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S.I.Bilenkaya, J.Bystrický, Z.Janout,
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N-N SCATTERING PHASE - SHIFT ANALYSIS AT ENERGIES NEAR 20 MeV WITH A NEW METHOD OF PHASE - SHIFT SETS TESTING

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N-N SCATTERING PHASE-SHIFT ANALYSIS AT ENERGIES NEAR 20 MeV WITH A NEW METHOD OF PHASE-SHIFT SETS TESTING

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1. Introduction

Recently, the N-N elastic scattering data at energies near 20 MeV was considerably increased. The accurate data (accuracy about 0.001) of polarization in p-p scattering /1/, the spin correlation coefficient C_{nn}^{pp} /2/, polarization and spin-correlation in the n-p scattering /3/ and of the total n-p cross section /4/ at energies about 20 MeV were measured.

It is interesting to note, that the Slobodrian's data measured in Berkeley /5/ does not coincide with the highly accurate data of Saclay /1/.

In the present paper the results of phase-shift analysis, performed, while taking into account the new data, are discussed. One of our aims is to show, which p-p polarization data agrees best with the whole set of the N-N experimental data.

A new method for the discrimination of the obtained phase-shifts sets previously published in ref. /6/ was performed. The description of the principles and the performance of this method (the so called r - criterion) for the concrete case of phase-shift analysis is given.

2. Experimental Data

The experimental quantities used in phase-shift analysis are given in Table 1. From the whole number of 84 points there are 40 points measured in p-p scattering and 44 points in n-p scattering. All the experimental data is given in Table 2.

The phase-shift analysis was performed at the effective energy of 23.1 MeV. Therefore it was necessary to transform the known data of effective cross section of p-p and n-p scattering to this energy. Since the total cross section at 23.1 MeV is unknown, we made the normalization of the p-p cross section data in the following way.

The data of σ_{pp} at 90° in the energy region 9.7 - 44 MeV (see Tab. 3) was approximated by the function

$$f(E, 90^{\circ}) = a_0 + \frac{a_1}{E} + \frac{a_2}{E^2}, \qquad (1)$$

where E is the energy in 10^{-2} MeV. The values of the coefficients: $a_0 = -1.185 \pm 0.196$, $a_1 = 4.572 \pm 0.088$ and $a_2 = 0.097 \pm 0.008$ were obtained by using the least squares method.

The obtained dependence of $f(E, 90^{\circ})$ on energy is shown in fig.1.

For the normalization of σ_{pp} data, measured at 25.63 MeV to the effective 23.1 MeV, we need the ratio of the calculated values

$$\frac{f(23.1\ \mathrm{MeV},\ 90^\circ)}{f(25.63\ \mathrm{MeV},\ 90^\circ)} = 1.126. \qquad (2)$$

All the data of σ_{pp} at 25.63 MeV was multiplied by this value.

The data of n-p effective cross section at 22.5 MeV/ 7,8/ and 27.5 MeV/ 8/ was normalized, so that the total cross section σ_{tot}^{np} is equal to (397.2 ± 1.7) mb/ 4/.

For this purpose the following functional has been minimized by the linearisation method/ 9/

$$\chi^2 = \mathrm{Min}\left[\sum_k \sum_i \left(\frac{\sigma_i^k - N_k f}{\Delta_i^k}\right)^2 + \left(\frac{\sigma_{tot} - 2\pi \int_0^\pi f \sin\nu \, d\nu}{\Delta_{tot}}\right)^2\right], \qquad (3)$$

where

$$f = f(\cos\nu) = a_0 + a_2 \cos^2\nu . \qquad (4)$$

The index i denotes the number of experimental points, k ($k = 1, 2, 3$) is the experiment number and a_0, a_2, N_k are the variable coefficients, for which we obtained the following values:

$a_0 = 30.89 \pm 0.34$

$a_2 = 2.14 \pm 0.94$

$N_1 = 1.065 \pm 0.015$

$N_2 = 1.048 \pm 0.020$

$N_3 = 0.865 \pm 0.014$

The experimental data σ_{np}, measured by Flynn at 22.5 MeV[7], by Scanlon et al. (at 22.5 MeV and 27.5)[8] was divided by N_1, N_2 and N_3, respectively.

The phase-shift analyses were performed at $\ell_{max} = 2$ and $\ell_{max} = 3$ according to the programme described in ref.[10]. The phase-shifts in the states with $\ell > \ell_{max}$ were taken into account by one pion exchange approximation.

The data of P_{pp}, measured in Saclay[1] was included also into the phase-shift analysis at 18.2 MeV, together with the previously known data[11,12]. To this data the total n-p cross section $\sigma_{tot}^{np} = 493.3 \pm 2.4$ mb measured at 19.565 MeV[4] and the last data of C_{nn} and A_{ss} measured in Saclay[2] were added.

3. *Phase-Shift Analysis*

The experimental data at 23.1 MeV is described by two different sets of phase-shifts with approximately equal values of χ^2. The first set corresponds to solution 1 of ref.[13], the second set corresponds to the solution C, of Arndt and Mac-Gregor[14]. Only 11 coeffeicients ($\ell_{max} = 2$, f^2 – fixed) are sufficient for the good description of experimental points. These can be seen from the values of $\chi^2/\bar{\chi}^2$, which are equal to 0.95 and 0.88 for the first and second sets, respectively. Both sets of phase-shifts are given in Table 4. From this table it follows that both the solutions differ, mainly in the wave $^3S_1$ and the other phase-shifts coincide within their errors. At

$\ell_{max}=2$ the χ^2- value is lower for the second set than for the first one.

