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Бельков А.А. и др. **E1 - 12643**

Парциально-волновой анализ реакции $K^- p \to \Lambda \pi^+ \pi^-$ в области резонанса $\Lambda(1520)$

Проведен парциально-волновой анализ реакции $K^- p \to \Lambda \pi^+ \pi^-$ в области резонанса $\Lambda(1520)$ с учетом вклада треугольного графика. Получены парциальные ширины распада резонанса $\Lambda(1520)$ по каналам: $\Lambda(1520) \to \Sigma(1385)\pi$ и $\Lambda(1520) \to \Lambda\pi\pi$. Получена оценка угла смешивания SU(3)-синглета с изоскалярной компонентой октета $J^P=(3/2)^-$. Допустимые значения угла смешивания $19^\circ \le |\theta_{CM}| \le 35^\circ$ находятся в согласии с предсказаниями SU(3) симметрии. Определена верхняя граница S-волновой длины $\pi\pi$-рассеяния a°_0. Величина $a^\circ_0 < 0.21(h/m_\pi c)$ с уровнем достоверности 95%.

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Belkov A.A. et al. **E1 - 12643**

Partial-Wave Analysis of the $K^- p \to \Lambda \pi^+ \pi^-$ Reaction near the $\Lambda(1520)$ Resonance

The partial-wave analysis of the $K^- p \to \Lambda \pi^+ \pi^-$ reaction near the $\Lambda(1520)$ resonance has been performed by taking into account the triangular graphs. The partial widths of the $\Lambda(1520)$ resonance decays have been obtained for the following channels: $\Lambda(1520) \to \Sigma(1385)\pi$ and $\Lambda(1520) \to \Lambda\pi\pi$.

The angle of mixing of the SU(3) singlet with the isoscalar component of the $(3/2)^-$ octet has been estimated to be $19^\circ \le |\theta_{min}| \le 35^\circ$. This is in good agreement with the predictions. The upper limit for S-wave scattering length has been determined to be $a^\circ_0 \le 0.21(h/m_\pi c)$ with a 95% confidence level.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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1. INTRODUCTION

The results of the partial-wave analysis of the $K^-p \to \Lambda \pi^+\pi^-$ reaction at the momenta of primary K^--mesons (370-420) MeV/c are reported. The same experimental data as in[1] are used in the present paper.

The main aim of our work was to estimate the S -wave scattering length in the I =0 isospin state by taking into account the contribution of a triangular graph. A similar analysis of the $\pi N \to \pi\pi N$ reaction taking into account the triangular graphs has been made previously in ref.[2,3]

The major information, concerning the structure and the properties of resonance states in three particle systems is extracted now from the partial wave analysis of the reactions with three particle production at intermediate energies. Some version of the isobar model proposed first by Lindenbaum and Sternheimer[4] and then developed by Ollson and Iodh[5], Deler and Valladas[6] and others, serves as a basis for such an analysis. The isobar model assumes that the amplitude of the reaction with a three-particle production

$$a + b \to 1 + 2 + 3 \qquad (1)$$

is saturated by two-particle intermediate states among which resonance states predominate. In other words, it is assumed that the diagrams of figs. 1a,b,c give the main contribution to the amplitude of reaction (1).However, in two particle subsystems of reaction (1) there are always both resonant states and nonresonant states which produce a comparatively smooth background. The contribution of nonresonant states into amplitude (1) can be schematically shown in fig.1d. To understand which intermediate states are to be taken into account in the amplitude of the $a + b \to 1 + 2 + 3$ transition, one should use the unitary condition. The contribution of three particle intermediate states to unitarity condition for three particle production amplitude T_{23} is

$$\mathrm{Im}\, T_{23} = \int T_{23} \cdot T_{33}^{*} \, d\Gamma_3 , \qquad (2)$$

where T_{33} is the amplitude of the process $3 \to 3$ and the integration in (2) covers the three particle phase space.

