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SIMULTANEOUS PHASE SHIFT ANALYSIS OF 210 MeV NUCLEON - NUCLEON SCATTERING

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I. Introduction

The phase shift analysis at 210 MeV was previously performed in refs.[1-4]. Since many new experimental data have been recently obtained it is necessary to take these data into account and find more accurate phase shifts at this energy.

Besides the specification of previously obtained solutions the following circumstance caused the performance of the new phase shift analysis. As is known, in the present programme of the phase shift analysis[1,5-7] the considerable part of the scattering amplitude corresponding to the interaction in high orbital momentum states $\ell > \ell_{max}$ is taken in one pion exchange approximation. Here, the limit of the application of this approximation (ℓ_{max}) is determined according to the best agreement with experimental data[1,8] by various statistical criteria. The increased accuracy of the previously known experimental data and the measurement of the new experimental values usually shifts ℓ_{max} in the direction of high orbital momenta. The number of varying parameters (phase shifts) determined on the basis of the experimental data increases and this in many cases causes the appearance of the new, previously unknown sets of phase shifts[9,10].

In the first works on the simultaneous phase shift analysis of n, p and p, p data at 210 MeV[1] solutions were searched for at $\ell_{max} = 3$. The obtained solutions were later specified for $\ell_{max} = 4$. Unambiguity of the phase shift analysis at the transition to the higher ℓ_{max} was not earlier investigated.

2. Experimental Data

The data used in the phase shift analysis at 210 MeV[11-12] are given in Table 1 and Table 2. The results of the measurements of the differential cross section in a small angle region[12], the polarization of p,p [12] and n,p [x] scattering, transfer triple scattering parameters D_t [21] and R_t [22] for the recoil particles in n,p scattering were not earlier included in the phase shift analysis.

The triple scattering parameters for the recoil particle are discussed in detail in the Appendix.

All the experimental data are concentrated in the energy region of 197-217 MeV. The measured values were not normalized to the mean energy of 210 MeV.

The unpublished data on the differential p,p cross section measured by Konradi[11] were taken from ref.[24] and were multiplied by the correction factor 0.986 according to ref.[4]. The used p,p polarization data have been obtained by Marshall et al.[12], Tinlot, Warner[13] and Baskire et al.[14] The polarization data from[14] which do not agree with more accurate values from ref.[13] in the angular region of $52^{\circ}-71^{\circ}$ were not included into the phase shift analysis.

The values of the parameter A_{pp} at 80° and 90° given in[15] were excluded because they make large contribution to χ^2. Instead of these points we used the data obtained by the authors of ref.[15]. They give the values $A_{pp}(80^{\circ})=0.167\pm 0.095$ and $A_{pp}(90^{\circ})=0.085\pm 0.135$ published in[23].

From the seven measured values of the parameter R'_{pp} (see ref.[16]) the two values at angles 60° and 70° had to be excluded because their contribution in χ^2 are equal 12.9 and 16.3, respectively.

In the n,p interaction the polarization data of Tinlot, Warner[13] and Thomas et al.[19] were used. The polarization in ref.[13] was measured in the quasielastic scattering of protons on the neutrons in deuteron nuclei. The measured values were corrected later by Koehler et al.[25]

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[x] The polarization data of P_{np} measured by Thomas et al.[19] have been published in the report by Thorndike[23].

to the final S -state interaction of scattered particles. The corrected data in the region of $40^{\circ}-90^{\circ}$ c.m.s. were included in the phase shift analysis, whereas for the angles of 100°, 110° and 120° the original data from ref.[13] were taken.

The new data of Thomas et al.[19] which have been measured in the scattering of neutrons on free protons are only preliminary. It may be seen in the present paper that except one point at 86.6° all the data are well described.

The triple scattering parameters D_{pn}, D_t, and R_t were also measured in the quasielastic p,d scattering. The values of these parameters were corrected to the final S -state interaction of the scattered particles[25] and used in the present phase shift analysis.

3. Phase Shift Analysis

The phase shift analysis was carried out according to the programme, described in details in ref.[1]. In the present work an additional condition was introduced, determining the behavior of the scattering am - plitude at $\theta = 0^{\circ}$. As is known, at $\theta = 0^{\circ}$, the scattering amplitude consist of three terms in accordance with three parameters determining the scattering in this case ($\vec{\sigma}_1, \vec{\sigma}_2$, $\vec{k}$, where $\vec{k}$ is the momentum of an incident particle and $\vec{\sigma}_1$, $\vec{\sigma}_2$ are Pauli matrices)

$$M(0)=\alpha + \beta (\vec{\sigma}_1, \vec{\sigma}_2) + \delta (\vec{\sigma}_1, \vec{k})(\vec{\sigma}_2, \vec{k}), \qquad (1)$$

where α, β, δ are the coefficients of the scattering matrix.

It is of interest to note, that this condition in the singlet - triplet representation means simply that $M_{1-1} = 0$. This condition is fulfilled authomatically if M_{1-1} is expressed in the series of the associated Legendre functions P^2_ℓ. However in the phase shift analysis programme the value M_{1-1} is obtained from the T -invariance condition

$$M_{1-1} = M_{11} - M_{00} - \sqrt{2}\, \text{cotg}\, \theta \,(M_{10} + M_{01}) \qquad (2)$$

The approximation in which the scattering matrix elements M_{11}, M_{00}, M_{10}, M_{01} are calculated, can in some cases cause the considerable errors in the determination of M_{1-1} at $\theta = 0^{o}$ [26].

The search for solutions from random initial parameters was performed at $\ell_{max} = 5$, i.e. the one pion exchange approximation was used for the momenta at $\ell_{max} \geq 6$. This was due to the fact that the previously obtained solution [1,2] describes sufficiently well the behavior of the differential cross section and polarization for p, p scattering at small angles [12] only at $\ell_{max} = 5$.

There were made 100 searches from random initial parameters from which three solutions with $\chi^2 = 127.3$, 159.8, and 190.4 were obtained in the $\chi^2 \leq 2\chi^2$ region (Table 3). It should be noted that all the obtained solutions correspond to the phase shifts sets obtained previously at the first analysis of p, p and n,p data at this energy [1] (solutions b, b_1, and c, respectively). The search for the solutions from a random initial parameters was performed with all the available data and only during the specification of the obtained solutions the points given above were excluded. These points did not agree with any of the three solutions in the region $\chi^2 < 2\chi^2$.

Solution 3 in Table 3 ($\chi^2 = 190.4$) can be rejected according to the χ^2 criterion with the mean square error smaller than 10^{-3}%. The error due to the rejection of the 2nd solution P ($\chi^2 \geq 159.8$) = 0.14%. It should be noted, that in rejecting solution 2 it would be more correct to use the another criterion [27].

It follows from the above, that it may be possible to state that the phase shift analysis at 210 MeV is unambiguous as previously.

4. Results

The set of the phase shifts with $\chi^2 = 127.3$ is given in Table 4. The previously known sets of Kazarinov et al.[2] and Arndt, MacGregor[3] are given in the same table for comparison. It may be seen, from the table, that the mean values of phase shifts have not been changed very much, but their errors have been considerably decreased.

The solutions with $\chi^2 = 159.8$ and 190.4 are given in Table 5. On the basis of phase shift set 1 the angular dependences of the experimental values were calculated together with their corridor of errors (see Fig. 1-9). The corridor of errors represents the mean square error at each point of the calculated curve. The measured values used in the present phase shift analysis are given in the graphs.

It is seen from Fig. 2 for A_{pp} that the new data are in good agreement with the calculated curve. Corrected values of R_t given in Fig. 6 are in worse agreement with the calculated values than the results of the measurement of the p,d quasielastic scattering. This may be due to the systematical error of about 14% contained in the corrections[23]. The angular dependences of the experimental data which can be measured only using the polarized proton target are given in Figs. 4,8,9. The components of the polarization tensor M_{plk} are discussed in refs.[28,29], the components of the asymmetry tensor A_{lk} are determined in[28] and the spin correlation coefficients C_{pqlk} are given in[30]. Here, the indeces p,q are related to the measured polarization components of the scattered and recoil particles, l and k to the initial polarization of the beam and the target, respectively.