It is interesting that for $\ell_{max}=3$ we get for the first set a lower χ^2 - value than for the second one, as it is shown in Table 5. The coupling constant f^2 was fixed and taken equal to 0.08.

Varying the coupling constant f^2, the χ^2-value for the second set remains approximately the same. On the contrary the χ^2-value for the first set considerably decreases (see Table 6). The calculated values of f^2 at 23.1 MeV in both solutions differ from the value of 0.08, measured in π - N scattering.

By specifying the three previously known sets of phase-shifts at 18.2 MeV /11,12/ the results given in Table 7 were obtained. From this table it follows that the χ^2-value for the first and the second sets remained nearly the same (15.75 and 15.78, respectively) but for the third set the χ^2- value considerably increased (up to 26.91).

The question of the unambiguity of the phase-shift analysis at both energies was discussed on the basis of the criterion/6/, described in the next section. Here, the principles of the new criterion are generally discussed.

4. Discrimination Criterion

We are speaking about an ambiguous phase-shift analysis if obtaining several relative maxima of the likelihood function with nearly equal amplitude (approximately equal χ^2 -values). For rejecting a false set of phase-shifts, various methods of mathematical statistics were used, mainly the χ^2-test. However this test is not sufficiently powerful, as has been shown previously (see ref./15,16/.) This means that the χ^2-test needs a superfluous amount of experiments for rejecting a false phase-shift set. It seems reasonable to base the test on the difference Δ of the χ^2-values, i.e. on the logarithm of the likelihood ratio/17,6/ (the ratio of the relative maxima of the likelihood function). Such a test is described in ref./16/ but calculation of the Type I error seems to be rather complicated and gives only a rough upper estimate of the Type I error.

In the present paper the r-criterion, described in detail in ref./6/, is used. This criterion is a modification of the likelihood ratio test, which is more effective than the one described in/16/. Comparison of these two criteria on a certain example will be described further.

Let us characterize the main features of the r-criterion for testing phase-shift sets. We denote by $y_1,\dots,y_n$ the experimental data (as effective errors section, polarization etc.), $\eta_1(\vec{\theta}),\dots\eta_n(\vec{\theta})$ the respective calculated values for the phase-shifts, $\vec{\theta}=(\theta_1,\dots,\theta_n)$, $\sigma_1,\dots\sigma_n$ the experimental errors, $\hat{\vec{\theta}}$ and $\hat{\vec{\Phi}}$ two discussed phase-shift sets. This means that for the vectors $\hat{\vec{\theta}}$ and $\hat{\vec{\Phi}}$ the relative minima of

$$\chi^2(\vec{\theta})=\sum_{i=1}^{n}\frac{1}{\sigma_i^2}[y_i-\eta_i(\vec{\theta})]^2 \qquad (5)$$

are reached. Let us suppose that $\chi^2(\hat{\vec{\theta}})>\chi^2(\hat{\vec{\Phi}})$.

For the correctness of the χ^2 -criterion, of the criterion described in ref./16/ and of the r -criterion, the following quasilinearity assumption is necessary. In the neighbourhoods of $\hat{\vec{\theta}}$ and $\hat{\vec{\Phi}}$ given by the respective ellipsoides of concentration, the non linear functions $\eta_i(\vec{\theta}), (i=1,..,n)$ can be approximated by:

$$\eta_i(\vec{\theta}) = \sum_{k=1}^{m} F_{ik}(\theta_k - \hat{\theta}_k) + \eta_i(\hat{\vec{\theta}})$$

$$\eta_i(\vec{\theta}) = \sum_{k=1}^{m} G_{ik}(\theta_k - \hat{\Phi}_k) + \eta_i(\hat{\vec{\Phi}}) \tag{6}$$

$$i = 1, \dots, n$$

in the neighbourhood of $\hat{\vec{\theta}}$ and $\hat{\vec{\Phi}}$, respectively.

Here

$$F_{ik} = \frac{\partial \eta_i(\vec{\theta})}{\partial \theta_k} \Big|_{\vec{\theta} = \hat{\vec{\theta}}}$$

$$G_{ik} = \frac{\partial \eta_i(\vec{\theta})}{\partial \theta_k} \Big|_{\vec{\theta} = \hat{\vec{\Phi}}} \tag{7}$$

Besides these actual sets $\hat{\vec{\theta}}$ and $\hat{\vec{\Phi}}$ an auxiliary set $\hat{\vec{\Phi}}_{\hat{\theta}}$ is used. This is the set for which the minimum

$$\min_{\vec{\theta}} \sum_{i=1}^{n} \frac{1}{\sigma_i^2} \left[\eta_i(\hat{\vec{\theta}}) - \eta_i(\hat{\vec{\Phi}}) - \sum_{k=1}^{m} G_{ik} (\theta_k - \hat{\Phi}_k) \right]^2 \tag{8}$$

is obtained.