In the framework of the isobar model it is assumed that the whole T_{33} amplitude is saturated by pair collisions of particles (i.e., three particle collisions are negligible). At intermediate energies of the primary beam (when the kinetic energy of the produced particles reaches some hundreds of MeV) the amplitudes of pair interaction do not probably contain smallness parameter. So taking into account different diagrams in the amplitude T_{33} is reasonable only if we can in any way distinguish diagrams between each other by their contributions to the full amplitude T_{23}. It becomes possible when one studies the analytical properties of the three particle amplitude.

The amplitudes, corresponding to intermediate states in T_{33} with resonance interaction of one or two pairs of particles (e.g., figs. 1i,q type diagrams) have specific analytical properties which are peculiar to such amplitudes only.

Namely, the amplitude corresponding to the Fig. 1q diagram has a singularity of logarithmic type (i.e., at some point it tends to infinity as a logarithm).

It is known that the Breit-Wigner pole describing the resonance, e.g., in the system of particles 2 and 3, is placed at definite $m_{23}=\sqrt{(p_2+p_3)^2}$ irrespective of the value of other variables on which the T_{23} amplitude depends. Unlike this logarithmic singularity of diagram 1q is placed under different m_{23} values depending on $W=\sqrt{(p_1+p_2+p_3)^2}$ which is the total energy of three particles in their c.m.s. (or under different W values depending on m_{23}). Its location is defined by two variables.

Analogously, the amplitude of graph 1i has a pole in the complex plane m_{23} which corresponds to the resonance interaction of particles 2 and 3, and logarithmic singularity of the type mentioned.

From the above said it follows that in order to satisfy the strict unitarity condition the isobar model must include both specific amplitudes corresponding to triangular graphs 1i,q and the resonance production amplitudes 1a,b,c and the "background" amplitudes 1d.

Unfortunately, these obvious arguments were doubted at the Chicago conference in 1972 /7/. G.Lovelace in his rapporteur talk devoted to the progress in resonance physics has noted that including a triangular graphs into isobar model leads to double-counting and breaks the unitarity condition.

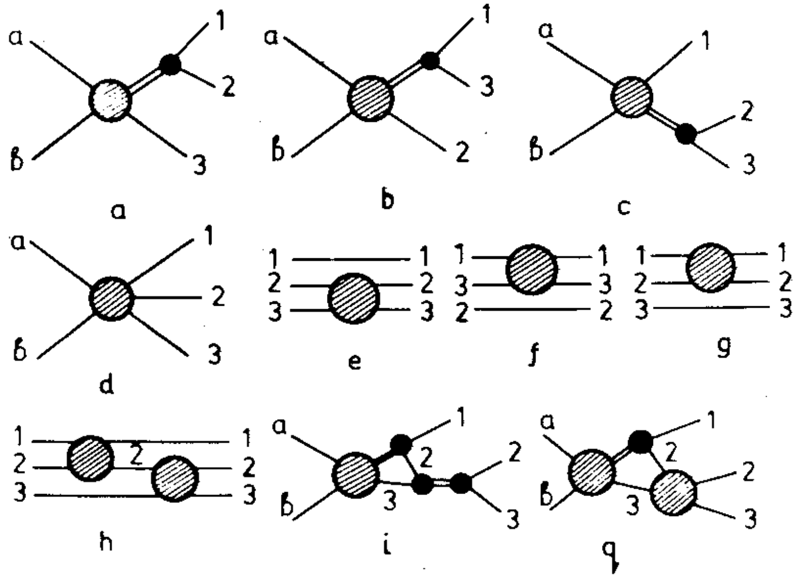


Fig. 1a,b,c Diagrams of the production of resonance amplitudes in the isobar model, d - background diagram in the isobar model, e,f,g,h - diagrams of pair interactions in the three particle amplitude, i,q - triangular diagrams corresponding to singular amplitudes in the isobar model.