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Appendix

Triple Scattering Parameters for Recoil Particles in the N-N Interaction

The N-N -scattering matrix $M(\vec{k}',\vec{k})$ satisfying invariance requirements with respect to rotations, space reflections and time invariance is given by [31,32]

$$M(\vec{k}',\vec{k}) = \frac{a+b}{2} + \frac{a-b}{2}(\vec{\sigma}_1,\vec{n})(\vec{\sigma}_2,\vec{n}) + \frac{e}{2}(\vec{\sigma}_1+\vec{\sigma}_2,\vec{n}) + \frac{c+d}{2}(\vec{\sigma}_1,\vec{m})(\vec{\sigma}_2,\vec{m}) + \frac{c-d}{2}(\vec{\sigma}_1,\vec{\ell})(\vec{\sigma}_2,\vec{\ell}). \tag{A.1}$$

The coefficients a , b , c , d , e are the energy and angular dependent complex function in c.m.s., $\vec{\sigma}_1$, $\vec{\sigma}_2$ are the Pauli matrices. The unit vectors $\vec{\ell}$, $\vec{m}$, $\vec{n}$ are defined by

$$\vec{\ell} = \frac{\vec{k}'+\vec{k}}{|\vec{k}'+\vec{k}|} ; \quad \vec{m} = \frac{\vec{k}'-\vec{k}}{|\vec{k}'-\vec{k}|} ; \quad \vec{n} = \frac{\vec{k}\times\vec{k}'}{|\vec{k}\times\vec{k}_n'|} , \tag{A.2}$$

where $\vec{k}$, $\vec{k}'$ are the unit vectors in the directions of incident and scattered particle momenta[x] in c.m.s., respectively.

We use the two following orthonormal systems of vectors in the laboratory system

$$\vec{n}_t = \frac{\vec{k}_L\times\vec{k}_t'}{|\vec{k}_L\times\vec{k}_t'|} ; \quad \vec{k}_L ; \quad \vec{s}_L = \vec{n}_t\times\vec{k}_L , \tag{A.3}$$

$$\vec{n}_t \quad ; \quad \vec{k}_t' ; \quad \vec{s}_t' = \vec{n}_t\times\vec{k}_t' . \tag{A.4}$$

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[x] The protons are the incident and scattered particles in the measurements of the parameters D_t and R_t in refs. [21,22] .

Here $\vec{k}_L$, $\vec{k}'_t$ are unit vectors in the directions of incident and recoil particles momenta, respectively, and $\vec{n}_t$ in the normal to the plane, determined by the vectors $\vec{k}_L$ and $\vec{k}'_t$. In this case $\vec{k}_L = \vec{k}$ and $\vec{n}_t = -\vec{n}$.

The measured polarization vector for the recoil particle is characterized by the components by system (A.4). The polarization of the incident nucleons is expressed by system (A.3).

The projection of the recoil particle polarization to the direction of an arbitrary unit vector $\vec{a}'_t$ measured in lab. system is given by /33–35/

$$\langle \vec{\sigma}_2 \rangle_L \vec{a}'_t = \frac{1}{\sigma} \, Sp \vec{\sigma}_2 (\vec{a}'_t)_R M \rho_0 M^+ , \tag{A.5}$$

where ρ_0 is the density matrix $\rho_0 = \frac{1}{4}(1 + \vec{\sigma}_1 \vec{P}_1)$, σ is the differential cross section in c.m.s. and

$$(\vec{a}'_t)_R = R_{\vec{n}} (\Omega'') \vec{a}'_t . \tag{A.6}$$

In the last formula $R_{\vec{n}}(\Omega'')$ is the operator of the rotation around the normal $\vec{n}$ at the angle $\Omega'' = 2\theta_t - \phi$; θ_t is the recoil angle in lab.system, $\phi = \pi - \theta$ is the recoil angle in c.m.s.

The relation for $(\vec{a}'_t)_R$ can be written as /35/

$$R_{\vec{n}} (\Omega'') \vec{a}'_t = (\vec{a}'_t, \vec{n}) \vec{n} (1 - \cos \Omega'') + \vec{a}'_t \cos \Omega'' + \vec{n} \times \vec{a}'_t \sin \Omega'' . \tag{A.7}$$

Using this formula for vectors (A.4) we obtain

$$\begin{aligned} (\vec{n}_t)_R &= R_{\vec{n}} (\Omega'') \vec{n}_t = \vec{n}_t , \\ (\vec{k}'_t)_R &= R_{\vec{n}} (\Omega'') \vec{k}'_t = - \vec{\ell} \cos \beta - \vec{m} \sin \beta , \\ (\vec{s}'_t)_R &= R_{\vec{n}} (\Omega'') \vec{s}'_t = - \vec{\ell} \sin \beta + \vec{m} \cos \beta , \end{aligned} \tag{A.8}$$

where

$$\beta = \theta_t + \frac{\theta}{2} .$$

Note, that in the non-relativistic case, where $\beta = 90^\circ$

$$(\vec{k}'_t)_R = -\vec{m} ; \quad (\vec{s}'_t)_R = -\vec{\ell} .$$

For vectors (A.3) we obtain

$$\begin{aligned} \vec{k}_L &= \vec{\ell} \cos \theta/2 - \vec{m} \sin \theta/2 , \\ \vec{s}_L &= -\vec{\ell} \sin \theta/2 - \vec{m} \cos \theta/2 . \end{aligned} \tag{A.9}$$

The polarization of the recoil particle in the scattering of the polarized beam at the unpolarized target in the lab. system is determined by

$$\begin{aligned} \sigma \langle \vec{\sigma}_2 \rangle_L = \sigma_0 \{ & [P_t + D_t (\vec{P}_1 , \vec{n}_t)] \vec{n}_t + \\ & + [A_t (\vec{P}_1 , \vec{k}_L) + R_t (\vec{P}_1 , \vec{s}_L)] \vec{s}'_t + \\ & + [A'_t (\vec{P}_1 , \vec{k}_L) + R'_t (\vec{P}_1 , \vec{s}_L)] \vec{k}'_t \} . \end{aligned} \tag{A.10}$$

Using formula (A.10) we obtain for the measured components of the recoil particles polarization the following expressions

$$\begin{aligned} \sigma \langle \vec{\sigma}_2 \rangle \vec{n}_t &= \sigma_0 [P_t + D_t (\vec{P}_1 , \vec{n}_t)] ; \\ \sigma \langle \vec{\sigma}_2 \rangle \vec{k}'_t &= \sigma_0 [A'_t (\vec{P}_1 , \vec{k}_L) + R'_t (\vec{P}_1 , \vec{s}_L)] , \\ \sigma \langle \vec{\sigma}_2 \rangle \vec{s}'_t &= \sigma_0 [A_t (\vec{P}_1 , \vec{k}_L) + R_t (\vec{P}_1 , \vec{s}_L)] , \end{aligned} \tag{A.11}$$

where P_t is the recoil particle polarization in scattering of the unpolarized beam, $\vec{P}_1$ is the polarization vector of the incident beam, σ_0 is the differential cross section in scattering of unpolarized particles on the unpolarized target $\sigma_0 = 1/4\, Sp\, MM^+$, σ is the differential cross section of the polarized beam $\sigma = \sigma_0 [1 + P_t (\vec{P}_1, \vec{n}_t)]$ and the parameters

$$D_t = (\vec{n}_t)_{R_i} K_{ik} (\vec{n}_t)_k ,$$

$$R_t = (\vec{s}'_t)_{R_i} K_{ik} (\vec{s}_L)_k ,$$

$$A_t = (\vec{s}'_t)_{R_i} K_{ik} (\vec{k}_L)_k , \tag{A.12}$$

$$R'_t = (\vec{k}'_t)_{R_i} K_{ik} (\vec{s}_L)_k , \quad A'_t = (\vec{k}'_t)_{R_i} K_{ik} (\vec{k}_L)_k ,$$

are called the triple scattering parameters for the recoil particles, determined with the account of the relativistic spin rotation.

The quantities K_{ik} are the components of the transfer polarization tensor

$$K_{ik} = 1/4\, \sigma_0\, Sp\, \sigma_{2i}\, M \sigma_{1k}\, M^+ . \tag{A.13}$$

We express the measured quantities (A.12) by the components of the tensor K_{ik} in c.m.s. Using (A.8) and (A.9) we have[x/]

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x/ In ref./35/ instead (A.3) and (A.4) the following systems of unit vectors are used

$$\vec{n}_L , \vec{k}_L , \vec{s} = \vec{n}_L \times \vec{k}_L , \tag{A.3'}$$

$$\vec{n}_L , \vec{k}''_L , \vec{s}''_L = \vec{n}_L \times \vec{k}''_L , \tag{A.4'}$$

where $\vec{n}_L = \vec{n} = -\vec{n}_t$, $\vec{s} = -\vec{s}_L$, $\vec{k}''_L = \vec{k}'_t$, $\vec{s}''_L = -\vec{s}'_t$.

