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POSSIBILITIES TO TEST QCD IN DEEP-INELASTIC MUON SCATTERING AT LARGE q^2

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POSSIBILITIES TO TEST QCD IN DEEP-INELASTIC MUON SCATTERING AT LARGE q^2

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Возможности проверки QCD в глубоконеупругих рассеяниях мюонов при больших q^2

Обсуждается возможность определения моментов структурной функции $F_2(x,q^2)$ при рассеянии мюонов на углероде на ускорителе СПС в ЦЕРНе. Предсказания QCD для кинематической области $q^2 \sim 100$ (ГэВ/с)^2 сравниваются с ожидаемыми статистическими ошибками.

С этой целью светимость установки НА-4 учитывается вместе со структурной функцией, полученной на дейтериевой мишени. Ограничение первичной энергии определяет пределы для q^2. Верхний предел определен из условия, что моменты аппроксимируются с точностью 10%. В области $q^2 \sim 100$ (ГэВ/с)^2 продемонстрировано влияние глюонов и кварков и поправок второго порядка. Анализ возможности эксперимента показывает, что моменты можно измерять до $q^2 \sim 50$, 160, 230 (ГэВ/с)^2 для $\mu = 2,4,6$. При этом статистическая точность должна быть достаточной для измерения параметров QCD.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Possibilities to Test QCD in Deep-Inelastic Muon Scattering at Large q^2

The possibilities are discussed to evaluate the moments of the structure function $F_2(x,q^2)$ in the muon carbon experiment NA-4 carried out at the CERN SPS. The predictions of QCD for the kinematic region of q^2 of about 100 (GeV/c)^2 are compared with the expected experimental accuracy.

For this purpose the luminosity of NA-4 set-up and the structure function obtained on deuterium target have been taken into account. The finite primary energies set a limit on q^2 reachable. The upper limits were determined requiring 10% accuracy of the approximation of the moments. The influence of the gluons and quarks as well as the second order corrections are illustrated in this q^2 range.

The analysis of the possibilities of the experiment has shown that the moments can be measured up to q^2 of 50, 160, 230 (GeV/c)^2 for $n = 2,4,6$ respectively. The statistical accuracy should be sufficient to determine QCD parameters.

The investigation has been performed at the Laboratory of High Energies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

1. INTRODUCTION

Deep-inelastic muon scattering offers the possibility to test QCD predictions by measuring the q^2 dependence of the moments of the structure function $F_2(x,q^2)$

$$M(n,q^2) = \int_0^1 dx\, x^{n-2} F_2(x,q^2). \qquad (1)$$

The moments of F_2 have been evaluated up to momentum transfers $q^2 \simeq 30\ (GeV/c)^2$. In this paper we analyze the possibility of testing QCD at the largest q^2 up to about $200 (GeV/c)^2$ reachable by the muon experiment NA-4 /2/. For this purpose we have estimated the influence of the gluon and quark contributions /3,4/ on $M(n,q^2)$ as well as that of second-order corrections /5,6/ in this new kinematic region.

2. EVALUATION OF $M(n,q^2)$ IN THE (q^2,x) REGION OF NA-4

In the anticipated region of large momentum transfers, the difference between the naive moments of eq. (1) and QCD moments including target mass effects becomes less important. This is illustrated in fig. 1 which gives the ratio r of the Nachtmann moments /3/ to the moments of eq. (1) as a function of q^2 for different n. Above $q^2 \approx 30\ (GeV/c)^2$ this ratio becomes larger than 94% depending only slightly on the specific F_2 chosen to calculate the moments. As expected, target mass effects are presumably negligible in the large q^2 region.