It is clear, however, that taking into account the amplitudes with a different analytical structure can never lead to double-counting. Rejecting the triangular praps 1i,q is equivalent to rejecting some definite items in the T_{33} amplitude in unitarity condition (2) and thus, strictly speaking, breaks the unitarity condition.

The analytical properties of the amplitude which corresponds to the triangular graph 1q, have been considered in detail by several authors (see, e.g., ref.[8,9]).

The direct calculation of the amplitude, corresponding to diagram 1q, shows that if its singularity is placed close to the physical region, the amplitude is proportional to the scattering amplitude of particles 2 and 3 with m_{23} close to the singularity position. Hence, when it is known "a priori" that the latter amplitude is negligible, it is possible to reject the triangular graph when constructing the isobar model. When there is no such information and the kinematics of the reaction allows the logarithmic singularity to come close to the physical region, then it seems to be necessary to take into account the corresponding contribution to the full amplitude.

The isolation of this contribution from the available experimental data could provide us with useful information about the $2+3 \to 2+3$ scattering amplitude. Bearing in mind, that the logarithmic singularity may occur close to the physical region only at the values of $\mathrm{Re}\, m_{23} \approx m_2 + m_3$(i.e., near the elastic threshold), one may apparently hope to determine the S-wave scattering length of particles 2 and 3.

Now consider some conditions which seem to be necessary for successful determination of the $\pi\pi$ -scattering amplitude through the isolation of the contribution of the logarithmic singularity in the amplitude of the reaction $K^- p \to \Lambda \pi^+ \pi^-$.

First of all, it is clear that the considered contribution should not be deliberately small. It requires first that the logarithmic singularity approach sufficiently close to the physical region and, second, that the resonance production amplitude (Fig. 1a) would not be too small.

The latter requirement is caused by the proportionality of the amplitude of the triangular graph to the resonance production amplitude.

In other words, it is desirable that the produced resonance would be sufficiently narrow and the amplitude of the production and the subsequent decay of the resonance give the dominating contribution to the cross section of the reaction $a+b \to 1+2+3$, the cross section being saturated by not too many partial waves. It is appropriate to call these conditions dynamic conditions.

In order to determine the contribution of the logarithmic singularity to the amplitude of reaction 1 correctly, it is necessary to understand clearly which kind of observable effects may be connected with such a singularity.

C.Schmid in ref. /10/ has claimed that the observable effect of the triangular singularity is, in general, extremely small and completely disappears in the differential cross section $d\sigma / dm_{23}^2$ of the reaction $a+b \to 1+2+3$.

Schmid's arguments are as following. The absorptive part of the amplitude of diagram 1q may be found with the help of the Cutkosky rule, which follows from the unitarity condition. Taking into account only the S-wave scattering of particles 2 and 3, one may write:

$$A_1 = q a_{23} \frac{1}{4\pi} \int_{-1}^{1} dz \int_{0}^{2\pi} d\phi \, \frac{G}{m_{12}^2 - M_R^2 + i M_R \Gamma_R}, \tag{3}$$

where q is a momentum of particles 2,3 in their c.m.s., a_{23} is the elastic scattering amplitude of these particles:

$$a_{23} = (e^{2i\delta} - 1)/2iq .$$

A_1 to an accuracy of the factor $q \cdot a_{23}$ is equal to the S-wave part of the Breit-Wigner amplitude A_R in the expansion:

$$A_R = \frac{G}{m_{12}^2 - M_R^2 + i M_R \Gamma_R} = \sum_{\ell=0}^{\infty} A_R^{(\ell)} \cdot P_\ell(\cos\theta)$$

$$A_1 = q a_{23} A_R^{(s)} .$$

Up to this point Schmid's arguments [10] are doubtless. However, his subsequent conclusions are inaccurate which has been pointed out by Valuev [11].