In systems (A.3') and (A.4') relations (A.14) for the D_t , R_t and A'_t are not changed, the parameters A_t and R'_t change their signs. The sign must be changed also in formula (A.15) and for relations (A.17) it follows, that $A_t = (\theta) = -A(\pi - \theta)$; $R'_t = (\theta) = -R'(\pi - \theta)$.

$$D_t = K_{nn} ,$$

$$R_t = K_{\ell\ell} \sin\beta \sin\theta/2 - K_{mm} \cos\beta \cos\theta/2 + K_{\ell m} \sin(\beta+\theta/2) ,$$

$$A_t = - K_{\ell\ell} \sin\beta \cos\theta/2 - K_{mm} \cos\beta \sin\theta/2 - K_{\ell m} \cos(\beta+\theta/2) , \qquad (A.14)$$

$$R'_t = K_{\ell\ell} \cos\beta \sin\theta/2 + K_{mm} \sin\beta \cos\theta/2 + K_{\ell m} \cos(\beta+\theta/2) ,$$

$$A'_t = - K_{\ell\ell} \cos\beta \cos\theta/2 + K_{mm} \sin\beta \sin\theta/2 + K_{\ell m} \sin(\beta+\theta/2) .$$

The four measured quantities (A.12) are related by

$$\frac{A_t + R'_t}{A'_t - R_t} = + \mathrm{tg}\,\theta_t . \qquad (A.15)$$

In the non-relativistic case, where $\beta = 90^{\circ}$ relations (A.14) are simplified as

$$D_t = K_{nn} ,$$

$$R_t = K_{\ell\ell} \sin\theta/2 + K_{\ell m} \cos\theta/2 ,$$

$$A_t = - K_{\ell\ell} \cos\theta/2 + K_{\ell m} \sin\theta/2 , \qquad (A.16)$$

$$R'_t = K_{mm} \cos\theta/2 - K_{\ell m} \sin\theta/2 ,$$

$$A'_t = K_{mm} \sin\theta/2 + K_{\ell m} \cos\theta/2 .$$

For the identical particles we have the following relations between the parameters D_t, R_t, A_t, R'_t, A'_t and the Wolfenstein triple scattering parameters[36]

$$D_t(\theta) = D(\pi-\theta),$$

$$R_t(\theta) = R(\pi-\theta),$$

$$A_t(\theta) = A(\pi-\theta), \qquad (A.17)$$

$$R'_t(\theta) = R'(\pi-\theta);$$

$$A'_t(\theta) = A'(\pi-\theta).$$

In the phase shift analysis it is convenient to express the experimentaly measured quantities (A.12) by the matrix elements M_{ik} of the scattering matrix in the singlet triplet representation, since the latter are directly related to phase shifts/37/.

The components (A.13) of the transfer polarization tensor K_{ik} depend on the scattering matrix coefficients as:

$$\sigma_0 K_{nn} = 1/2\,(|a|^2-|b|^2+|c|^2-|d|^2+|e|^2),$$

$$\sigma_0 K_{\ell\ell} = \mathrm{Re}\,(a^*c-b^*d),$$

$$\sigma_0 K_{mm} = \mathrm{Re}\,(a^*c+b^*d), \qquad (A.18)$$

$$\sigma_0 K_{\ell m} = -\sigma_0 K_{m\ell} = -\,\mathrm{Im}\,ce^*,$$

$$\sigma_0 = 1/2\,(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2).$$

The scattering matrix coefficients a, b, c, d and e depend on the elements M_{ik} in the singlet triplet representation/37/ as:

$$a = 1/2\,(M_{11}+M_{00}-M_{1-1}),$$

$$b = 1/2\,(M_{11}+M_{ss}+M_{1-1}),$$

$$e = 1/2\, i\sqrt{2}\,(M_{10} - M_{01}),$$

$$c = 1/2\,(M_{11} + M_{1-1} - M_{ss}),$$

$$d = 1/2\,(-M_{11} + M_{1-1} + M_{00})\sec\theta\,. \tag{A.19}$$

The components $\sigma_0 K_{\ell\ell}$, $\sigma_0 K_{mm}$, $\sigma_0 K_{\ell m}$ of the transfer polarization tensor expressed by the scattering matrix elements M_{ik} have the form

$$\sigma_0 K_{\ell\ell} = 1/4\,\mathrm{Re}\{[(\cos\theta+1)(\frac{\sqrt{2}M_{10}}{\sin\theta}+\frac{\sqrt{2}M_{01}}{\sin\theta})+2M_{00}](M_{11}+M_{1-1}-M_{ss})^*+2(\frac{\sqrt{2}M_{10}}{\sin\theta}+\frac{\sqrt{2}M_{01}}{\sin\theta})M^*_{ss}\},$$

$$\sigma_0 K_{mm} = 1/4\,\mathrm{Re}\{[(\cos\theta-1)(\frac{\sqrt{2}M_{10}}{\sin\theta}+\frac{\sqrt{2}M_{01}}{\sin\theta})+2M_{00}](M_{11}+M_{1-1}-M_{ss})^*-2(\frac{\sqrt{2}M_{10}}{\sin\theta}+\frac{\sqrt{2}M_{01}}{\sin\theta})M^*_{ss}\},$$

$$\sigma_0 K_{\ell m} = \frac{\sqrt{2}}{4}\,\mathrm{Re}\,(M_{10}-M_{01})(M_{11}+M_{1-1}-M_{ss})^*\,. \tag{A.20}$$

We can directly express the experimental quantities (A.14) using (A.20) in the terms of the scattering matrix in the singlet triplet representation.

For the non-relativistic case we have

$$\sigma_0\,(1-D_t) = 1/4\{|M_{11}+M_{1-1}+M_{ss}|^2+|M_{11}-M_{1-1}-M_{00}|^2+2|M_{10}+M_{01}|^2\},$$

$$\frac{\sigma_0 R_t}{\sin\theta/2} = 1/2\,\mathrm{Re}\{[M_{00}+(\cos\theta+1)\frac{\sqrt{2}M_{10}}{\sin\theta}](M_{11}+M_{1-1}-M_{ss})^*+(\frac{\sqrt{2}M_{10}}{\sin\theta}+\frac{\sqrt{2}M_{01}}{\sin\theta})M^*_{ss}\},$$

$$\frac{\sigma_0 A_t}{\cos\theta/2} = -1/2\,\mathrm{Re}\left\{\left[M_{00} + (\cos\theta - 1)\frac{\sqrt{2}M_{10}}{\sin\theta}\right](M_{11} + M_{1-1} - M_{ss})^* + \left(\frac{\sqrt{2}M_{10}}{\sin\theta} + \frac{\sqrt{2}M_{01}}{\sin\theta}\right)(M_{11} + M_{1-1})^*\right\},$$

$$\frac{\sigma_0 R'_t}{\cos\theta/2} = 1/2\,\mathrm{Re}\left\{\left[M_{00} + (\cos\theta - 1)\frac{\sqrt{2}M_{10}}{\sin\theta}\right](M_{11} + M_{1-1} - M_{ss})^* - \left(\frac{\sqrt{2}M_{10}}{\sin\theta} + \frac{\sqrt{2}M_{01}}{\sin\theta}\right)M^*_{ss}\right\},$$

$$\frac{\sigma_0 A'_t}{\sin\theta/2} = 1/2\,\mathrm{Re}\left\{\left[M_{00} + (\cos\theta + 1)\frac{\sqrt{2}M_{10}}{\sin\theta}\right](M_{11} + M_{1-1} - M_{ss})^* - \left(\frac{\sqrt{2}M_{10}}{\sin\theta} + \frac{\sqrt{2}M_{01}}{\sin\theta}\right)(M_{11} + M_{1-1})^*\right\}. \qquad (A.21)$$

Since the relativistic rotation has no influence on the quantity D_t the corresponding relativistic and non-relativistic formulae are equal.

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Table 1

The experimental data, used in phase-shift analysis at 210 MeV.

Measured quantity	Actual experimental energy /MeV/	Number of points	Refs.
σ_{pp}	213	7	/11/
	213	13	/12/
P_{pp}	210	7	/13/
	210	14	/14/
	213	13	/12/
	217	7	/13/
R_{pp}	213	7	/15/
A_{pp}	213	7	/15/
R'_{pp}	213	5	/16/
D_{pp}	213	7	/17/
σ_{np}	200	21	/18/
P_{np}	199	7	/19/
P_{pn}	215	9	/13/
D_{pn}	212	5	/20/
D_t	197	3	/21/
R_t	203	5	/22/
σ^t_{np}	200	1	/18/

Number of points in pp-scattering: 87,
number of points in np-scattering: 51,
total number of points: 138.