A limited primary energy of $E \leq 300$ GeV and the kinematics of deep-inelastic scattering determine a lower limit of the Bjorken variable x to be $x_{min} = q^2/2ME$ for a given q^2. Therefore the integral over x (eq. 1) can be calculated only approximately since data for small x and large q^2 will not be available. This restriction is important for $n=2$ (energy-momentum sum rule). For $n > 2$ the integrand of $M(n,q^2)$ is small at low x due to the weight factor x^{n-2} as illustrated in fig.2. We have estimated a certain minimum $x_n = x_{min}(n)$ (see fig. 2) which allows one to approximate $M(n,q^2)$ by

$$\int_{x_{min}}^{0.9} x^{n-2} F_2(x,q^2)\, dx \text{ with an accuracy of } \epsilon = \int_0^{x_{min}} x^{n-2} F_2\, dx/M \qquad (2)$$

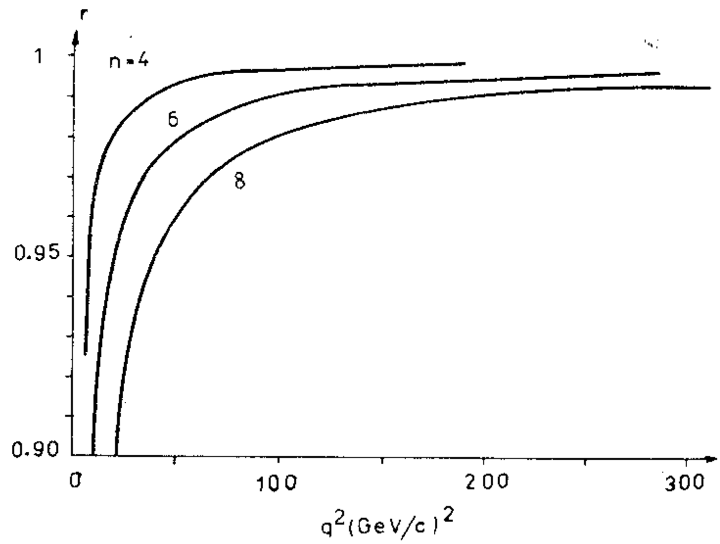

Fig. 1. Ratio of Nachtmann to naive moments (eq.1) for different n as a function of q^2; $M(n,q^2)$ are calculated with F_2 taken from ref. [9].

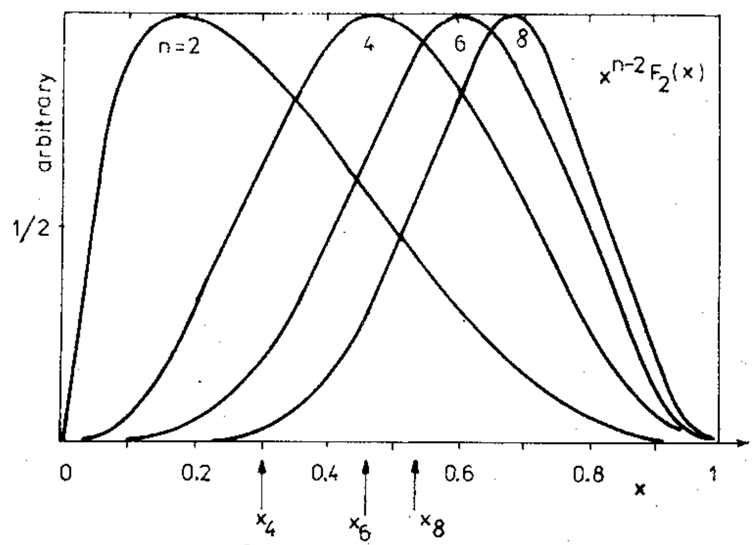

Fig. 2. Integrand $x^{n-2}F_2$ as given by the scale-invariant valence-quark parametrization of [8]; x_n is the minimum value of x to be included for the calculation of $M(n,q^2)$.