The r - test is based on the following quantity/ 6/

$$r = \frac{\sum_{i=1}^{n} \frac{1}{\sigma_i^2} [y_i - \eta_i(\hat{\vec{\theta}})] x_i}{\left[\sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} - \sum_{k,\ell=1}^{m} \left(\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} F_{ik} \right) D_{k\ell} \left(\sum_{j=1}^{n} \frac{x_j}{\sigma_j^2} F_{j\ell} \right) \right]^{1/2}}, \tag{9}$$

where

$$x_i = \eta_i(\hat{\vec{\Phi}}_{\hat{\theta}}) - \eta_i(\hat{\vec{\theta}}) \tag{10}$$

and $D_{k\ell}$ $(k, \ell = 1, \ldots m)$ are the elements of the matrix of errors which correspond to the set $\hat{\vec{\theta}}$. In ref/ 6/ the following basic result is proved. The probability

$$P\{r > k\} = \frac{1}{\sqrt{2\pi}} \int_k^{\infty} e^{-\frac{x^2}{2}} dx \tag{11}$$

under the hypothesis $\mathcal{H}_\theta$ that $\hat{\vec{\theta}}$ is the true solution (precisely that the true value of $\vec{\theta}$ is in the neighbourhood of $\hat{\vec{\theta}}$).

The integral in (11) is a well known normal distribution function tabulated in detail, e.g. in ref./18/.

We emphasize that $\hat{\vec{\theta}}$ is the set with the larger χ^2-value. That is why instead of (11) we have to use the inequality

$$P\{r > k \quad \text{and} \quad \chi^2_{\hat{\theta}} > \chi^2_{\hat{\Phi}}\} \leq \alpha \tag{12}$$

under the hypothesis $\mathcal{H}_\theta$.

Formula (12) follows from (11) and from the fact that for arbitrary events A and B the relation $P(A \text{ and } B) \leq P(A)$ is valid.

It can be shown, that for large values of k the probabilities in (11) and (12) differ only slightly.

The decision procedure is as follows. We can choose an available value of the Type I error α, and compute the respective value of k from the equation:

$$\frac{1}{\sqrt{2\pi}} \int_k^\infty e^{-\frac{x^2}{2}} dx = \alpha \tag{13}$$

If the computed value of r is larger than k, $\hat{\theta}$ is to be rejected, i.e. $\hat{\Phi}$ can be accepted. The probability that this decision is false (Type I error) is equal to α. If r is smaller than k, it is necessary to continue the experiments. The probability that this second decision is false, that is: $\hat{\Phi}$ is true and we are making unnecessary suplementary experiments, is the Type II error. Evidently the Type II error defines the power (the efficiency) of the test, and it is sufficient to know it only approximately. It is shown in /6/ that asymptotically (for a large number of experiments) the Type II error is minimal, i.e. the proposed test is asymptotically optimal, which is not true for the χ^2-test.

For the calculation of the value r the existing programme of the phase-shift analysis with a small additional part was used /10/. The block-scheme of this addition is given in fig. 2.

It is interesting to compare the time which is necessary for the calculation of the value r with that for the value Δ /16/.

For example, if we have 25 parameters and 153 experimental points, we need about 10 hours on a computer of M-20 type for the calculation of Δ, while for the calculation of r under the same conditions only about 7 minutes are necessary. These calculations were made for two sets of the phase-shift analysis at 400 MeV /19/ in order to compare the properties of χ^2, Δ and r-criteria.

The results are as follows:

criterion:	χ^2	Δ	r
α (see formula (12)):	52.8%	0.59%	0.02%

It can be seen, that according to the χ^2 -criterion no solution is to be preferred, while using the r -criterion, we can reject one of the solutions with a negligible small Type I error. This error is much smaller then the estimation of the Type I error obtained using the Δ - criterion.

5. Results

On the basis of the r - criterion an attempt was made to reject one of the existing solutions at 23.1 MeV. It appeared that for $\ell_{max}=2$ it is possible to reject the "worse" set (with the higher χ^2 -value) only with a probability of Type I error $\alpha = 1{,}05\%$. This seems to be rather a great probability and therefore discrimination of the two solutions needs additional experiments.

With the help of the two sets of phase-shifts the angular dependences of experimental quantities were calculated. The most interesting of them are plotted in figs. 3, 4. From the graphs it can be seen that the discrimination of the two sets will be possible only after measurements of n-p triple scattering parameters. The measurements at D_{np} and R_t seem to be most efficient.

The experimental data of the p-p polarization together with the calculated curve are shown in fig. 3a, (the curves for both the sets coincide). The Berkeley data /5/ is the cause of the increase of the χ^2 -value up to 72.72 and 71.58 for the first and second sets for $\ell_{max}= 3$, respectively. This fact gives evidence, that the data in question does not coincide with the other measurements.[x)]

At energy 18.2 MeV the second and third sets of phase-shifts were compared with the first one separately. The third set has the probability of Type I error equal to 2.11%. The same probability for the second set is equal to 77.4%. Both these probabilities are too high for rejecting the corresponding solutions and so need a continuation of experiments. As in the case discussed above, the triple scattering parameters could also give the possibility to discriminate the three remaining sets at energy 18.2 MeV.

Also at this energy the P_{pp} Berkeley data has not been described with any of the obtained sets of phase-shifts. (The χ^2 -value increases up to 52.5 for the first and second sets and up to 48.1 for the third set.)