Table 2

The Experimental Data Used in Phase-Shift Analysis for pp - and np - Scattering at Energy 210 MeV.

Parameter	Energy /MeV/	$\vartheta_{c.m.s.}$	Measured value	Statistical error ±	Refs.
σ_{pp} mb/sterad	213	30^{0}	3.80	0.06	/11/
		40	3.83	0.05	
		50	3.73	0.04	
		60	3.65	0.04	
		70	3.66	0.04	
		80	3.66	0.03	
		90	3.61	0.04	
σ_{pp} mb/sterad	213	8.9	4.86	0.12	/12/
		9.8	4.12	0.10	
		10.4	3.67	0.09	
		12.2	3.49	0.06	
		13.2	3.47	0.07	
		14.8	3.50	0.04	
		17.2	3.55	0.06	
		18.5	3.59	0.04	
		19.4	3.55	0.04	
		21.7	3.77	0.06	
		24.2	3.72	0.03	
		29.0	3.76	0.03	
		38.7	3.67	0.05	
P_{pp}	210	30	0.312	0.009	/13/
		40	0.319	0.011	
		50	0.303	0.010	
		60	0.240	0.009	
		70	0.163	0.008	
		80	0.084	0.007	
		90	-0.002	0.007	

Parameter	Energy (MeV)	θ c.m.s.	Measured value	Statistical error $\pm$	Refs.
P_{pp}	210	13°42′	0.217	0.080	/14/
		21°04′	0.250	0.021	
		21°04′	0.286	0.010	
		31°32′	0.311	0.010	
		31°32′	0.362	0.027	
		31°32′	0.323	0.027	
		42°00′	0.321	0.010	
		42°00′	0.338	0.028	
		47°12′	0.302	0.007	
		52°22′	0.289	0.011	
		72°53′	0.178	0.020	
		83°02′	0.077	0.012	
		92°58′	-0.006	0.027	
		112°36′	-0.175	0.032	
P_{pp}	213	8.9	0.061	0.035	/12/
		9.8	0.120	0.027	
		10.4	0.133	0.019	
		12.2	0.173	0.013	
		13.2	0.215	0.015	
		14.8	0.218	0.012	
		17.2	0.255	0.012	
		18.5	0.269	0.020	
		19.4	0.255	0.010	
		21.7	0.299	0.015	
		24.2	0.277	0.011	
		29.0	0.321	0.010	
		38.7	0.340	0.006	
P_{pp}	217	60	0.246	0.010	/13/
		70	0.153	0.009	
		80	0.079	0.008	
		90	0.014	0.011	
		100	-0.090	0.009	
		110	-0.153	0.010	
		120	-0.218	0.011	

Table 2 - Continuation

Parameter	Energy (MeV)	θ c.m.s.	Measured value	Statistical error $\pm$	Refs.
P_{pp}	210	13°42′	0.217	0.080	/14/
		21°04′	0.250	0.021	
		21°04′	0.286	0.010	
		31°32′	0.311	0.010	
		31°32′	0.362	0.027	
		31°32′	0.323	0.027	
		42°00′	0.321	0.010	
		42°00′	0.338	0.028	
		47°12′	0.302	0.007	
		52°22′	0.289	0.011	
		72°53′	0.178	0.020	
		83°02′	0.077	0.012	
		92°58′	-0.006	0.027	
		112°36′	-0.175	0.032	
P_{pp}	213	8.9	0.061	0.035	/12/
		9.8	0.120	0.027	
		10.4	0.133	0.019	
		12.2	0.173	0.013	
		13.2	0.215	0.015	
		14.8	0.218	0.012	
		17.2	0.255	0.012	
		18.5	0.269	0.020	
		19.4	0.255	0.010	
		21.7	0.299	0.015	
		24.2	0.277	0.011	
		29.0	0.321	0.010	
		38.7	0.340	0.006	
P_{pp}	217	60	0.246	0.010	/13/
		70	0.153	0.009	
		80	0.079	0.008	
		90	0.014	0.011	
		100	-0.090	0.009	
		110	-0.153	0.010	
		120	-0.218	0.011	

T a b l e 2- Continuation

Parameter	Energy (MeV)	θ c.m.s.	Measured value	Statistical error $\pm$	Refs.
R_{pp}	213	30	-0.203	0.012	/15/
		40	-0.133	0.017	
		50	-0.041	0.018	
		60	0.071	0.026	
		70	0.147	0.029	
		80	0.248	0.042	
		90	0.223	0.055	
A_{pp}	213	30	-0.400	0.019	/15/
		40	-0.317	0.019	
		50	-0.205	0.021	
		60	-0.102	0.025	
		70	-0.012	0.036	
		80	0.167	0.095	/23/
		90	0.085	0.135	
R'_{pp}	213	30	0.491	0.025	/16/
		40	0.390	0.024	
		50	0.177	0.022	
		60	0.120	0.025	
		70	-0.277	0.045	
		80	-0.208	0.068	
		90	-0.340	0.104	
D_{pp}	213	30	0.200	0.016	/17/
		40	0.232	0.026	
		50	0.240	0.018	
		60	0.319	0.021	
		70	0.297	0.030	
		80	0.360	0.070	
		90	0.50	0.18	

Parameter	Energy (MeV)	θ c.m.s.	Measured value	Statistical error $\pm$	Refs.
σ_{np}	200	6.25	9.5	2.5	/18/
mb/sterad		10.5	8.3	0.8	
		21.3	4.7	0.7	
		31.5	4.1	0.5	
		41.7	3.0	0.4	
		62.7	2.4	0.4	
		67.3	2.16	0.16	
		77.3	1.91	0.07	
		87.0	1.87	0.08	
		97.0	2.20	0.08	
		109.3	2.79	0.16	
		117.5	3.51	0.24	
		129.6	3.85	0.16	
		139.3	4.63	0.16	
		148.5	5.79	0.12	
		159.0	7.02	0.13	
		163.0	7.78	0.24	
		165.0	9.22	0.26	
		169.5	10.33	0.23	
		173.75	11.29	0.24	
		180.0	11.4	0.4	
P_{np}	199	77.0	0.128	0.026	/19/ x)
		86.6	0.030	0.015	
		96.3	-0.079	0.014	
		117.1	-0.117	0.011	
		127.3	-0.132	0.010	
		137.8	-0.132	0.008	
		148.1	-0.123	0.011	
		158.0	-0.075	0.012	

x/ The preliminary data have been tabulated in ref./23/.

Table 2 - Continuation

Parameter	Energy (MeV)	θ^0 c.m.s.	Measured value	Exp. error	Corr. value	Statistical error	Refs.
P_{pn}	215	40	0.469	0.028	0.501	0.035	/13,25/
		50	0.460	0.031	0.466	0.038	
		60	0.372	0.041	0.362	0.044	
		70	0.258	0.033	0.240	0.035	
		80	0.032	0.036	0.012	0.038	
		90	-0.069	0.032	-0.087	0.034	
		100	-0.124	0.029			/13/
		110	-0.184	0.029			
		120	-0.170	0.030			
D_{pn}	212	40	0.70	0.07	0.79	0.09	/20/,/25/
		50	0.85	0.08	0.90	0.09	
		60	0.79	0.08	0.82	0.08	
		70	0.99	0.14	1.01	0.14	
		80	1.05	0.45	1.06	0.45	
D_t	197	126.9			0.058	0.103	/21/
		138.6			-0.014	0.071	
		147.4			0.095	0.068	
R_t	203	139.0			-0.607	0.124	/22/
		148.8			-0.953	0.061	
		158.9			-0.929	0.070	
		169.2			-0.540	0.096	
		179.2			-0.269	0.095	
σ^{tot}_{np} mb/sterad	200		42.5	0.9			/18/

T a b l e 3

The Solutions, Obtained Searching for Random Initial Parameters

i - number of solution	χ^2_i	$\chi^2_i \; / \overline{\chi^2}$	$P(\chi^2 \geq \chi^2_i)$
1	127.3	1.15	14.01 %
2	159.8	1.44	0.14 %
3	190.4	1.72	3x10^{-4} %

Table 4

The Phase-Shifts in Degrees (the Stapp et al.[7] Parametrization) for 210 MeV Nucleon-Nucleon Scattering

