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Z. Janout, Yu.M. Kazarinov, F. Lehar, I. Ulehla

PHASE-SHIFT ANALYSIS AND PLANNING OF EXPERIMENTS AT 18.2 AND 9.7 MEV

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The use of a polarized proton target (PPT) in experiments below 30 Mev opens new possibilities for investigating deviations from pure S -scattering, occurring at these energies. It seems highly probable, that a detailed investigation of P and D state nucleon interactions at 10 - 20 MeV, will make it possible to extend the region, in which the scattering amplitude is determined unambiguously to energies, at which the nucleons interact practically only in the S state, so that the phase-shift analysis is considerably simpler. It is therefore important to perform a phase-shift analysis of the existing experimental data in order to obtain information necessary for the planning of future experiments. This has been done previously at 14.5 MeV /1/, the results of the data processing at 9.7 and 18.2 MeV are given below.

Treating np and pp data simultaneously at relatively low energies (< 20 MeV) it is necessary to bear in view that the accuracy, with which the Coulomb effects are taken into account by the method used generally in the phase-shift analysis /2,3/ becomes worse as the energy decreases. In this paper an attempt was made in the framework of the existing notions, to estimate the deviations in the value of the total phase-shift, due to the assumed additivity of the Coulomb and nuclear scattering effects at various energies.

Another reason why the energy region up to 20 MeV is interesting is that it is possible to obtain intensive monoenergetic polarized beams of nucleons from nuclear reactins with proton or neutron emission. Nuclear reactions like $T(d,n)He^3$ et al are realized on cyclotrons and Van de Graaf generators, available in many laboratories.

The treated data are given in Table 1. It should be noted that at 9.7 and 18.2 MeV the existing experimental data are not sufficient for performance of the phase-shift analysis taking the D - wave into account, so that it was necessary to use data obtained by extrapolating of experimental results at near energies in the calculation. Estimated values were taken for the pp polarization and np spin correlation.

The phase-shift analysis was performed according to a program, described in detail in /4/. The scattering amplitude was taken in the one-pion approximation for $\ell = 3$. The Coulomb effects were considered assuming the Coulomb and nuclear phase-shifts to be additive. The coupling constant was set equal to 0,08 and fixed. In a search for solutions starting from random initial conditions 43 and 50 trials were performed at 18.2 and 9.7 MeV, respectively. This lead to solutions with positive $^1S_0$ phase-shifts, given in Tables 2,3. All the solutions describe the experimental data equally adequately. Each of the found solutions was repeated 3 or 4 times. The results obtained at 18.2 MeV show, that the phase-shifts of the $^3D_2$ and $^3D_3$ waves are small and are determined with large errors. Therefore at 9.7 MeV they were set equal to zero and fixed.

The angular dependences of the experimental quantities given on figs. 1-12 were calculated using the obtained phase-shift at both energies. The vertical lines shown the errors, calculated from the error matrix of the phase-shifts.

In view of the short range of the nuclear forces it is possible to present the pp scattering phase-shifts as

$$\delta_\ell^{pp} = \delta_\ell^N + \sigma_\ell$$

where σ_ℓ is the Coulomb phase-shift /2,3,5/, δ_ℓ^N the nuclear one /2/. It was assumed that δ_ℓ^N is equal to the np -scattering phase-shift δ_ℓ^{np} so that phase-shifts at $T = 1$ may somewhat differ from the "true" values of both δ_ℓ^N and δ_ℓ^{np}. The $T = 0$ phase-shift would be exact if all the parameters under consideration were non-correlated. Actually the correlations between the parameters prove to be small, so that the mentioned assumption should have little influence on the $T = 0$ phase-shifts.

The deviations from pure S -scattering are small already at 18.2 MeV and the P and D phase-shifts do not exceed several degrees. It is therefore interesting to estimate the possible error in the $T = 1$ phase-shifts, due to the assumed additivity of the Coulomb and nuclear scattering effects. This value can of course only be estimates under definite assumptions concerning the nuclear forces. It was assumed that the nuclear forces can be described by a potential

$V(r)$ and that the result of the interaction (the phase-shift) in a given state or group of states can be calculated, using the Schrödinger equation with the potential $V(r)$ for energues up to the pion production threshold.

It proved convenient to connect the potential $V(r)$ with the phase-shift by means of a non-linear equation of the first order, the which the radial part of the Schrödinger equation can be reduced /6/:

$$\frac{dy_\ell}{dr} = \frac{-V(r)}{k}[\hat{j}_\ell \cos y_\ell + \hat{n}_\ell \sin y_\ell]^2 \tag{1}$$

where

$$\lim_{r\to\infty} y_\ell(k,r) = \delta_\ell$$

$$y_\ell(0,r) = 0$$

and k is the relative motion momentum, $V(r)$ is the properly normalized potential and j_ℓ and n_ℓ are the spherical Bessel and Neuman functions, respectively.

If both Coulomb and nuclear interactions are present, the sum of the Coulomb and nuclear potential $V_{\mathcal{C}}+V$ figures in the Schrödinger equation and equation (1) has to be replaced by /7/

$$\frac{dy_\ell}{dr} = -\frac{V(r)}{k}[F_\ell \cos y_\ell + G_\ell \sin y_\ell]^2 \tag{2}$$

where F_ℓ and G_ℓ are the continuons spectrum Coulomb wave functions. The problem was solved in two stages. Firstly expression (2) was used to determine the potential $V(r)$ from the energy dependence of the phase-shifts, known from the phase-shift analysis. This was done assuming that

$$V(r) = \frac{\ell^{-r}}{r}\sum_{n=0}^{n_{max}}\{a_n + b_n J(J+1)\}\ell^{-nr} \tag{3}$$

where J is the total momentum. The coefficients a_n and b_n in (3) were determined using the least squares method, so as to obtain an optimal description of the energy dependence of the phase-shifts, obtained from the simultaneous pp and np phase-shift analysis for a given orbital momentum state (or group of states) in the energy region up to the pion-production threshold.

Secondly, equation (2) with the nuclear potential found from (1) was solved, so as to find the value of the nuclear phase-shift δ_ℓ^N in the presence of the Coulomb field.

The difference

$$\Delta_\ell = \delta_\ell^{np} - \delta_\ell^{N} \tag{4}$$

characterizes the error made in the simultaneous phase-shift analysis.