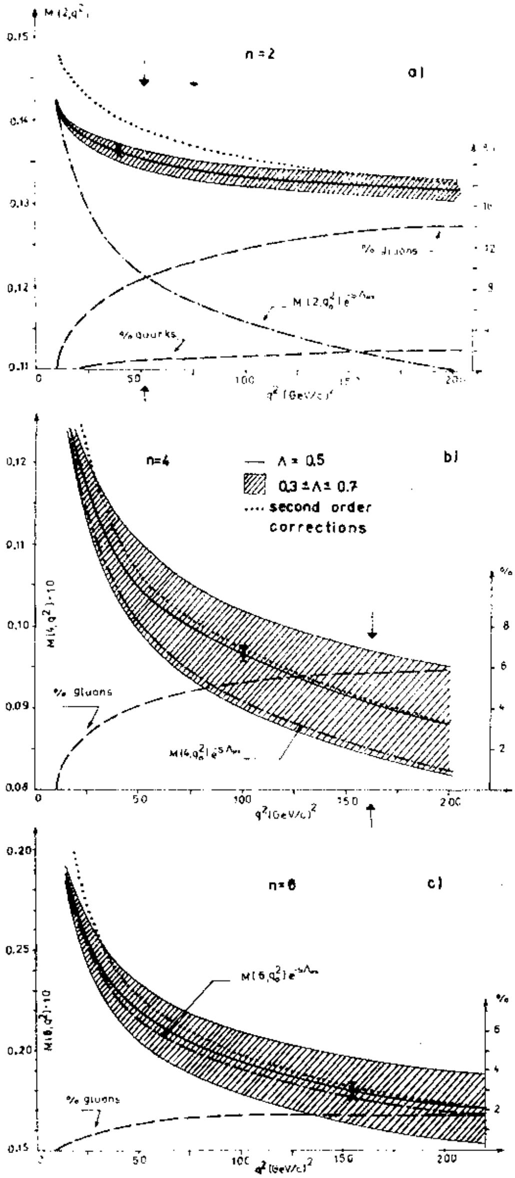

Fig.3. QCD moments as given by eq.(3) for a) n =2, b) n =4 and c) n =6 using M,G,Q at q_0^2 =10 (GeV/c)^2 from[1]. Solid curves: leading order M(n,q^2) for λ =0.5 GeV; dashed areas: variation of the leading order M for λ between 0.3 and 0.7 GeV; dashed-dotted curves: the leading term for λ =0.5; dotted curves: second-order corrections according to[5]; dashed curves: relative contributions of gluons and quarks (eq.3) in %. The error bars are the estimated statistical errors typical for the Na-4 experiment. The arrows indicate the q^2 limits set by the maximum SPS muon beam.

Using ϵ ~ 10%, the maximum values of q^2, up to which the moments of F_2 can be calculated in the NA-4 experiment, are given in the table. Note that in practice the accuracy of the approximation of the moments M(n,q^2) will be better than ϵ since the integrand of eq. (1) can be extrapolated

Table

Estimated maximum q^2 reachable at the CERN SPS to calculate the moments $M(n,q^2)$ of $F_2(x,q^2)$

n	q^2_{max} (GeV/c)^2 E = 200 GeV	q^2_{max} (GeV/c)^2 E = 280 GeV
2	40	50
4	110	160
6	160	230
8	200	280

to the region below x_{min} and above 0.9, respectively.

The expected statistical errors [7] of the moments have been estimated for $1.2 \cdot 10^{11}$ incoming muons and a 50 m carbon target [2] and $F_2(x,q^2)$ taken from [8]. These errors are included in fig. 3 to be discussed in the next section.

3. TESTING OF QCD AT LARGE q^2

Testing of QCD requires to compare the possibilities of the experiment with the sensitivity of the moments to parameters of the theory, to non-leading contributions and to second-order effects in the new q^2 range. In the notation of ref. [4] the n-th moment of F_2 can be written as

$$M(n,q^2) = M(n,q_0^2)e^{-s\lambda_{NS}} + X_n(s)G(n,q_0^2) + Y_n(s)Q(n,q_0^2), \qquad (3)$$

where $G(n,q_0^2)$ and $Q(n,q_0^2)$ are the moments of the gluon and quark distributions at a reference value q_0^2. The coefficients $X_n(s)$ and $Y_n(s)$ can be found for the leading order in [4] and for the next to the leading order in [5]. The "anomalous" dimensions λ_{NS} are specified by the theory, $s = \ln[\bar{g}(q_0^2)/\bar{g}(q^2)]$

$$\bar{g}(q^2) = \frac{16\pi^2}{\beta_0 \ln(q^2/\lambda^2)}\left[1 - \frac{\beta_1 \ln(\ln(q^2/\lambda^2))}{\beta_0^2 \ln(q^2/\lambda^2)}\right]. \qquad (4)$$

Here $\beta_0 = 11 - 2m_f/3$ and $\beta_1 = 102 - 38m_f/3$, where the number of flavours m_f is supposed to be 4. The parameter λ has to be determined along with the moments G and Q. For our purpose we have used G and Q at $q_0^2 = 10\,(\mathrm{GeV}/c)^2$ as given in [1]. As the NA-4 experiment will be carried out with a carbon target at first, we give results only for isoscalar combinations of moments.