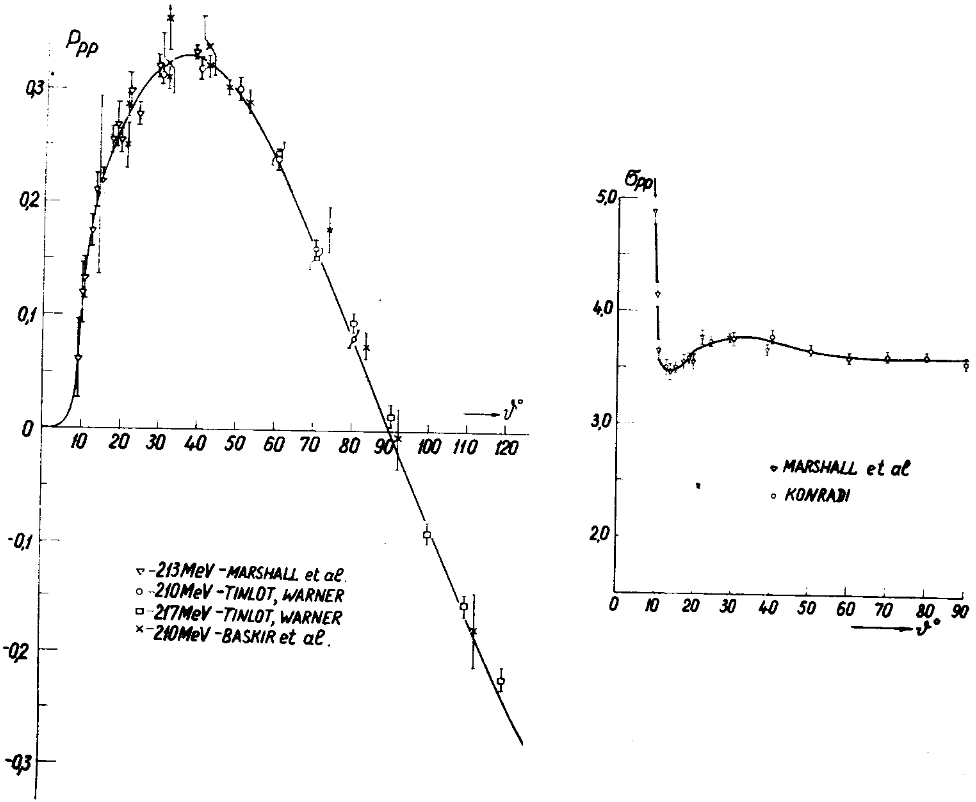
Phase Shifts	Present Solution δ^0 $\Delta\delta^0$	Solution from[2] δ^0 $\Delta\delta^0$	Solution from[3] δ^0 $\Delta\delta^0$
$^1S_0$	4.99 $\pm$ 0.46	5.14 $\pm$ 0.49	5.18 $\pm$ 0.57
$^3S_1$	15.44 $\pm$ 1.84	17.11 $\pm$ 2.87	18.23 $\pm$ 3.10
$^3P_0$	-1.34 $\pm$ 0.50	-1.02 $\pm$ 0.51	-0.79 $\pm$ 0.61
$^1P_1$	-23.99 $\pm$ 2.56	-30.34 $\pm$ 1.80	-23.37 $\pm$ 8.01
$^3P_1$	-22.46 $\pm$ 0.20	-22.04 $\pm$ 0.15	-21.59 $\pm$ 0.60
$^3P_2$	15.73 $\pm$ 0.14	16.14 $\pm$ 0.12	15.89 $\pm$ 0.27
ε_1	5.77 $\pm$ 0.81	1.22 $\pm$ 1.88	3.13 $\pm$ 2.87
$^3D_1$	-19.84 $\pm$ 1.64	-22.40 $\pm$ 2.58	-22.98 $\pm$ 4.04
$^1D_2$	6.96 $\pm$ 0.24	7.10 $\pm$ 0.18	7.02 $\pm$ 0.32
$^3D_2$	27.09 $\pm$ 2.06	22.47 $\pm$ 3.58	23.39 $\pm$ 4.33
$^3D_3$	4.20 $\pm$ 0.91	2.36 $\pm$ 1.36	2.82 $\pm$ 1.66
ε_2	-2.62 $\pm$ 0.12	-2.76 $\pm$ 0.11	-2.78 $\pm$ 0.19
$^3F_2$	1.32 $\pm$ 0.21	1.56 $\pm$ 0.25	1.58 $\pm$ 0.34
$^1F_3$	-4.72 $\pm$ 1.35	-2.98 $\pm$ 1.42	-5.53 $\pm$ 2.78
$^3F_3$	-2.62 $\pm$ 0.17	-2.44 $\pm$ 0.17	-2.58 $\pm$ 0.21
$^3F_4$	2.19 $\pm$ 0.11	2.14 $\pm$ 0.16	2.32 $\pm$ 0.20
ε_3	6.63 $\pm$ 0.83	5.87 $\pm$ 1.04	7.09 $\pm$ 0.96
$^3G_3$	-1.66 $\pm$ 0.95	0.13 $\pm$ 1.11	-0.42 $\pm$ 1.50
$^1G_4$	1.10 $\pm$ 0.09	1.08 $\pm$ 0.13	1.04 $\pm$ 0.16
$^3G_4$	5.48 $\pm$ 1.13	4.08 $\pm$ 1.66	4.40 $\pm$ 2.60
$^3G_5$	-0.12 $\pm$ 0.67	1.55 $\pm$ 1.15	0.00 $\pm$ 1.50
ε_4	-0.87 $\pm$ 0.08		-0.94 $\pm$ 0.09
$^3H_4$	-0.06 $\pm$ 0.21		0.47 $\pm$ 0.36
$^1H_5$	-0.54 $\pm$ 0.66		— —
$^3H_5$	-0.96 $\pm$ 0.17		-0.64 $\pm$ 0.22
$^3H_6$	0.00 $\pm$ 0.13		0.41 $\pm$ 0.27
r^2	0.063 $\pm$ 0.008	0.064 $\pm$ 0.006	0.071 fix.

T a b l e 5

The Phase-Shifts in Degrees (the Stapp et all.[7] Parametrization) for 210 MeV Nucleon-Nucleon Scattering