In the calculation above, the phase-shifts obtained from the phase-shift analysis were assumed to be equal to δ_ℓ^{np}. This is actually a limit case, since the parameters determined in the simultaneous phase-shift analysis are known to lie between δ_ℓ^{N} and δ_ℓ^{np}. The calculations showed that the second limit case, i.e. the assumption that the quantities determined in the phase-shift analysis, are equal to δ_ℓ^{N} gives the same result for Δ_ℓ. It should be noted, that the reliability with which Δ_ℓ can be determined decreases, when going over to lower energies, since at very low energies the pp and np phase-shift energy dependences cannot be described by the same potential $V(r)$ at all[8]. The energy dependence of Δ_ℓ is shown on fig. 13 for the S, P, D and F waves at $T = 1$.

The vertical lines give the errors in the phase-shift analysis. It can be seen from the figure 13 that the deviation from the additivity above 20 MeV does not exceed the accuracy, of the phase-shift determination. The situation is significantly worse below 20 MeV.

The errors in accounting for the Coulomb effects significantly disturb the $^1S_0$ shift, as well as the $^3P_1$ a $^3P_0$ ones (these, however, within the experimental errors).

The planning of experiments using the method suggested by Sokolov[9], showed that the best way to decrease the errors in the determination of the phase-shifts, is to perform the following experiments.

At 18.2 MeV it is advisable the measure the differential cross section σ_{np} for angles larger than 60°. If a monoenergetic polarized neutron beam and a PPT are available, the simplest further experiments are those, determining the asymmetry tensor component A_{ss}^{np} and the spin correlation C_{nn}^{np} in the region 60 -120°. Experiments measuring the polarization P_{np} can only give useful information, if the experimental errors will be smaller than 0.01.

At 9.7 MeV the simplest experiments are those determining A_{ss}^{np} and C_{nn}^{np} using a polarized neutron beam and a PPT. The quantity C_{nn}^{np} will probably be small and it will be necessary to measure the cross section with high accuracy (0.005).

The measurement of triple scattering parameters is very helpful in the determination of the most probable set of phase-shifts. However, at present these experiments are extremely difficult. It should be noted, that as soon as the accuracy with which the phase-shifts are determined, will be considerably improved, it will be possible to determine the most probable set on the basis of the monotonous energy dependence of the phase-shifts.

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T a b l e 1

Quantity	9.7 MeV			18.2 MeV		
	Number of points	Energy MeV	Ref.	Number of points	Energy MeV	Ref.
σ_{pp}	26	9.69	[10]	8	18.2	[17]
P_{pp}	4	(**)		2	16.2; 17.7	[18, 19]
C_{nn}^{pp}	1	10	[11]	1	18.2	[11]
A_{ss}^{pp}	1	18.2 - 25.7 (*)	[11]	1	18.2	[11]
σ_{np}	14	14.1 (*)	[12, 13]	14	14.5 - 22.5 (*)	[12,13,14]
P_{np}	3	(**)		8	16.4 - 20; 0	[15, 16]
C_{nn}^{np}	1	(**)		1	(**)	
σ_{np}^{tot}				1	18.2	[20]

(*) The interpolated or extrapolated values.

(**) The estimated value.

T a b l e 2

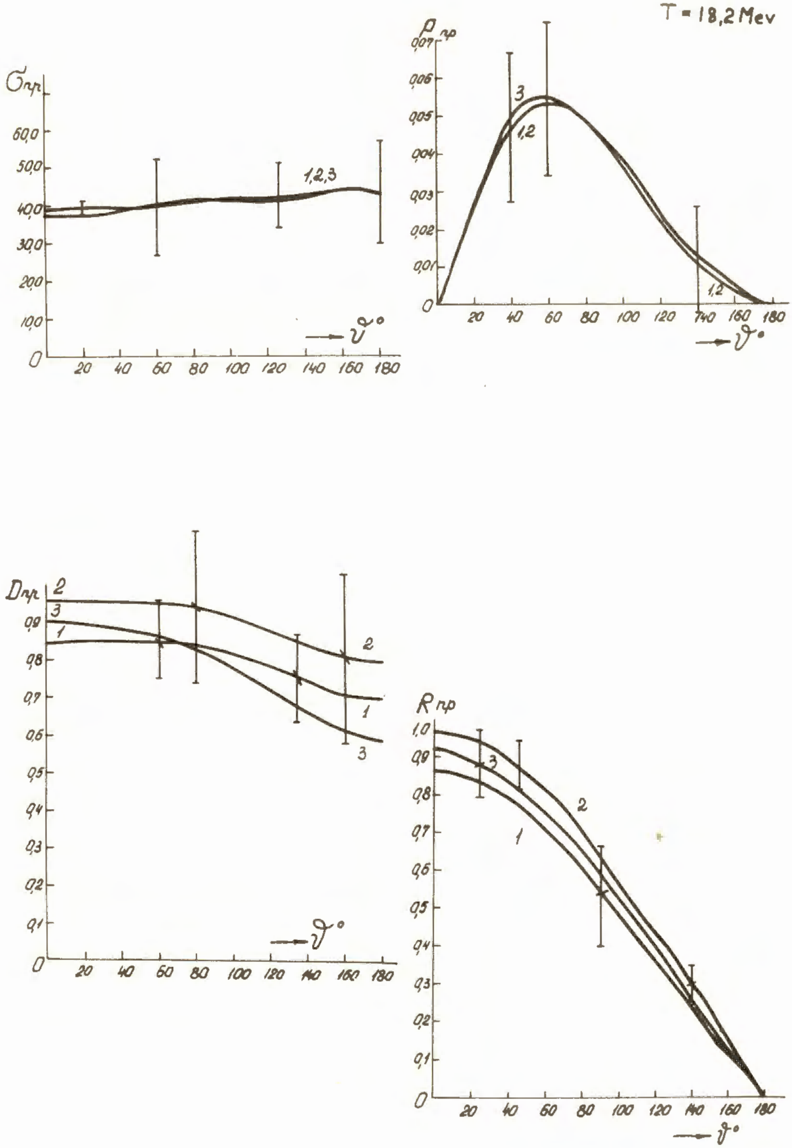
The phase-shifts in degrees (the Stapp parametrization[2]) for 18.2 MeV nucleon-nucleon scattering.