In fig. 3 we show the behaviour of the QCD moments for $n = 2,4,6$. The dependence on λ is illustrated by the shaded areas giving the variation of $M(n,q^2)$ when λ changes from 0.3 to 0.7 GeV. The solid curves represent the moments (eq.3) at $\lambda = 0.5$ GeV for the leading order. The sensivity of the second moment to changes of λ is about 1% in the considered range, i.e., it is almost negligible in the relevant q^2 region (see fig. 3a). On the other hand, the higher moments change within 10% with λ (figs. 3b, c).

Second-order corrections to $M(n,q^2)$ were included according to ref. [5]. For $n=2$ the second-order effects seem to be important up to $q^2 \sim 100$ $(GeV/c)^2$. For $n > 2$ they contribute to $\sim 2\%$ for $q^2 > 30$ $(GeV/c)^2$ (see the dotted lines in fig.3).

The relative gluon and quark contributions (eq. (3) are shown as dashed lines in fig. 3. The gluon contributions rise with q^2 and reach $\sim 15\%$ and $\sim 5\%$ for $n=2$ and $n>2$, respectively. Due to the smallness of the quark coefficient function $Y_n(s)$ relative to $X_n(s)$ (see fig. 4), the quark terms in eq.(3) can be neglected. Therefore eq.(3) multiplied by $\exp(s \cdot \lambda_{NS})$ gives

$$M'(n,q^2) = M(n,q_0^2) + X_n'(s) \cdot G(n,q_0^2). \qquad (5)$$

Thus QCD predicts a linear dependence of M' on X_n' and the moments of the gluon distribution can be determined as the corresponding slopes.

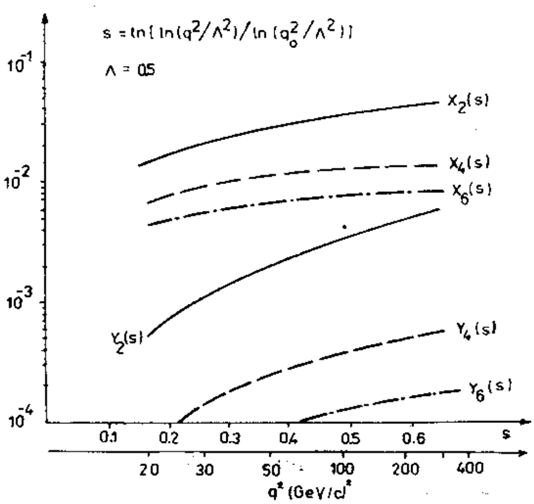

Fig. 4. Coefficients X_n and Y_n of the gluon and quark contributions to $M(n,q^2)$ (eq.(3) as a function of q^2 and s.

4. SUMMARY

From the above arguments the following conclusions can be drawn:

(1) The measurement of $M(n,q^2)$ in the NA-4 experiment can be performed up to q^2 equal to about 50, 160, 230 $(\mathrm{GeV/c})^2$ for $n=2,4,6$ respectively. Target mass effects are almost negligible in this q^2 region. The expected statistical accuracy should be sufficient for a rather precise determination of the QCD parameters.

(II) The second moment $M(2,q^2)$ appeared to be insensitive to changes of λ. It is, however, strongly affected by the gluon contribution to the leading term and by second-order effects. The situation changes with rising n: the sensitivity to λ increases whereas gluons and higher corrections contribute less.

(III) Eq.(5) could offer the possibility to test QCD predictions directly and to calculate moments of the gluon distribution since NA-4 will reach the largest possible range of $s = \ln(\bar{g}(q_0^2)/\bar{g}(q^2))$.

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