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x) *The contribution of the Saclay P_{pp} data to the χ^2 -value is equal to 0.091 and 0.86 and that of the Berkeley is equal to 33.84 and 31.88 for the first and second sets respectively.*

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T a b l e 1

The Experimental Data Used in Phase-Shift Analysis at 23.1 MeV.

Measured quantity	Energy [MeV]	Angular Range c.m.s.	Number of Points	$\Delta\chi^2_{I}$ *)	$\Delta\chi^2_{II}$ *)	Authors	Refs.
σ_{pp}	25.63**)	10°–91°	23	9.236	9.342	Jeong et al.	[20]
P_{pp}	20.2	34°–90°	8	0.857	0.799	Catillon	[1]
	27.4***)	45°	1	0.034	0.045	Christmas	[21]
A_{pp}	27.6	23°–55°	3	1.676	1.760	Ashmore	[22]
R_{pp}	27.6	23°–55°	3	4.133	4.045	Ashmore	[22]
C^{pp}_{nn}	23.45	90°	1	0.160	0.053	Catillon	[2]
A^{pp}_{ss}	23.45	90°	1	0.014	0.001	Catillon	[2]
σ_{np}	22.5 +)	65°–175°	12	9.187	7.253	Flynn	[7]
	22.5 +)	7°–51°	6	7.163	9.389	Scanlon	[8]
	27.5 +)	7°–72°	8 ++)	2.224	1.312	Scanlon	[8]
σ^{np}_{t}	23.951		1	0.318	0.427	Groce	[4]
P_{np}	23.1	50°–150°	6	2.048	2.122	Perkins	[23]
	23.1	140°	1	0.053	0.030	Malanify	[24]
	23.1	70°–150°	3	0.141	0.071	Simmons	[3]
D_{np}	23.1	70°–110°	3	1.638	3.498	Perkins	[25]
C^{np}_{nn}	23.1	140°, 174°	2	0.211	0.259	Malanify	[24]
	23.1	130°, 150°	2	0.649	0.119	Simmons	[3]

*) The contributions to χ^2 for the 1-st and 2-nd sets (f^2 - fixed, $\ell_{max}=3$).

**) Corrected according to the ratio of the cross sections at 90° (see fig. 1)

***) This value should be multiplied by a factor of 0.89 according to O.N.Jarvis and B.Rose (see ref./26/).

+) All data were normalized according to the total cross section (ref./4/)

++) The values at angles 159°, 166° and 173° were not included in the phase-shift analysis.

T a b l e 2

The Experimental Data Used in Phase-Shift Analysis for pp- and np-Scattering at Energy 23.1 MeV.

Quantity	Energy [MeV]	ϑ° c.m.s.	Measured value	Statistical error ±	Refs.
σ_{pp}	25.63	10.07	109.6	1.8 *)	[20]
		12.08	56.31	1.1	
[mb/sr]		14.09	33.20	0.6	
		16.11	23.76	0.5	
		18.12	19.90	0.5	
		19.13	18.70	0.5	
		20.13	17.98	0.5	
		22.15	17.33	0.5	
		24.16	17.09	0.5	
		25.16	17.16	0.5	
		26.17	17.17	0.5	
		28.18	17.30	0.5	
		30.19	17.43	0.5	
		32.21	17.68	0.5	
		34.22	17.80	0.5	
		36.23	17.93	0.5	
		40.25	18.20	0.5	
		44.27	18.33	0.5	
		50.30	18.52	0.5	
		60.34	18.56	0.5	
		70.37	18.65	0.5	
		80.38	18.60	0.5	
		90.39	18.59	0.5	
P_{pp}	27.4	45.0	0.0031	0.0046	[21]**)

*) The relative errors in percent are given. The errors must be multiplied by a factor of 1.5 (see ref. /27/).

**) This value should be multiplied by a factor of 0.89 according to O.N. Jarvis and B. Rose (see ref. /26/).

T a b l e 2

Continuation

Quantity	Energy [MeV]	ϑ^o c.m.s.	Measured value	Statistical error ±	Refs.
P_{pp}	20.2	34.6	−0.0024	0.0030	[1]
		42.6	−0.0002	0.0012	
		46.0	0.0000	0.0016	
		49.6	0.0008	0.0016	
		60.0	0.0013	0.0010	
		65.6	0.0003	0.0016	
		66.0	0.0000	0.0020	
		90.0	−0.0001	0.0006	
A_{pp}	27.6	23.2	0.012	0.030	[22]
		39.0	0.037	0.025	
		54.6	0.090	0.022	
R_{pp}	27.6	23.2	−0.324	0.063	[22]
		39.0	−0.187	0.030	
		54.6	−0.243	0.026	
C_{nn}^{pp}	23.45	90.0	−0.791	0.019	[2]
A_{ss}^{pp}	23.45	90.0	−0.945	0.022	[2]
σ_{np} [mb/sr]	22.5	175	35.9	12.0 *)	[7]**)
		165	34.3	7.5	
		155	34.0	5.7	
		145	37.0	4.7	
		135	35.6	4.3	
		125	34.7	4.0	
		115	32.4	4.0	
		105	32.2	3.8	
		95	33.2	3.7	
		85	32.7	3.7	

*) The errors of relative measurements in percent are given.