	1 - st set		2 - nd set		3 - rd set	
f^2	0.08 fix.		0.08 fix.		0.08 fix.	
	δ°	$\pm\,\Delta\delta^\circ$	δ°	$\pm\,\Delta\delta^\circ$	δ°	$\pm\,\Delta\delta^\circ$
$^1S_0$	52.21	0.57	52.22	0.57	51.71	0.70
$^3S_1$	101.80	5.05	78.07	4.86	102.01	3.80
$^3P_0$	7.14	1.14	7.28	1.46	-8.49	1.35
$^1P_1$	-3.89	3.52	-6.25	13.17	-3.28	4.78
$^3P_1$	-3.09	0.76	-3.04	0.74	4.89	0.95
$^3P_2$	2.98	0.45	2.89	0.45	1.76	0.48
ε_1	-1.38	6.88	0.79	14.89	-2.38	8.44
$^3D_1$	1.63	12.94	-2.42	28.92	1.74	8.05
$^1D_2$	0.66	0.06	0.65	0.06	0.20	0.09
$^3D_2$	-0.64	10.75	0.27	32.59	0.08	4.69
$^3D_3$	1.31	4.74	-0.96	7.49	1.66	4.03
χ^2	8.503		9.30		10.35	

Table 3

The phase-shifts in degrees (the Stapp parametrization [2]) for 9.7 MeV nucleon-nucleon scattering.

	1 - st set		2 - nd set		3 - rd set		4 - th set	
f^2	0.08 fix.		0.08 fix.		0.08 fix.		0.08 fix.	
	δ°	$\pm \Delta\delta^\circ$	δ°	$\pm \Delta\delta^\circ$	δ°	$\pm \Delta\delta^\circ$	δ°	$\pm \Delta\delta^\circ$
$^1S_0$	55.64	0.28	55.39	0.34	55.64	0.28	55.38	0.34
$^3S_1$	102.65	1.88	102.70	1.79	77.37	1.77	77.32	1.85
$^3P_0$	4.17	1.20	-5.58	1.16	4.31	1.15	-5.43	1.20
$^1P_1$	-2.56	1.61	-1.84	1.62	-1.47	1.53	-2.48	1.57
$^3P_1$	-2.13	0.88	2.21	0.94	-2.05	0.92	2.31	0.88
$^3P_2$	0.13	0.58	-0.44	0.60	0.06	0.60	-0.53	0.58
ε_1	-6.55	1.74	-6.58	1.69	-6.78	1.67	-6.61	1.75
$^3D_1$	0.05	1.38	0.23	1.39	-0.26	1.39	-0.13	1.38
$^1D_2$	0.14	0.03	0.04	0.04	0.14	0.04	0.04	0.04
$^3D_2$	0 fix.		0 fix.		0 fix.		0 fix.	
$^3D_3$	0 fix.		0 fix.		0 fix.		0 fix.	
χ^2	25.82		25.69		25.34		26.09	



T = 18,2 Mev
σnp
60,0
50,0
40,0
30,0
20,0
10,0
0
1,2,3
20 40 60 80 100 120 140 160 180
ϑ°
Pnp
0,07
0,06
0,05
0,04
0,03
0,02
0,01
0
3
1,2
1,2
20 40 60 80 100 120 140 160 180
ϑ°
Dnp
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
2
3
1
20 40 60 80 100 120 140 160 180
ϑ°
Rnp
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
20 40 60 80 100 120 140 160 180
ϑ°

Fig. 1.

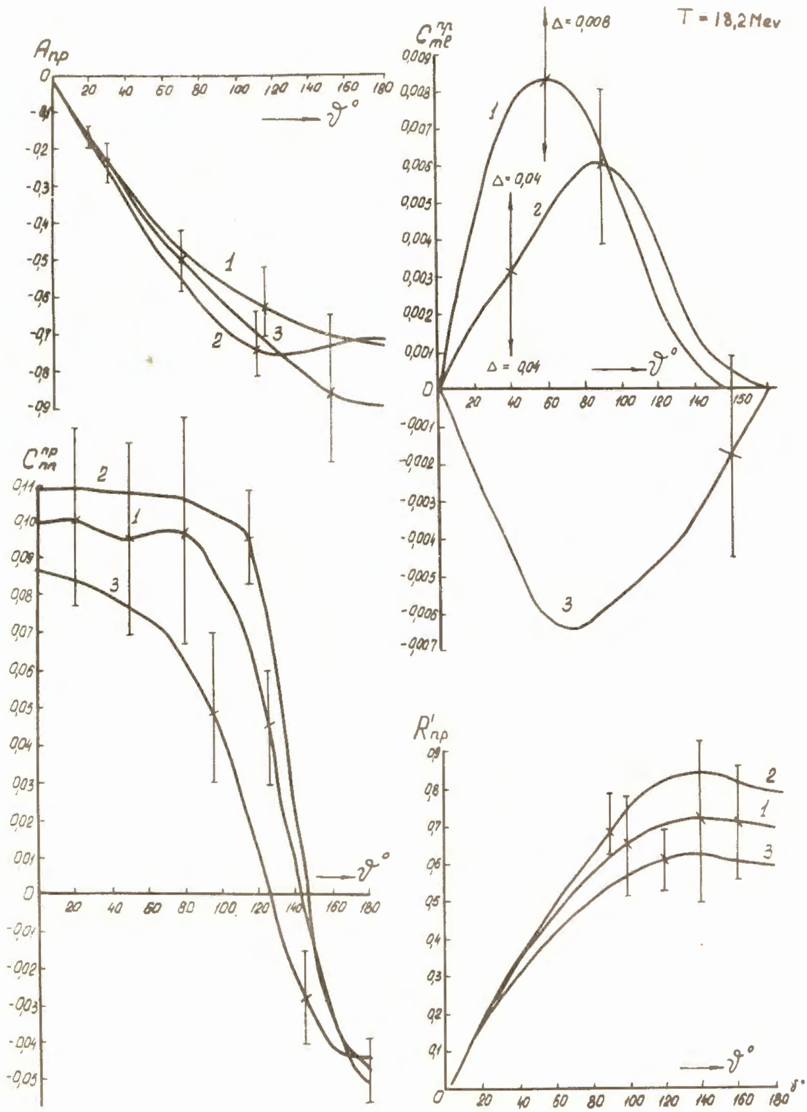


Fig. 2.

