right)^{1 / 2} \cos \left(\phi_{2}-\phi_{3}\right) \exp \left(\frac{\mathrm{t}+\frac{\mathrm{t}^{2}}{4 \mathrm{p}^{2}}}{\left.\frac{72}{\frac{13}{}\left\langle\mathrm{p}^{2}\right.}\right)}\right)\right]\left[1+\frac{\mathrm{t}}{2 \mathrm{p}^{2}}\right] .
$$

where $p$ is the momentum in the c.m.s. This formula has been obtained on the assumption of the interference of partial waves due to discrete effective scattering radii. This is equivalent to the condition:

$$
\begin{equation*}
<P_{1}^{2}>_{1}^{1 / 2}:<P_{\perp}^{2}>_{2}^{1 / 2}:<P_{1}^{2}>_{3}=1: 2: 3 \tag{2}
\end{equation*}
$$

where $\left\langle P_{L}^{2}\right\rangle^{1 / 2}$ is the root-mean-square transversal momenta. In ref. $/ 3 / p-p$ elastic scattering has been studied at 15,20 and 30 $\mathrm{GeV} / \mathrm{c}$. In this case the region $/ \mathrm{t} / \approx 1(\mathrm{GeV} / \mathrm{c})^{2}$ has been inverstigated in detail. Figure 1 presents the curve obtained for the following parameters by formula (1): $\left\langle\mathrm{p}_{1}^{2}\right\rangle^{1 / 2}=0.345 \mathrm{GeV} / \mathrm{c} ; \quad \mathrm{C}_{1}, \quad \mathrm{C}_{2}, \quad \mathrm{C}_{3}$ are equal to $88 \cdot 10^{-27} \mathrm{~cm}^{2} /(\mathrm{GeV} / \mathrm{c})^{2} ; 0.15 \cdot 10^{-27} \mathrm{~cm}^{2} /(\mathrm{GeV} / \mathrm{c})^{2} ; 0.001 \cdot 10^{-27}$ $\mathrm{cm}^{2} /(\mathrm{GeV} / \mathrm{C})^{2}$, respectively; $\phi_{2}=150^{\circ} ; \quad \phi_{3}=0^{\circ}$. The experimental data in Fig. 1 are taken at $20 \mathrm{GeV} / \mathrm{c}_{\text {from }} / 3 /$ and at $19.2 \mathrm{GeV} / \mathrm{c}$ from $/ 4 /$. Figure 2 shows the curves illustrating contributions of ditferment exponents from formula (1) at $19.2 \mathrm{GeV} / \mathrm{C}$. As is seen from Fig. 2, exponent 1 , involving $C_{1}$, gives a main contribution to $\frac{d \sigma}{d|t|}$ for $P_{L} \approx 0.5 \mathrm{GeV} / \mathrm{c}$; for $P_{L} \approx 1.5 \div 2 \mathrm{GeV} / \mathrm{c} \frac{\mathrm{d} \sigma}{\mathrm{d} . \mathrm{t} . \mid}$ is in the main determined by exponent 2 with $\mathbf{C}_{2}{ }^{\circ}$, and for $\mathbf{P}_{1}>2.5 \mathrm{GeV} / \mathrm{c}-$ by exponent 3 with $\mathrm{C}_{3}$.
$\overline{x /}$ The general view of formula (1) at $\left\langle P_{L^{2}}^{1 / 2}: \ldots:\left\langle P^{2}\right\rangle=1 \ldots . n\right.$

$$
\begin{aligned}
& \frac{d \sigma}{d|l|}=\sum_{k=1}^{n} C_{k} \exp \left(\frac{t+\frac{t^{2}}{4 p^{2}}}{\left\langle P^{2}\right\rangle}\right)+2 \sum_{k=2}^{n}\left(C_{1} C_{k}\right)^{1 / 2} \cos \phi_{k} \exp \left[\frac{t+\frac{t^{2}}{4 p^{2}}}{\left\langle P_{1}^{2}\right\rangle}\left(1+\frac{1}{k^{2}}\right)\right]+ \\
& +\sum_{k=2, m=1}^{k+m<n}\left(C_{k} C_{k+m}\right)^{1 / 2} \cos \left(\phi_{k}-\phi_{k+m}\right) \exp \left[\left(\frac{t+\frac{t^{2}}{4 p^{2}}}{\left\langle P_{1}^{2}\right\rangle}\right)\left(-\frac{1}{k^{2}+} \frac{1}{(k+m)^{2}}\right]\right\}\left[1+\frac{1}{2 p^{2}}\right] .
\end{aligned}
$$



Fig. 1.


Fig. 2.