**) The differential cross section was normalized to the total cross section 425 mb for 22.5 MeV, assuming the capture cross section to be negligible. (Neutron cross section - see Report BNL 325 (1958).)

Table 2

Continuation

Quantity	Energy [MeV]	ϑ^o c.m.s.	Measured value	Statistical error ±	Refs.
σ_{np}	22.5	75	32.8	4.5	[7]
		65	33.3	5.7	
[mb/sr]		7	32.8	3.6 *)	[8]
		14	32.6	4.7	
		21	35.6	5.1	
		31	34.4	3.3	
		41	32.8	4.1	
		51	35.5	3.0	
σ_{np}	27.5	7	28.5	3.7 *)	[8]
		14	28.5	4.7	
[mb/sr]		21	29.6	5.2	
		31	28.3	3.5	
		41	27.5	4.3	
		51	27.3	3.2	
		62	26.5	4.6	
		72	27.0	3.6	
		159	25.3	3.2	
		166	26.8	2.8	
		173	29.9	3.1	
σ_t^{np} [mb]	23.951		397.2	1.7 [mb]	[4]
P_{np}	23.1	50	0.0492	0.0140	[23]
		70	0.0529	0.0100	
		90	0.0522	0.0071	
		110	0.0310	0.0071	
		130	0.0247	0.0090	
		150	-0.0036	0.0090	
P_{np}	23.1	140	0.011	0.006	[24]

*) Only the statistical errors in percent are given. The systematical and normalization errors are about 3% (see ref. [27] p. 258).

T a b l e 2

Continuation

Quantity	Energy [MeV]	ϑ^{o} c.m.s.	Measured value	Statistical error $\pm$	Refs.
P_{np}	23.1	70	0.0587	0.0081	[3]
		90	0.0513	0.0065	
		150	0.0059	0.0102	
D_{np}	23.1 *)	70	0.56	0.20	[25]
		90	0.81	0.17	
		110	0.81	0.20	
C_{nn}^{np}	23.1	140	0.074	0.024	[24]
		174	-0.014	0.011	
C_{nn}^{np}	23.1	130	0.131	0.042	[3]
		150	0.052	0.021	

*) This data was taken from the graph (see ref./3/).

T a b l e 3

Experimental Data of σ_{pp} at the Angle 90° c.m.s.

Energy [MeV]	$\sigma_{pp} \pm \Delta \sigma_{pp}$		Authors	Refs.
9.73	56.11	0.23	Cork et al.	[28]
18.2	27.32	0.38	Yntema et al.	[29]
21.95	21.13	0.11	Batty et al.	[30]
25.62	18.30	0.09	Batty et al.	[30]
25.63	18.59	0.14 *)	Jeong et al.	[20]
28.2	16.27	0.21	Johnston et al.	[31]
30.33	15.01	0.08	Batty et al.	[30]
31.1	14.68	0.15	Johnston et al.	[31]
34.2	13.36	0.13	Johnston et al.	[31]
34.27	12.82	0.06	Batty et al.	[30]
36.9	12.14	0.12	Johnston et al.	[31]
39.4	11.16	0.08	Johnston et al.	[32]
39.6	11.19	0.11	Johnston et al.	[31]
40.75	10.54	0.05	Batty et al.	[30]
44.7	9.51	0.11	Johnston et al.	[31]

*) The value of the error is corrected according to the note of Wilson (see ref. /27/).

T a b l e 4

The N-N Scattering Phase Shifts in Degress (the Stapp Parametrization) for $\ell_{max} = 2$ at 23.1 MeV.

	1- st set		2- nd set	
f^2	0.08 fix.		0.08 fix.	
Phase Shifts	δ	$\pm \Delta\delta$	δ	$\pm \Delta\delta$
$^1S_0$	50.43	0.20	50.41	0.20
$^3S_1$	104.84	1.15	74.82	2.34
$^3P_0$	7.21	0.40	7.57	0.43
$^1P_1$	-1.59	0.91	-1.84	2.26
$^3P_1$	-3.94	0.20	-3.89	0.20
$^3P_2$	2.58	0.07	2.48	0.07
ε_1	-0.81	0.83	-0.46	2.07
$^3D_1$	2.07	2.14	-3.38	4.73
$^1D_2$	0.80	0.03	0.80	0.03
$^3D_2$	-3.85	1.34	0.40	2.93
$^3D_3$	0.56	1.05	-0.25	2.15
χ^2	68.80		63.43	
$\chi^2/\bar{\chi^2}$	0.95		0.88	

Table 5

The N-N Scattering Phase Shifts in Degrees (the Stapp Parametrization) for $\ell_{max} = 3$ at 23.1 MeV.