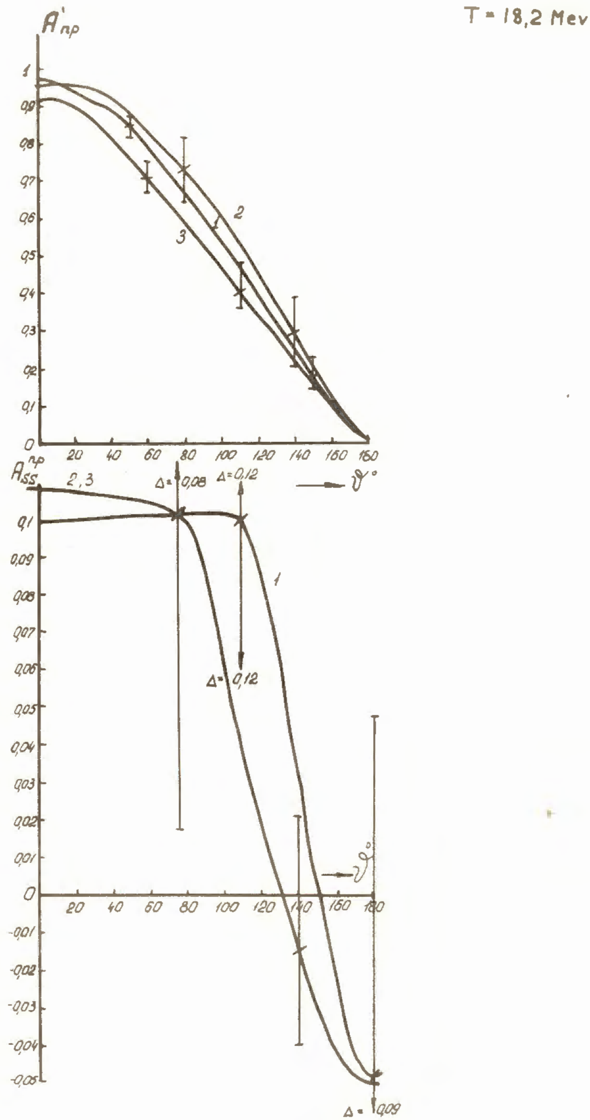


Fig. 3.

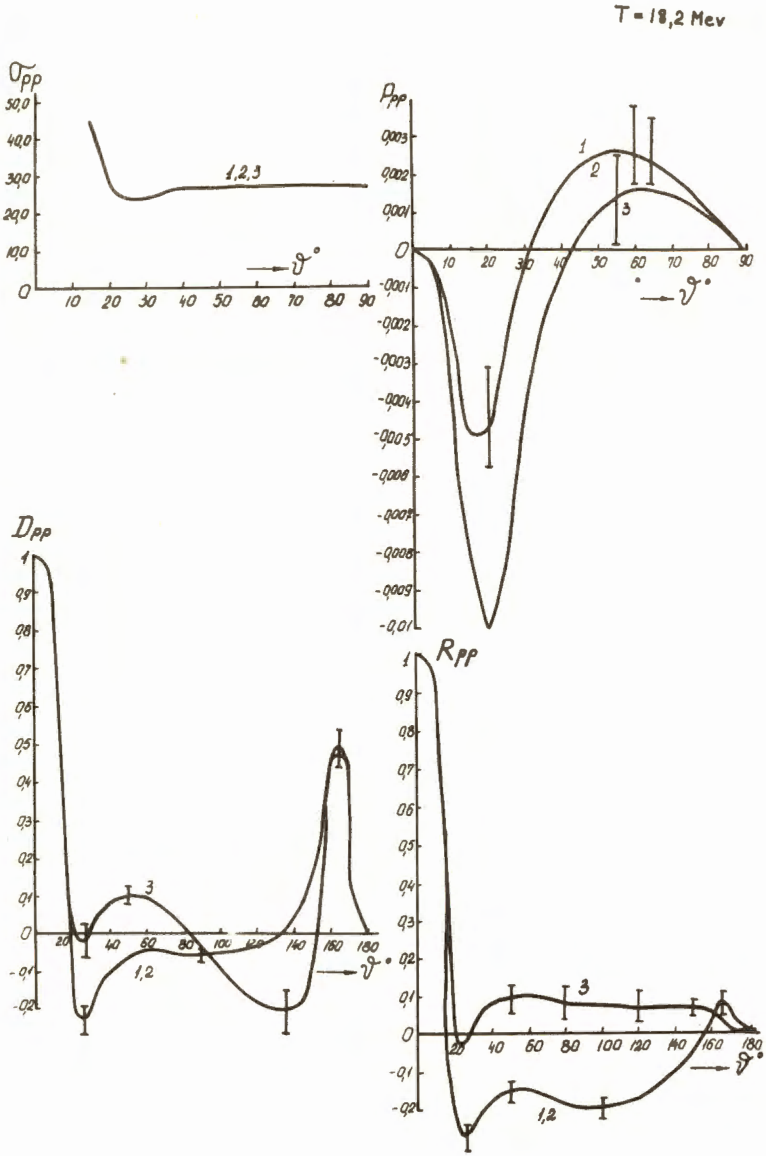


Fig. 4.

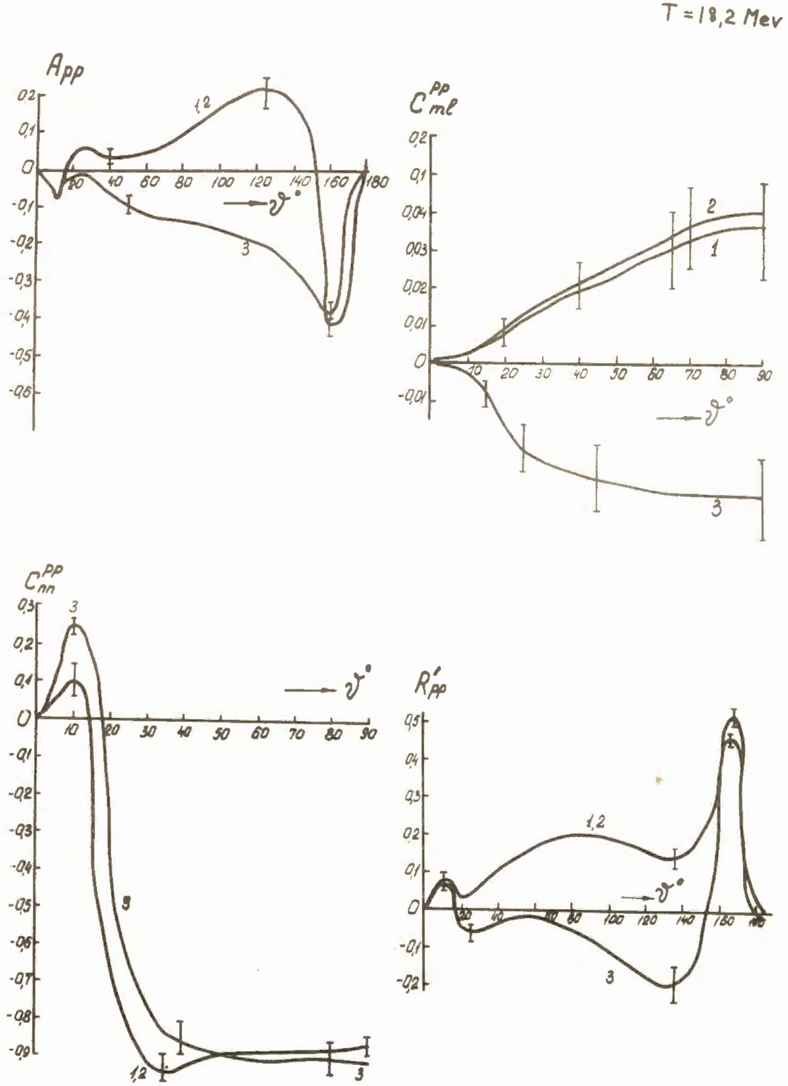


Fig. 5.