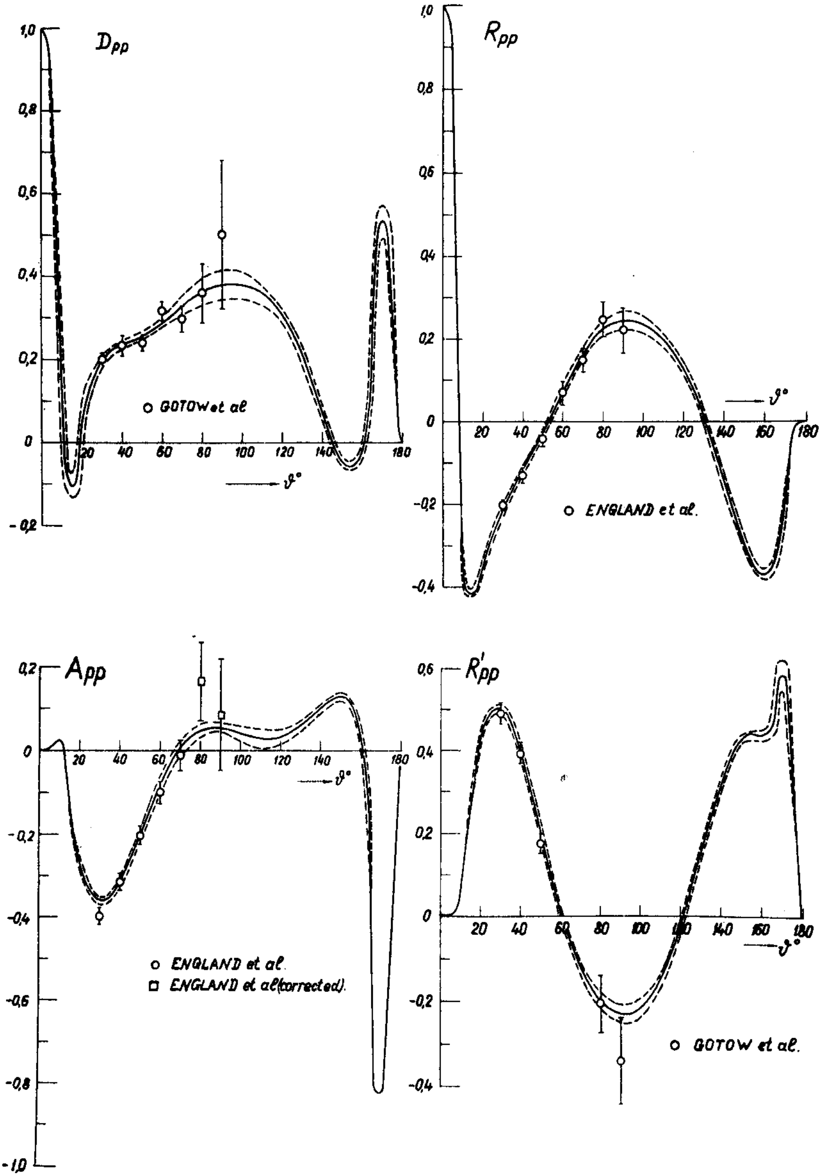
The Rejected Solutions

Phase Shifts	2-nd set δ° $\Delta\delta^\circ$	3-rd set δ° $\Delta\delta^\circ$
$^1S_0$	5.41 ± 0.50	-15.27 ± 0.71
$^3S_1$	14.20 ± 2.15	8.77 ± 4.17
$^3P_0$	3.13 ± 0.54	-25.71 ± 0.65
$^1P_1$	-19.92 ± 2.34	-0.02 ± 2.06
$^3P_1$	-20.26 ± 0.19	-2.40 ± 0.41
$^3P_2$	17.51 ± 0.15	18.83 ± 0.36
$\mathcal{E}_1$	6.78 ± 0.82	29.65 ± 1.46
$^3D_1$	-17.47 ± 1.31	-9.73 ± 2.73
$^1D_2$	7.95 ± 0.20	3.96 ± 0.18
$^3D_2$	29.26 ± 1.63	16.32 ± 1.22
$^3D_3$	0.92 ± 1.20	0.91 ± 1.22
$\mathcal{E}_2$	-1.83 ± 0.15	-6.65 ± 0.20
$^3F_2$	-1.67 ± 0.19	0.58 ± 0.47
$^1F_3$	-6.08 ± 1.43	2.81 ± 1.27
$^3F_3$	-2.01 ± 0.16	-1.64 ± 0.16
$^3F_4$	-0.02 ± 0.14	1.25 ± 0.36
$\mathcal{E}_3$	6.33 ± 0.79	10.08 ± 0.72
$^3G_3$	-4.13 ± 0.72	-2.44 ± 0.81
$^1G_4$	0.86 ± 0.09	1.24 ± 0.17
$^3G_4$	6.62 ± 0.89	4.57 ± 0.56
$^3G_5$	0.08 ± 0.73	-0.47 ± 0.53
$\mathcal{E}_4$	-1.07 ± 0.08	-0.78 ± 0.12
$^3H_4$	0.33 ± 0.15	-0.21 ± 0.16
$^1H_5$	-0.44 ± 0.60	-2.00 ± 0.81
$^3H_5$	-0.64 ± 0.15	-0.62 ± 0.09
$^3H_6$	0.51 ± 0.10	-0.22 ± 0.15
r^2	0.077 ± 0.008	0.067 ± 0.010



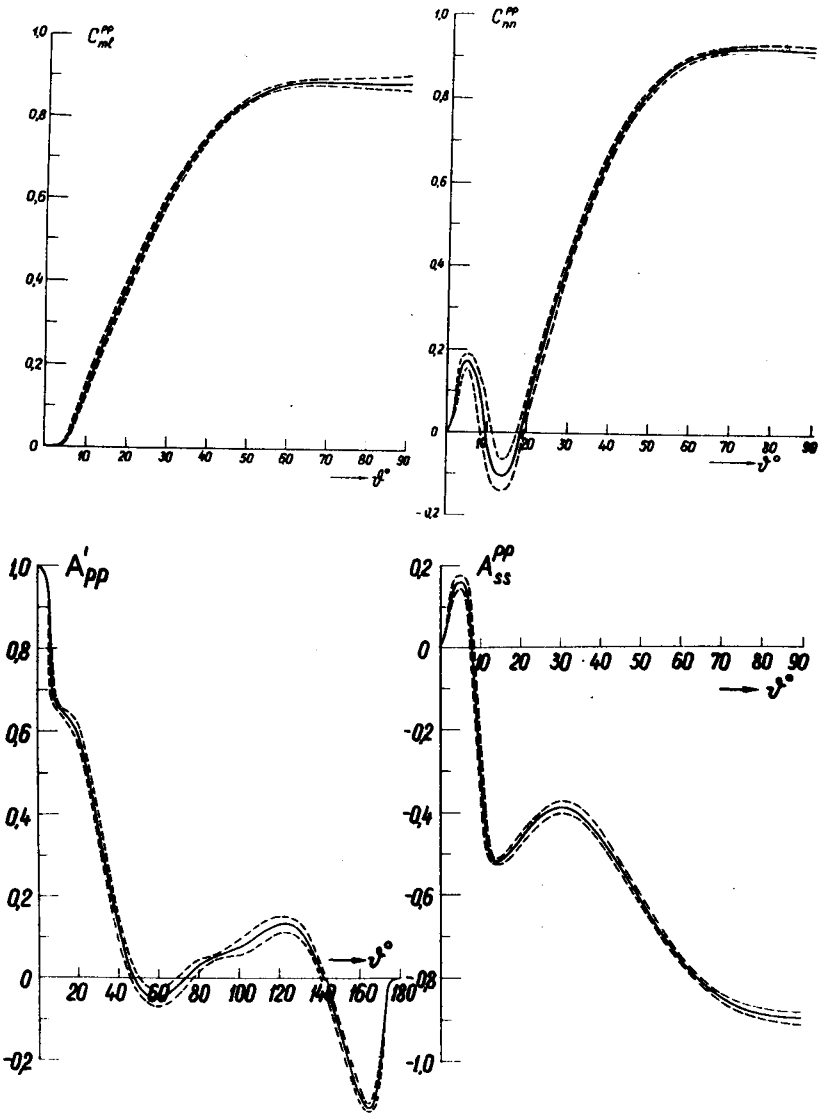
P_{pp}
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
10 20 30 40 50 60 70 80 90 100 110 120
→ $\vartheta°$
▽ -213MeV - MARSHALL et al.
○ -210MeV - TINLOT, WARNER
□ -217MeV - TINLOT, WARNER
× -210MeV - BASKIR et al.
σ_{pp}
5,0
4,0
3,0
2,0
0 10 20 30 40 50 60 70 80 90
→ $\vartheta°$
MARSHALL et al
KONRADI

Fig. 1.



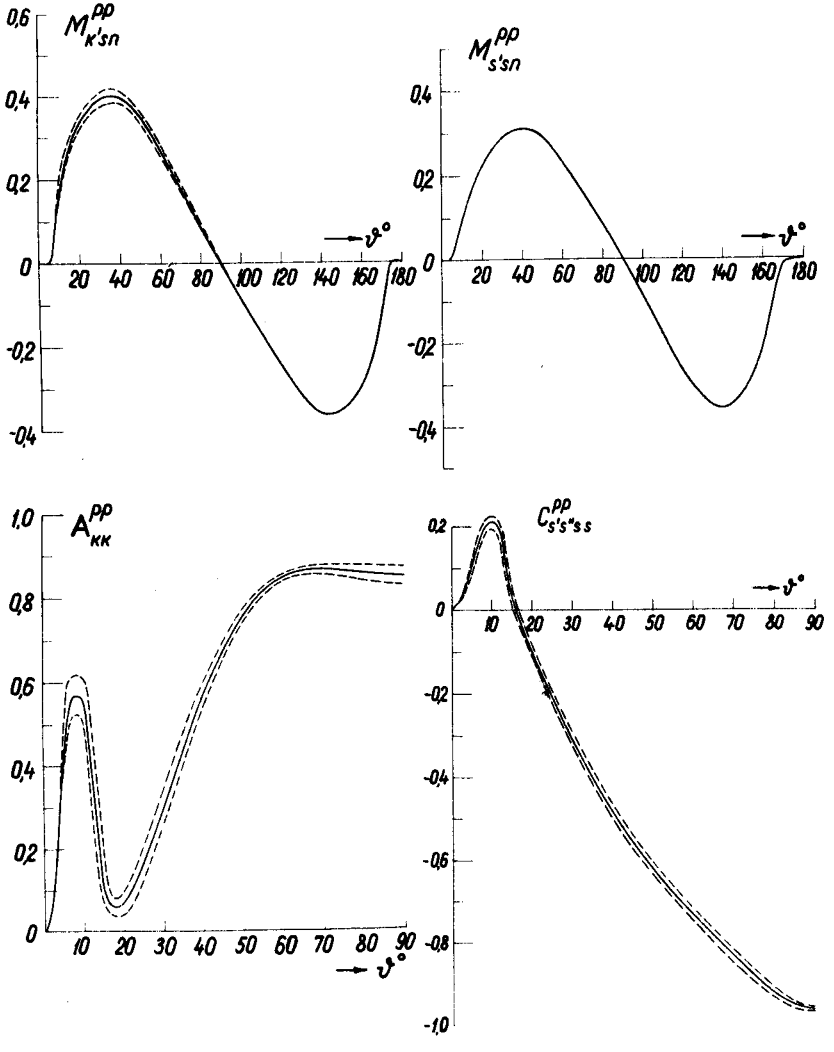
Dpp
GOTOW et al
Rpp
ENGLAND et al.
App
ENGLAND et al.
ENGLAND et al(corrected).
R'pp
GOTOW et al.