	1- st set		2- nd set	
f^2	0.08 fix.		0.08 fix.	
Phase Shifts	δ	$\pm \Delta\delta$	δ	$\pm \Delta\delta$
$^1S_0$	50.36	0.26	50.41	0.25
$^3S_1$	104.37	1.06	75.64	1.27
$^3P_0$	6.31	0.57	6.48	0.59
$^1P_1$	-0.68	1.22	-1.11	1.48
$^3P_1$	-4.19	0.52	-4.08	0.51
$^3P_2$	2.08	0.37	2.03	0.38
ε_1	-0.74	0.83	-0.30	1.21
$^3D_1$	0.60	3.24	-1.85	3.61
$^1D_2$	0.99	0.08	0.98	0.08
$^3D_2$	-3.14	2.20	2.85	2.56
$^3D_3$	-0.21	1.39	0.32	1.53
ε_2	-1.22	0.27	-1.23	0.28
$^3F_2$	0.10	0.28	0.11	0.30
$^1F_3$	-0.94	0.63	-0.94	0.58
$^3F_3$	0.34	0.70	0.31	0.72
$^3F_4$	0.08	0.10	0.07	0.10
χ^2	39.79		40.56	
$\chi^2/\bar{\chi}^2$	0.59		0.60	

Table 6

The N-N Scattering Phase Shifts in Degrees (the Stapp Parametrization) for ($\ell_{max} = 3$ at 23.1 MeV. The Coupling Constant f^2 Is Taken as a Variable Parameter.

	1- st set		2- nd set	
f^2	0.450 ± 0.107		0.202 ± 0.107	
Phase Shifts	δ	$\pm \Delta\delta$	δ	$\pm \Delta\delta$
$^1S_0$	50.33	0.22	50.38	0.25
$^3S_1$	105.63	1.30	75.55	1.30
$^3P_0$	6.87	0.52	6.52	0.60
$^1P_1$	1.92	1.10	-0.80	1.32
$^3P_1$	-3.63	0.45	-4.17	0.44
$^3P_2$	2.60	0.21	2.00	0.34
ε_1	-0.40	0.96	-0.81	1.09
$^3D_1$	-4.41	1.84	-2.16	3.28
$^1D_2$	1.07	0.09	0.98	0.07
$^3D_2$	2.14	1.99	2.84	2.49
$^3D_3$	-1.52	0.59	0.11	1.37
ε_2	-1.59	0.31	-1.18	0.23
$^3F_2$	0.43	0.19	0.04	0.26
$^1F_3$	-1.78	0.51	-0.92	0.65
$^3F_3$	-0.59	0.42	0.45	0.62
$^3F_4$	0.08	0.10	0.06	0.10
χ^2	32.32		39.69	
$\chi^2/\bar{\chi}^2$	0.48		0.59	

Table 7

The N-N Scattering Phase Shifts in Degrees (the Stapp Parametrization) for ($\ell_{max} = 2$ at 18.2 MeV.

	1 - st set		2 - nd set		3 - rd set	
f^2	0.08 fix.		0.08 fix.		0.08 fix.	
Phase Shifts	δ	$\pm \Delta\delta$	δ	$\pm \Delta\delta$	δ	$\pm \Delta\delta$
$^1S_0$	52.41	0.28	52.39	0.28	51.65	0.33
$^3S_1$	107.70	4.49	70.78	7.43	-72.90	2.18
$^3P_0$	6.22	0.86	6.40	0.93	-7.99	0.65
$^1P_1$	-7.09	2.84	-8.68	17.28	-4.98	5.54
$^3P_1$	-3.17	0.50	-3.16	0.49	5.53	0.64
$^3P_2$	2.54	0.30	2.28	0.30	1.60	0.18
ε_1	2.11	4.87	2.97	19.70	-0.25	8.18
$^3D_1$	3.31	9.54	-5.78	27.98	3.10	6.67
$^1D_2$	0.58	0.06	0.58	0.06	0.18	0.05
$^3D_2$	-0.70	8.19	1.39	37.53	-0.79	5.43
$^3D_3$	2.32	3.73	-2.83	5.64	2.05	2.86
χ^2	15.75		15.78		26.91	
$\chi^2/\bar{\chi}^2$	0.48		0.48		0.82	