Schmid supposed that the singular part of the triangular graph amplitude may be presented in the form:

$$A_t = 2 i A_1 = 2 i q a_{23} A_R^{(s)} = (e^{2i\delta} - 1) A_R^{(s)} .$$

Then the total amplitude of reaction I would take the form:

$$A_I = A_R + A_t = A_t + A_R^{(s)} + \sum_{\ell=1}^{\infty} A_R^{(\ell)} P_\ell(\cos\theta)$$

and

$$A_t + A_R^{(s)} = e^{2i\delta} A_R^{(s)} .$$

Hence, according to Schmid, the effect of a triangular graph consists only in changing the phase of the S-wave part of A_R amplitude. In the differential cross section $d\sigma/dm_{23}^2$ which is proportional to the form

$$\frac{d\sigma}{dm_{23}^2} \sim \sum_{\ell=1}^{\infty} |A_R^{(\ell)}|^2 + |A_t + A_R^{(s)}|^2 = \sum_{\ell=0}^{\infty} |A_R^{(\ell)}|^2$$

any observable effect disappears completely. The inaccuracy of this conclusion is that the singular part of the triangular graph cannot be written in the form $A_t = 2iA_1$.

In the correct form A_t contains one additional item (besides $2iA_1$) which has a logarithmic singularity close to the point $m_{23} = (W - m_1)^2$. This singularity cancels out the same singularity of A_1 and the total amplitude A_t is regular at the mentioned point. It is impossible, of course, to neglect such a singular item in the total amplitude A_t and hence we obtain

$$A_t \approx (e^{2i\delta} - 1)(A_R^{(s)} + A_2),$$

where A_2 stands for the mentioned singular item which has been rejected by Schmid. Then

$$|A_t + A_R^{(s)}|^2 = |A_R^{(s)}|^2 - 2\,\mathrm{Re}\,[(e^{2i\delta} - 1)A_R^{(s)} \cdot A_2^*] + |(e^{2i\delta} - 1)A_2|^2 \quad (4)$$

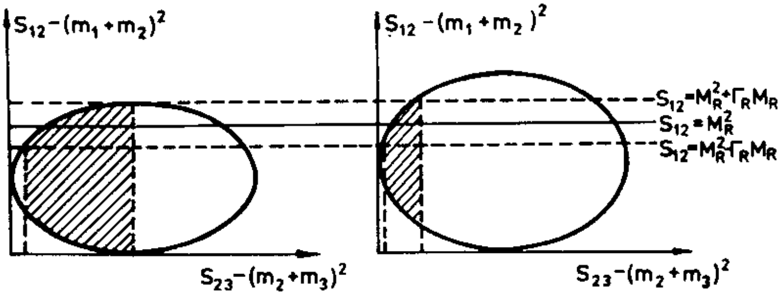
and the differential cross section $d\sigma\ dm_{23}^2$ contains the observable effect of the triangular singularity, the value of the effect being proportional to the 2-3 particle scattering amplitude. The effect is caused by the second item in (4). Consider now in detail the kinematic conditions which are necessary for the most evident appearance of the effect of the logarithmic singularity. The simplest way for it is to study the Dalitz diagrams.

The physical region for the variables $s_{12} = m_{12}^2$ and $s_{23} = m_{23}^2$ (the value of W^2 being fixed) is shown in Fig. 2a by a closed curve. Every possible kinematic configuration in the reaction $a + b \to 1 + 2 + 3$ corresponds to a single point in the Dalitz plot.

The production amplitude with a resonance in the 1,2 particle system is large in the band, corresponding to the resonance with $M_R^2 - \Gamma_R M_R \le s_{12} \le M_R^2 + \Gamma_R M_R$. In order to determine the region in the Dalitz plot where the effect of the logarithmic singularity can be considerable, it is necessary to drop the perpendiculars on the absciss axis from the points where the lines $s_{12} = M_R^2 - \Gamma_R M_R$ and $s_{12} = M_R^2 + \Gamma_R M_R$ cross the physical region boundary. One may hope, that in the shaded band in Fig. 2a, the effect from the triangular graph singularity is considerable.