Consequently, condition (2) is a decisive factor in determining $\frac{d \sigma}{d|i|}$ for the given values of $P_{1}$. In a region of $P_{1} \approx 1 \mathrm{GeV} / \mathrm{c}$ according to Fig. 2 three exponents: 1,2 and (1,2), which involve the interference term $\left(\begin{array}{llll}C_{1} & C_{2}\end{array}\right)^{1 / 2} \quad \cos \phi_{2}$, come closer together. In ref. $/ 3 /$ at $29.7 \mathrm{GeV} / \mathrm{c}$ and $/ \mathrm{t} / \approx 1.3$ the minimum is pronounced for $\frac{\mathrm{d} \sigma}{\mathrm{d}|\mathrm{t}|}$. However, as the authors note, it is not statistically presented.


Fig. 3.
Figure 3 gives the experimental data and diagrams from ref. $/ 3 /$ with parameters $C_{1}, C_{2}$ and $\phi_{3}$ having the above mentioned values; $\left\langle P_{L}^{2}\right\rangle^{1 / 2}=0.340 ; \quad \phi_{2}=160^{\circ} ; \quad C_{2}=0.13$ for curve 2 and 0.10 for
for curve 3. Curve 1 presents (in a large scale) a part of the curve from Fig. 1 and calculations as well:

FM is the $30-\mathrm{BeV} / \mathrm{C}$ prediction of Frautschi and Margolis; $C F$ is the $25-B e V / C$ cross section of Chiu and Finkelstein; ADG is the asymptotic cross section of Abarbanel, Drell and Gilman.
In a diffraction peak region $\frac{d \sigma}{d|t|}$ was described by the phenomenological formula:

$$
\begin{equation*}
\frac{\mathrm{d} \sigma}{\mathrm{~d}} \mathrm{t}=\exp \left(\mathrm{a}+\mathrm{bt}+\mathrm{ct}{ }^{2}\right) \tag{3}
\end{equation*}
$$

At $/ \mathrm{t} /<1 \mathrm{GeV} / \mathrm{c}$ the factor $1+\frac{i}{2 \mathbf{p}^{2}}$ in (1) is close to 1 and comparing formula (1) with (3) we obtain the relation between " b " and "c":

$$
\begin{equation*}
\mathrm{C}=\frac{\mathrm{b}}{4 \mathrm{p}^{2}} \tag{4}
\end{equation*}
$$

Table presents the data given in $/ 3 /$.

Table

| Refs. | Momentum $\mathrm{GeV} / \mathrm{c}$ | $\begin{aligned} & \text { Range } \\ & / \mathrm{t} / \mathrm{GeV} / \mathrm{c})^{2} \end{aligned}$ |  |  | b | c |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 15.1 | 0.22 | 7 T | 0.78 | $7.89 \pm 0.59$ | -0.43+0.59 |
| 5 | 18.4 | 0.2 | /t/ | 0.5 | $8.58 \pm 0.24$ |  |
| 6 | 19.84 | 0.2 | /t/ | 0.8 | $8.68 \pm 0.79$ | $0.70 \pm 0.92$ |
| 3 | 20.0 | 0.2 | /t/ | 0.8 | $9.15 \pm 0.45$ | $0.72 \pm 0.45$ |
| 6 | 24.63 | 0.25 | /t/ | 0.75 | $7.97 \pm 1.56$ | $0.82 \pm 1.83$ |
| 3 | 29.7 | 0.21 | /t/ | 0.73 | $8.02 \pm 0.60$ | -0.64 0.63 |

It follows from the table that in a range of $15 \div 30 \mathrm{GeV} / \mathrm{c}$ and $0.2</ \mathrm{t} /<0.8 \mathrm{bb"}$ is constant within the limits or errors, and its weight-average value is $8.5 \pm 0.4$. The weight-average value of " c " is $\approx 0.2$ (this value agrees with the estimate from (4); however, the averaging is rough because of a large spread in values). The value of "b" being equal to $8.5 \pm 0.4$ is in agreement with the estimate of $b \approx \frac{1}{\left\langle P^{2}\right\rangle} 8.3$, with the help of which the experimental data in Figs. 1 and 3 are fairly well described. In ref. $\mid 7 /$ when measuring the slope parameter "b" at $12+70 \mathrm{GeV}$ the following formula was defined:

$$
\begin{equation*}
b=\left[(6,8+0,3)+(0,94+0,18) \ln \frac{s}{s_{0}}\right](\mathrm{GeV} / \mathrm{c})^{2}, \tag{5}
\end{equation*}
$$

where "S" is the square of the total energy in the c.m.s. $s_{0}=1 \mathrm{GeV} / \mathrm{c}_{\text {. }}$ From (5) at $30 \mathrm{GeV} / \mathrm{c}$ we obtain $b=10.6 \pm 0.8$ i.e. the difference of about two root-mean-square errors with the above-mentioned values of " b ". This difference is possibly connected "with increasing "b" at small values of $/ t /$, since in ${ }^{/ 7 /}$ the measurements have been mäde at $0.01 \mathrm{lt} / 0.12(\mathrm{GeV} / \mathrm{c})^{2}$. This fact was also noted in $/ 3 /$.

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