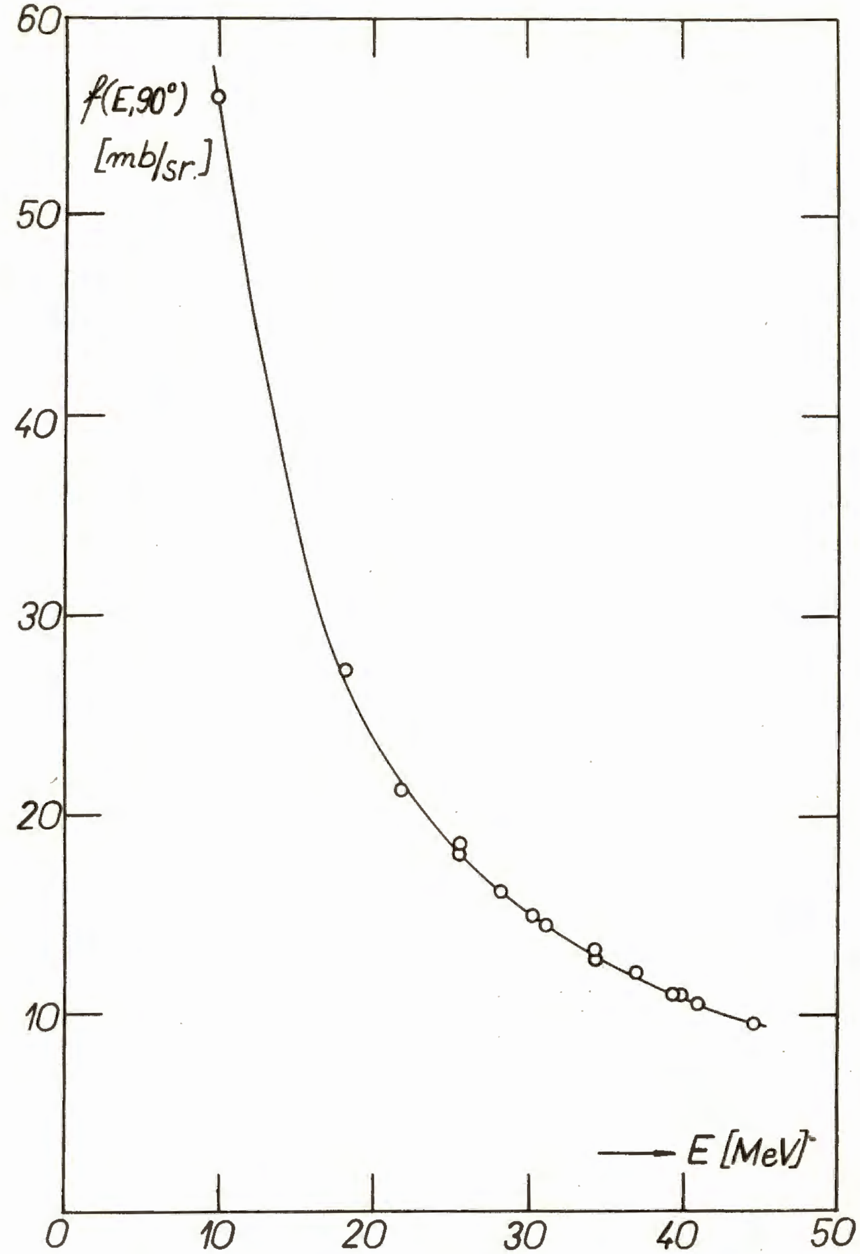

Fig. 1. The calculated dependence of the p-p elastic scattering effective cross - section f(E, 90°) on energy. The experimental points are given in Table 3.

$$y_i - \eta_i(\hat{\vec{\theta}}) \longrightarrow z_i \qquad ^{*)}$$

$$\eta_i(\hat{\vec{\theta}}) \longrightarrow y_i$$

$i = 1, \dots, n$

$$\hat{\Phi}_{\theta k} - \hat{\Phi}_k \longrightarrow \Delta\hat{\Phi}_k \qquad ^{*)}$$

$k = 1, \dots, m$

$$\frac{x_i}{\sigma_i^2} = \left(\sum_{k=1}^{m} \frac{\partial \eta_i}{\partial \theta_k}(\hat{\vec{\Phi}}) \Delta \hat{\Phi}_k + \eta_i(\hat{\vec{\Phi}}) - \eta_i(\hat{\vec{\theta}}) \right) \frac{1}{\sigma_i^2} \longrightarrow y_i \qquad ^{*)}$$

$i = 1, \dots, n$

$$\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} z_i \longrightarrow \pi \qquad \sum_{i=1}^{n} \frac{x_i^2}{\sigma_i^2} \longrightarrow \rho$$

$$\sum_{i=1}^{n} \frac{x_i}{\sigma_i^2} \frac{\partial \eta_i}{\partial \theta_k}(\hat{\vec{\theta}}) \longrightarrow \varphi_k \qquad ^{*)}$$

$k = 1, \dots, m$

$$\sum_{k,l=1}^{m} \varphi_k D_{kl} \varphi_l \longrightarrow \sigma$$

$$\frac{\pi}{\sqrt{\rho - \sigma}} \longrightarrow \tau$$

Print of results

Fig. 2. The block scheme of the additional standard programme for the τ - criterion test.
* This evaluation is made inside the arithmetical part of the programme of the phase - shift analysis if only one (the initial) step of the minimization is realized.