Fig. 2.



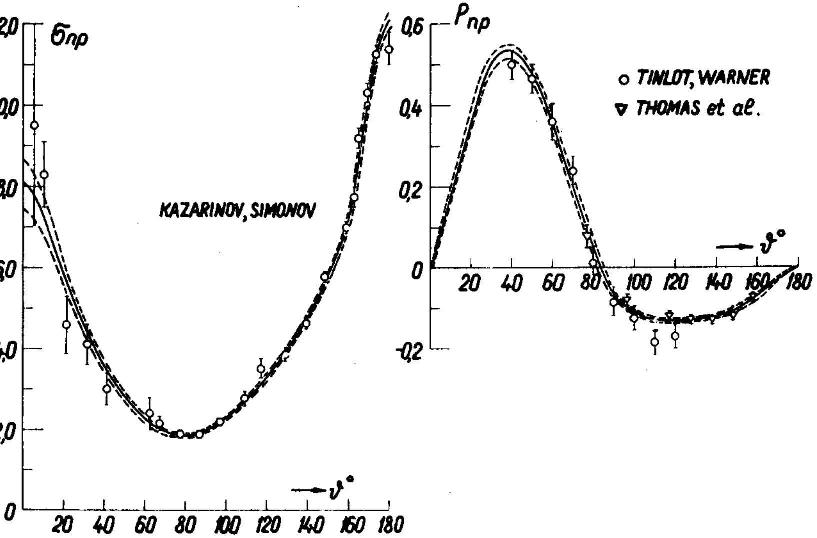
C_{ml}^{pp}
1,0
0,8
0,6
0,4
0,2
0
10 20 30 40 50 60 70 80 90
$\longrightarrow \vartheta^\circ$
C_{nn}^{pp}
1,0
0,8
0,6
0,4
0,2
0
-0,2
10 20 30 40 50 60 70 80 90
$\longrightarrow \vartheta^\circ$
A'_{pp}
1,0
0,8
0,6
0,4
0,2
0
-0,2
20 40 60 80 100 120 140 160 180
$\longrightarrow \vartheta^\circ$
A_{ss}^{pp}
0,2
0
-0,2
-0,4
-0,6
-0,8
-1,0
10 20 30 40 50 60 70 80 90
$\longrightarrow \vartheta^\circ$

Fig. 3.



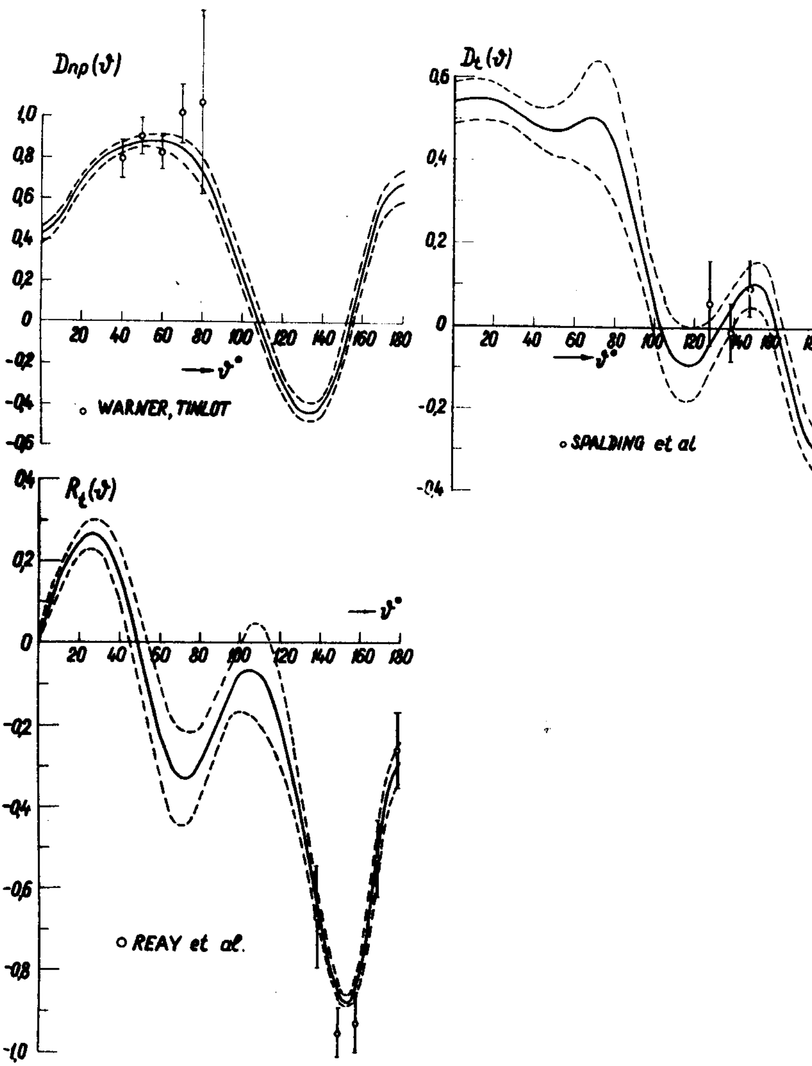
$M^{pp}_{k's n}$
$M^{pp}_{s's n}$
A^{pp}_{kk}
$C^{pp}_{s's''s s}$
ϑ°
0,6
0,4
0,2
0
-0,2
-0,4
20 40 60 80 100 120 140 160 180
1,0
0,8
0,6
0,4
0,2
-0,6
-0,8
-1,0
10 20 30 40 50 60 70 80 90

Fig. 4.



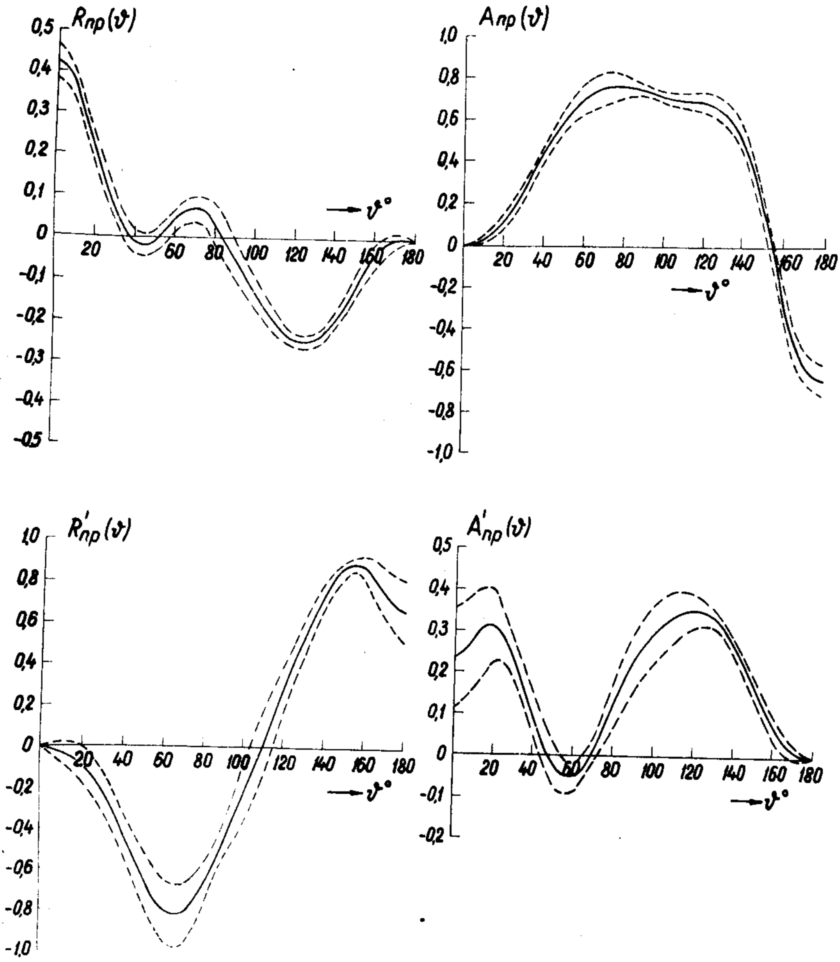
σnp
12,0
10,0
8,0
6,0
4,0
2,0
0
KAZARINOV, SIMONOV
→ ϑ°
20 40 60 80 100 120 140 160 180
Pnp
0,6
0,4
0,2
0
-0,2
○ TINLOT, WARNER
▽ THOMAS et al.
→ ϑ°
20 40 60 80 100 120 140 160 180

Fig. 5.