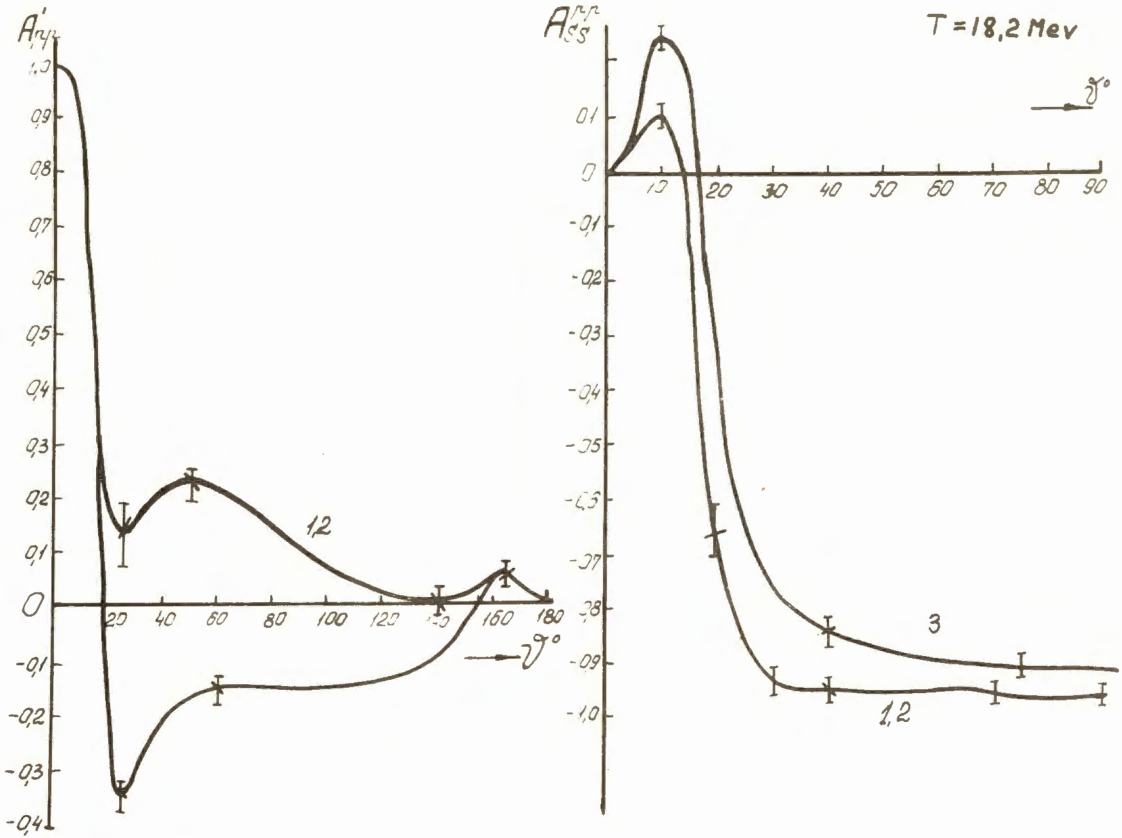

T = 18,2 Mev
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
20
40
60
80
100
160
180
1,2
0,1
-0,1
-0,2
-0,3
-0,4
-1,0
20
30
40
50
60
70
80
90
3
1,2

Fig. 6.

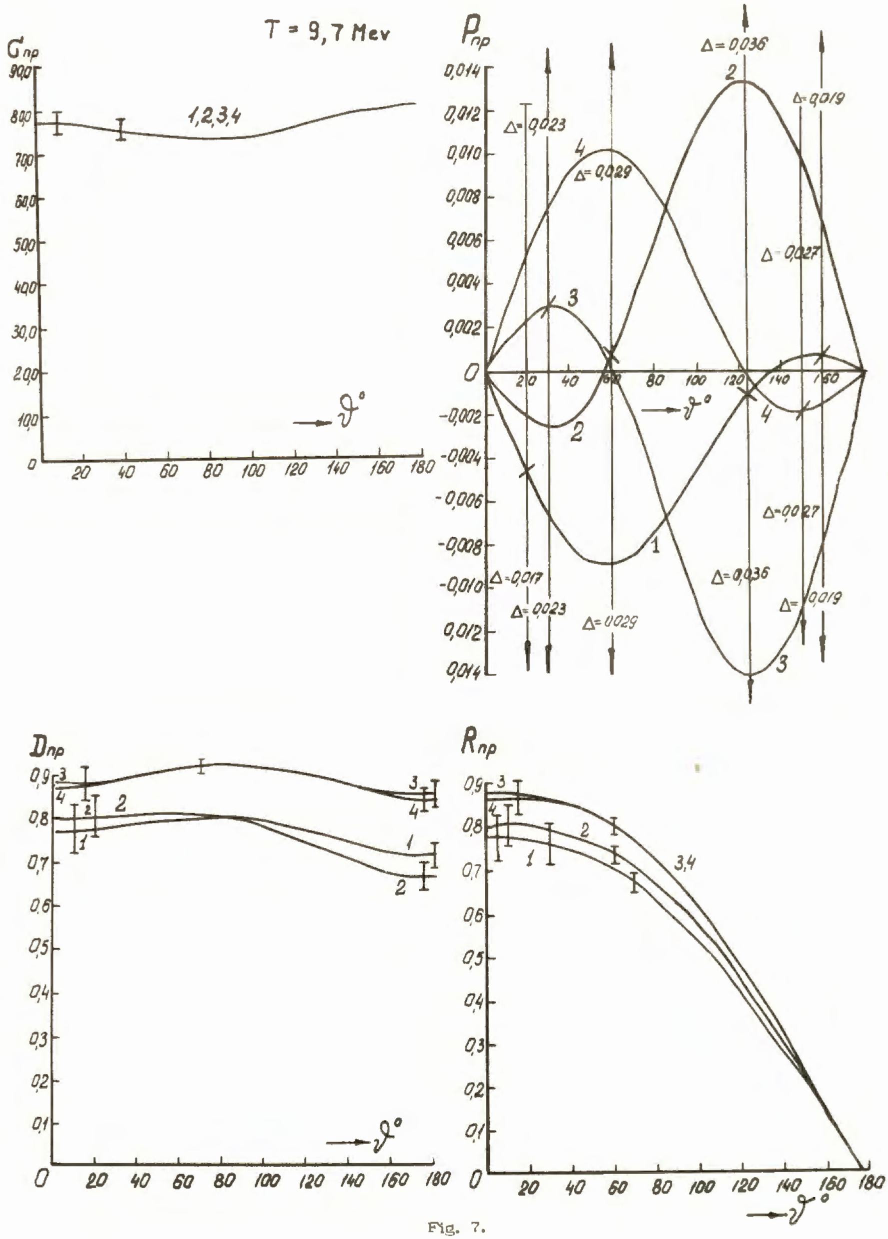


Fig. 7.