Hence, the most advantageous kinematic conditions for isolation of the triangular singularity contribution occur when this band of the singularity covers the highest possible area on the Dalitz-plot. Such a configuration of variables corresponds to Fig. 2a, where the line $s_{12} = M_R^2 + \Gamma_R M_R$ touches the physical region boundary, i.e., the whole resonant band comes into the physical region and the value $s_{12} = M_R^2 + \Gamma_R M_R$ is the largest possible value of this variable. In this case the total energy of three final particles in their c.m.s. is equal to $W_0 = \sqrt{s_{12}} + m_3 = \sqrt{M_R^2 + \Gamma_R M_R} + m_3$. Bearing in mind that usually $\Gamma_R \ll M_R$ we obtain $W_0 \simeq M_R + m_s + \Gamma_R/2$.

As W grows up, the band of logarithmic singularity influence becomes narrower, and its center tends to the lower va-



a) b)

Fig. 2. Dalitz plot of the reaction $a + b \to 1 + 2 + 3$. The interval of values $M_R^2 - \Gamma_R M_R \leq s_{12} \leq M_R^2 + \Gamma_R M_R$ corresponds to the region of resonance production with the mass M_R and the width Γ_R in the system of particles 1,2; influence of logarithmic singularity may be important in the shaded region.

lues of s_{23} (as it is seen from Fig. 2b). Then the kinematic conditions for the isolation of the singular item in the three-particle production amplitude becomes somewhat worse.

Generally speaking, it is possible to try to distinguish triangular singularity contribution at the values of the total energy $W \lesssim W_0$. In the latter case, however, it is difficult to make any definite prediction about the band of influence of the singularity; one may only think that the band width would be of the same order of magnitude as for $W = W_0$, but the effect would appear less evident.

Finally, for the correct interpretation of the results, obtained by distinguishing the logarithmic singularity contribution in the amplitude of reaction 1 it is necessary to take into account the following. The considered contribution is proportional to the elastic scattering amplitude of particles 2,3, averaged over the band of influence of the singularity. Hence, in order to obtain more definite results, this band must be narrower which, in turn, requires the increase of the initial beam energy.

It is easy to see that the latter requirement contradicts the two former ones (kinematic and dynamic). Thus, the optimal conditions for the isolation of the logarithmic singularity contribution and the determination of the scattering amplitude of particles 2,3 must be selected on the grounds of some compromise decision.

One of the reactions where it is possible to expect essential contribution of the logarithmic singularity is the reaction $K^- p \to \Sigma(1385)_{\pi^-} \to \Lambda\pi^+\pi^-$ well-studied experimentally near the $\Sigma(1385)$ production threshold.

Indeed, at the initial K^- energy of 400-500 MeV the total cross section of the reaction $K^- p \to \Lambda\pi^+\pi^-$ is sufficiently large, $\sigma \sim$ 1-2 mb, and the produced resonance $\Sigma(1385)$ is sufficiently narrow. According to the analysis of ref. [1], in the region of the K^- momentum 365÷415 MeV/c the cross section $K^- p \to \Lambda\pi^+\pi^-$ is saturated by a small number of partial waves, the amplitude with $\Sigma(1385)$ production dominates over them.

The kinematic conditions for the isolation of the logarithmic singularity contribution in the reaction $K^- p \to \Lambda\pi^+\pi^-$ in the considered energy region appear somewhat less favourable. The most favourable configuration (corresponding to Fig. 2a) takes place at the K^- momentum $p_{K^-} \cong$ 455 MeV/c, and the best conditions for the interpretation of the obtained value of the a_0-S-wave scattering amplitude occur at still larger values of $p_{K^-} \sim$ 500-550 MeV/c.

Nevertheless, taking into account the rich statistics of the data, obtained in ref. [1], it seemed reasonable to carry out the corresponding analysis of these data to isolate the logarithmic singularity contribution and to determine the $\pi\pi$ S-wave scattering amplitude.