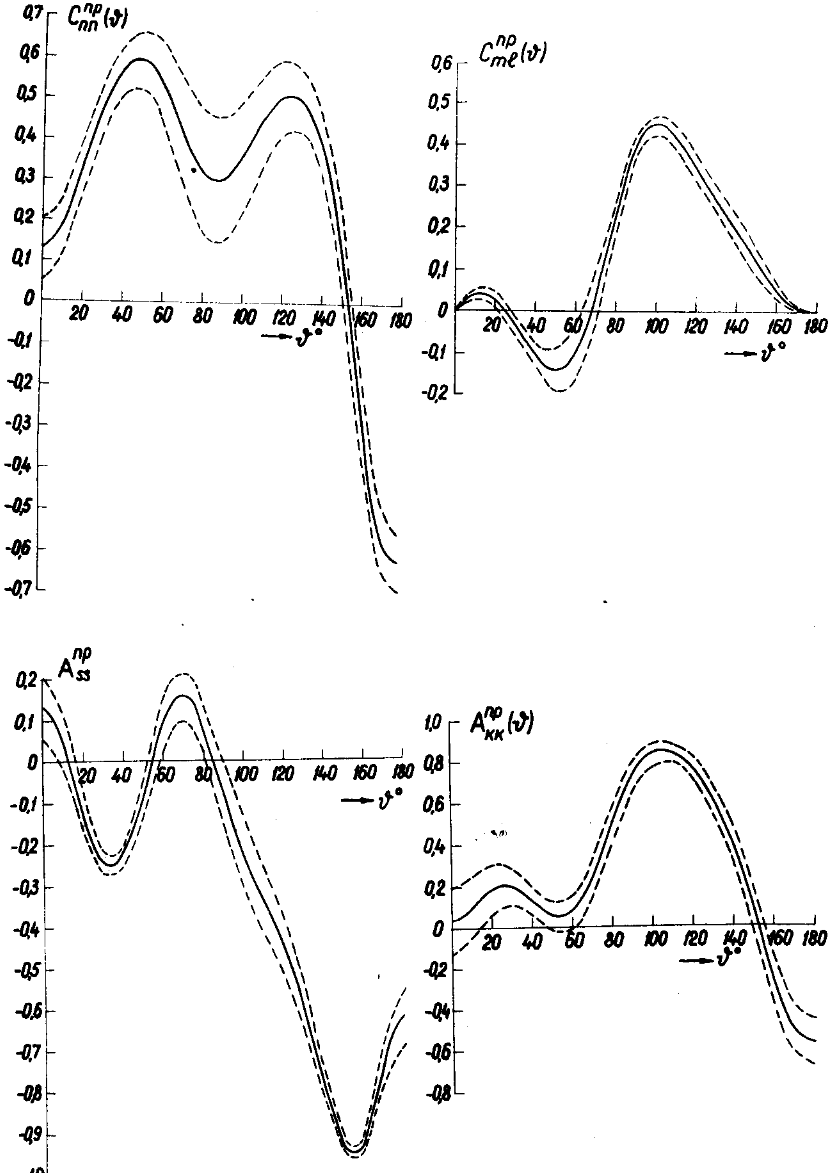
Dnp(ϑ)
1,0
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
20 40 60 80 100 120 140 160 180
ϑ°
WARNER, TINLOT
Dt(ϑ)
0,6
0,4
0,2
0
-0,2
-0,4
20 40 60 80 100 120 140 160 180
ϑ°
SPALDING et al
Rt(ϑ)
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
-1,0
20 40 60 80 100 120 140 160 180
ϑ°
REAY et al.

Fig. 6.



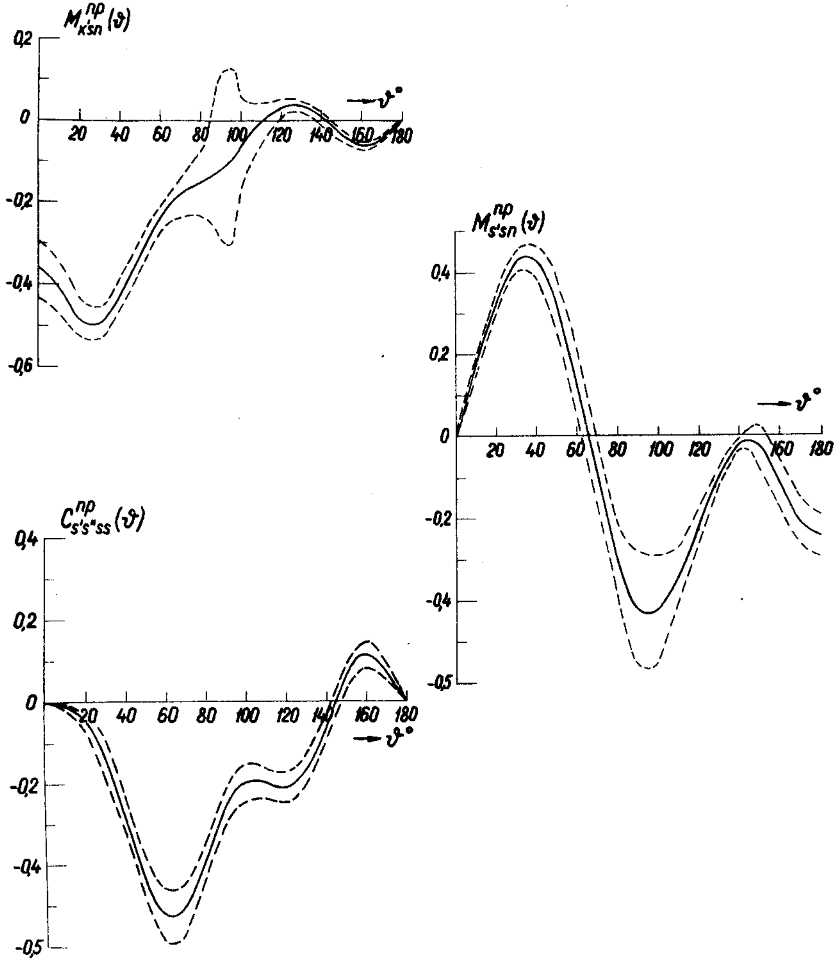
$R_{np}(\vartheta)$
$A_{np}(\vartheta)$
$R'_{np}(\vartheta)$
$A'_{np}(\vartheta)$
$\vartheta°$

Fig. 7.



$C_{nn}^{np}(\vartheta)$
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
-0,7
20 40 60 80 100 120 140 160 180
$\vartheta°$
$C_{m\ell}^{np}(\vartheta)$
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
20 40 60 80 100 120 140 160 180
$\vartheta°$
A_{ss}^{np}
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
-0,7
-0,8
-0,9
-1,0
20 40 60 80 100 120 140 160 180
$\vartheta°$
$A_{KK}^{np}(\vartheta)$
1,0
0,8
0,6
0,4
0,2
0
-0,2
-0,4
-0,6
-0,8
20 40 60 80 100 120 140 160 180
$\vartheta°$

Fig. 8.



$M^{np}_{x'sn}(\vartheta)$
0,2
0
-0,2
-0,4
-0,6
20 40 60 80 100 120 140 160 180
$\vartheta°$
$M^{np}_{s'sn}(\vartheta)$
0,4
0,2
0
-0,2
-0,4
-0,5
20 40 60 80 100 120 160 180
$\vartheta°$
$C^{np}_{s's''ss}(\vartheta)$
0,4
0,2
0
-0,2
-0,4
-0,5
20 40 60 80 100 120 140 160 180
$\vartheta°$

Fig. 9.