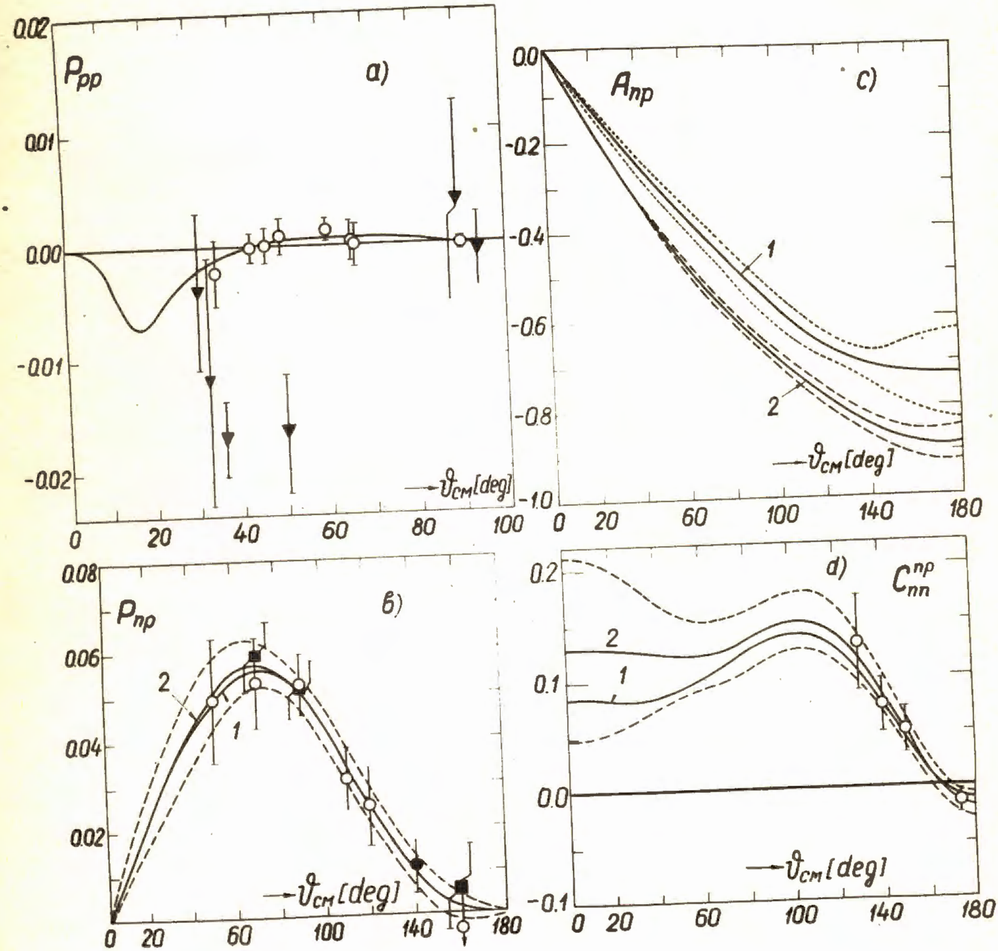

Fig.3. Curves 1 and 2 are the angular dependences of the experimental quantities calculated for the 1-st and 2-nd solutions at 23.1 MeV, respectively. The dashed curves denote the corridors of errors. The experimenral points are denote by the following way:

a) – data from ref./1/
– data from ref./5/

b) – data from ref./23/
– point from ref/24/
– data from ref./3/

d) – data from ref./3,24/

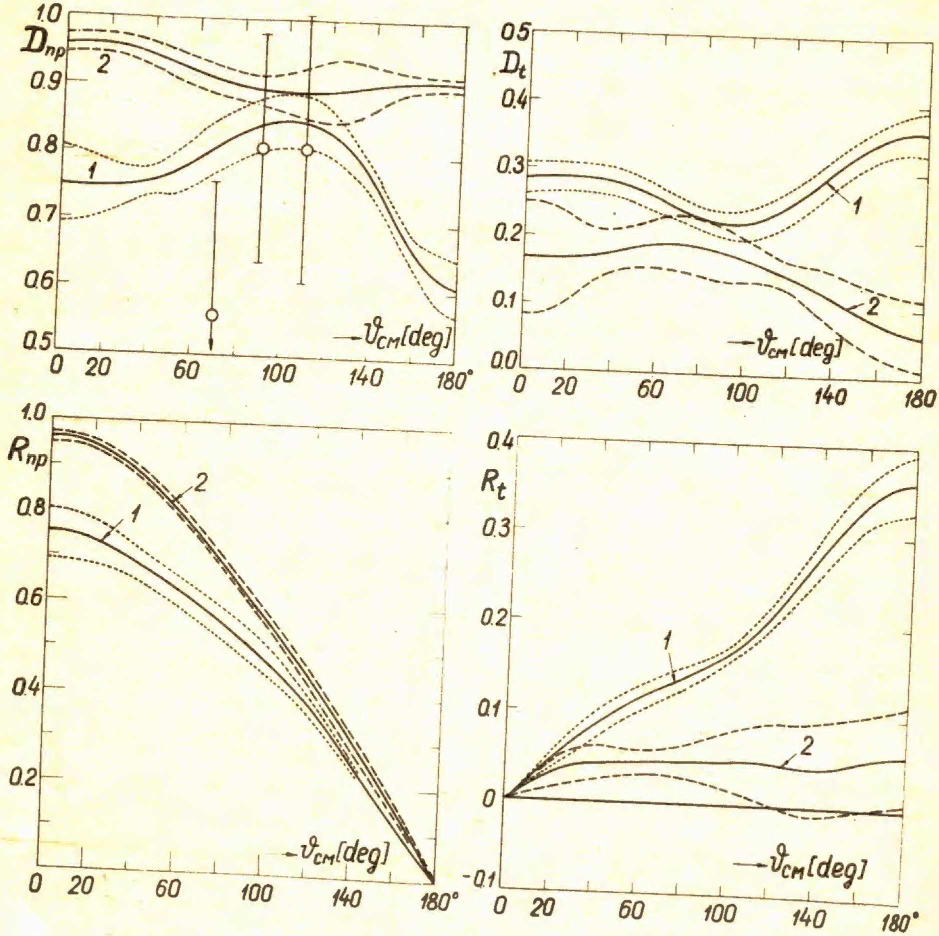

Fig. 4. Curves 1 and 2 are the angular dependence of the experimental quantities calculated for the 1-st and 2-nd solutions at 23.1 MeV, respectively. The dashed curves denote the corridors of errors. The experimental points of D_{np} are taken from ref./25/.