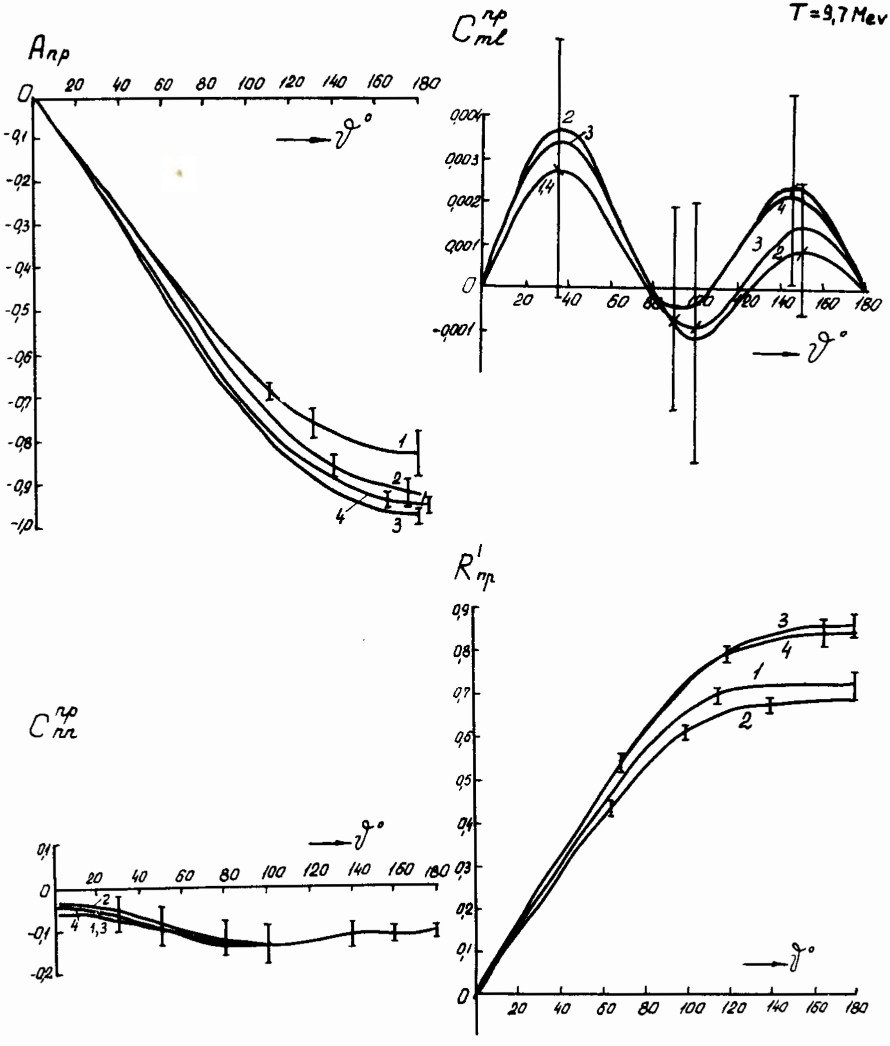


Fig. 8.

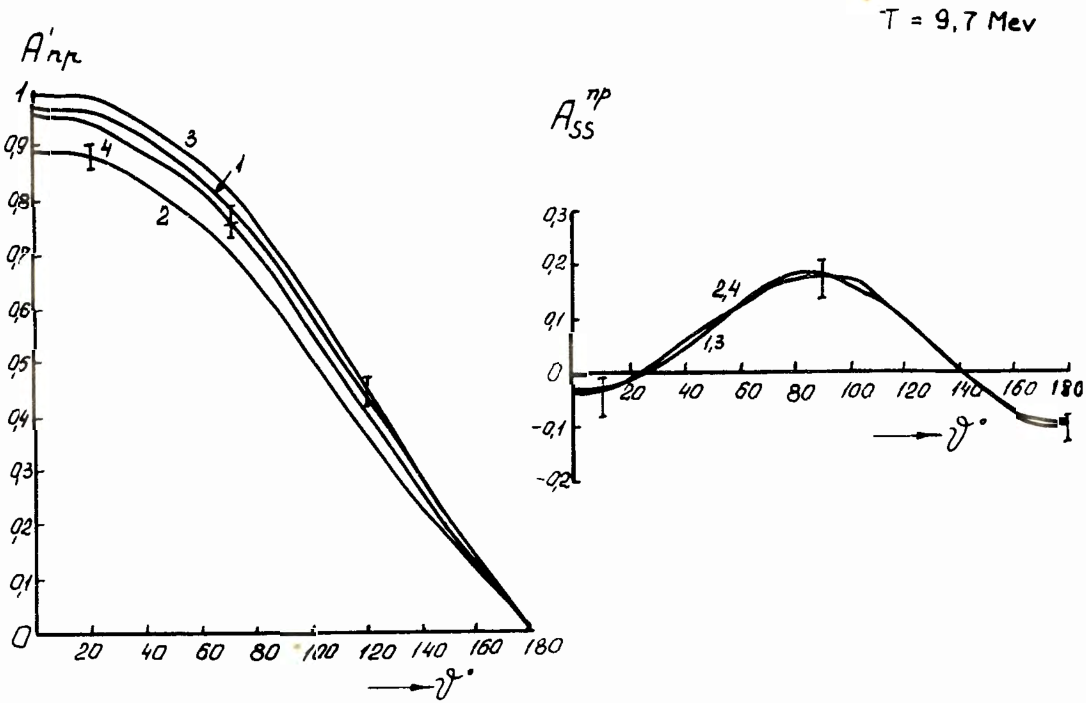

T = 9,7 Mev
A'np
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
20
40
60
80
100
120
140
160
180
ϑ°
1
2
3
4
A_{ss}^{np}
0,3
0,2
0,1
0
-0,1
-0,2
2,4
1,3
20
40
60
80
100
120
140
160
180
ϑ°

Fig. 9.

T = 9,7 Mev

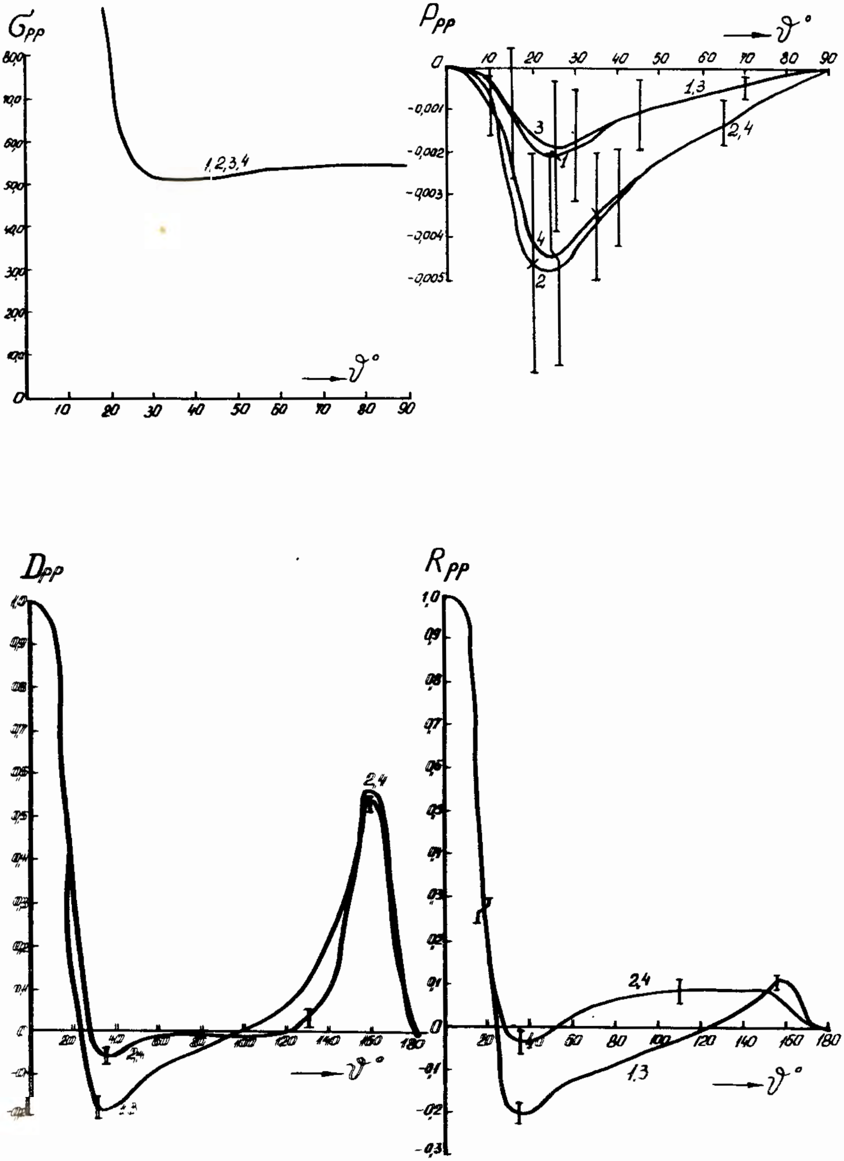


Fig. 10.

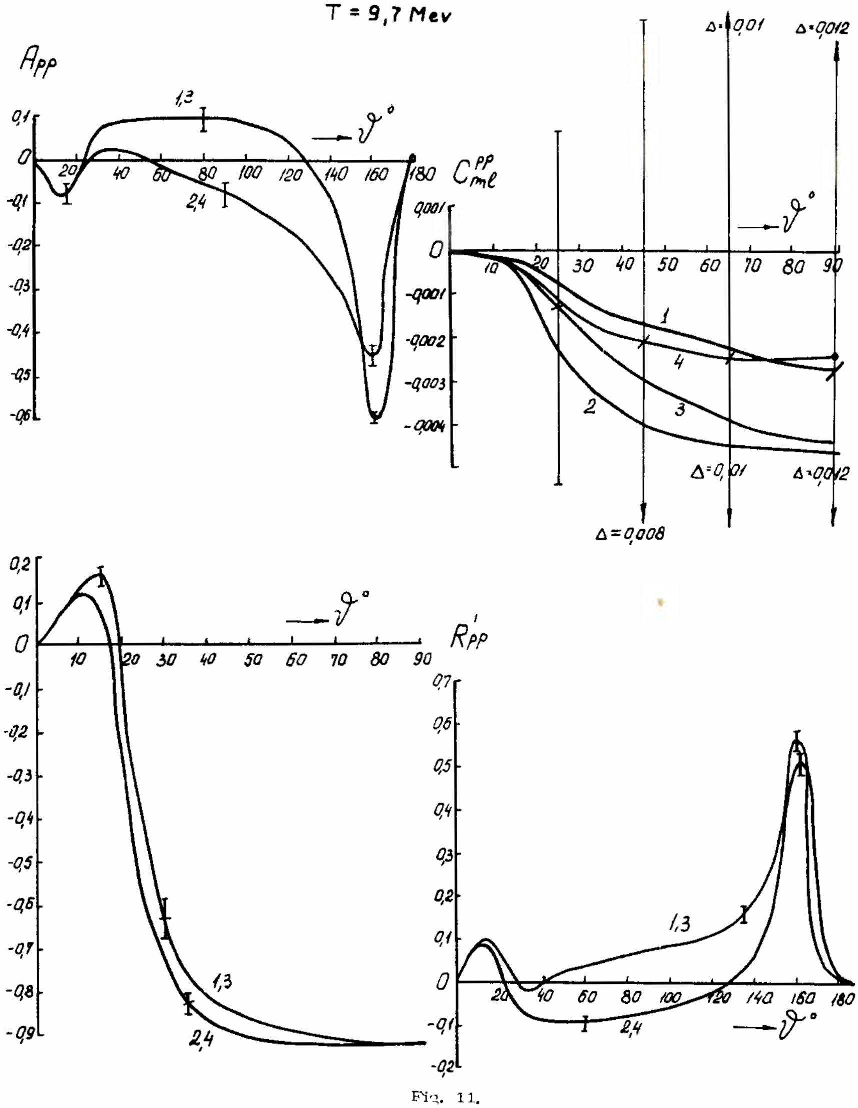


Fig. 11.

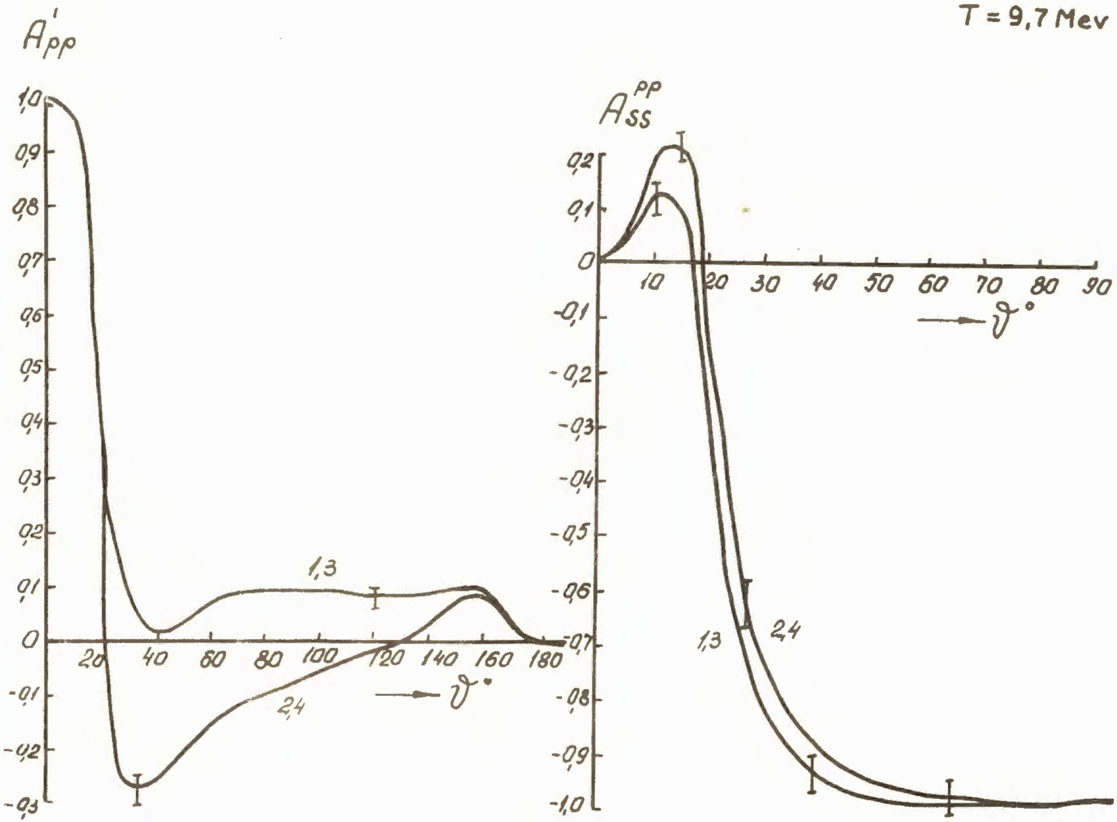

T = 9,7 Mev
A'_{pp}
1,0
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
20
40
60
80
100
120
140
160
180
1,3
2,4
$\longrightarrow \vartheta^\circ$
A_{ss}^{pp}
0,2
0,1
0
-0,1
-0,2
-0,3
-0,4
-0,5
-0,6
-0,7
-0,8
-0,9
-1,0
10
20
30
40
50
60
70
80
90
$\longrightarrow \vartheta^\circ$
1,3
2,4

Fig. 12.

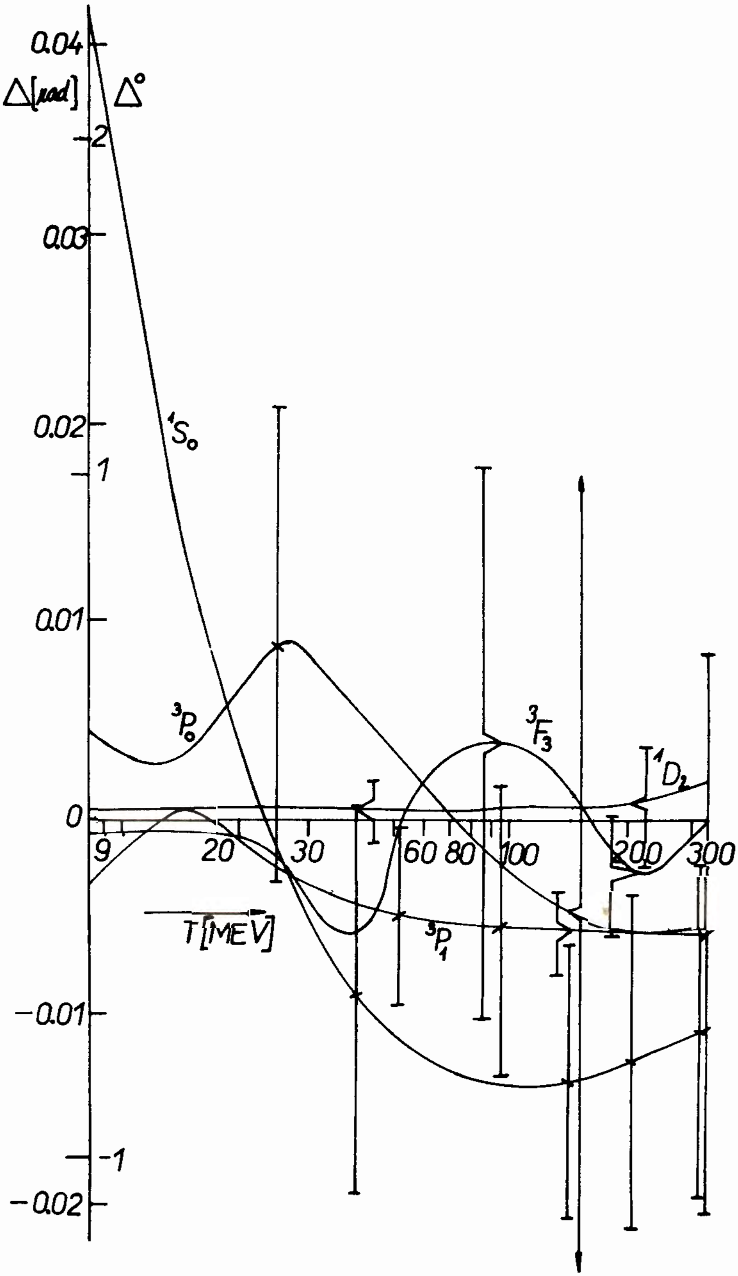


Fig. 13. The dependence of Δ_ℓ on the energy for S, P, D, and F waves at $\bar{T} = 1$. The vertical lines denote the phase- shift errors.