We used the expression, obtained in [12] for the triangular graph amplitude for numerical calculations. The used expression slightly differs from the ones listed in refs. [2,3,9]. Generally speaking, the singular part of the amplitude of graph 1q is defined only within some smooth function of W and $m^2_{\pi\pi}$. In order to estimate the influence of such an uncertainty on the estimation of the $\pi\pi$-scattering amplitude, some special subtractive procedures have been used when performing the analysis. These procedures are considered in detail below.

II. MODEL OF THE PARTIAL-WAVE ANALYSIS

The authors of ref.[1] have drawn the conclusion that the reaction $K^- p \to \Lambda \pi^+ \pi^-$ is well described by a model in which only six partial waves are taken into account, namely,- if we use their notations - the waves DSO3(Y*), DS13(Y*), PPO1(Y*), PPO3(Y*), PSO1(σ), and DPO3(σ). So initially we decided to use the same six waves but with an addition of a triangular graph to the amplitudes DSO3(Y*), PPO1(Y*) and PPO3(Y*). The results of our analysis differ essentially from those of ref.[1] where the value 0.82±0.10 has been obtained for

$$R = \frac{\sigma[\Lambda(1520) \to \Sigma \quad (1385)\pi^{\pm}]}{\sigma[\Lambda(1520) \to \Lambda\pi^+\pi^-]} . \qquad (5)$$

Our estimation of R for the momentum K^- 395 MeV/c is equal to about 0.40. Naturally, such a great difference has made us put a question about the correctness of the model with the mentioned six waves. We have decided to repeat the analysis of ref.[1] in full taking into account where it is necessary the contribution of the triangular graph.

Table 1 presents all partial waves which have been used in the analysis. The notations are the same as in ref.[1], i.e., each state is noted in the form LL′(I)2J, here L is the angular momentum of the initial $K^- p$ system, L′ is the angular momentum of the subsystem of two final particles relative to the third one, I is the total isotopic spin, J is the total angular momentum. The model takes into account S, P and D waves in the initial state (i.e., in the system of primary $K^- p$) and S, P waves in the final states. The total angular momentum $J \leq 3/2$.

Additionally, to the earlier accepted notations of states we accept a new notation characterizing the reaction from the point of view of the initial $K^- p$ system. We note the state by LI(2J), where L, I, J have previous meanings.

Then the model may be shown as follows:

D03 $\to$ DS03(Y*) + DP03(σ) + DP03($\Lambda\pi$)

D13 $\to$ DS13(Y*) + DP13($\Lambda\pi$) + DS13(ρ)

P01 $\to$ PP01(Y*) + PS01(σ) + PS01($\Lambda\pi$)

P03 $\to$ PP03(Y*)

$$P11 \to PP11(Y^*) + PS11(\Lambda\pi)$$

$$P13 \to PP13(Y^*)$$

$$S01 \to SP01(\Lambda\pi) + SP01(\sigma)$$

$$S11 \to SP11(\Lambda\pi) + SS11(\rho)$$

Table I

The waves used in the partial-wave analysis

	Wave type	LL'(I)(2J)
1	Y^*	DS03
2		DS13
3		PP01
4		PP11
5		PP03
6		PP13
7	$\Lambda\pi$	PS01
8		PS11
9		SP01
10		SP11
11		DP03
12		PD13
13	σ	PS01
14		SP01
15		DP03
16	ρ	SS11
17		DS13

The number of the waves is 17 (if we use notations of [1]), each wave is characterized by the complex parameter (i.e., two real parameters). Due to the uncertainty in the common phase the imaginary part of the DS03 (Y^*) wave parameter is taken zero and fixed. Moreover, the term taking into account the normalization on the total cross section was added to the minimized functional.

The main aim of ref.[1] consisted in estimating the value

$$R = \frac{\sigma[\Lambda(1520) \to \Sigma^{\pm}(1385)\pi^{\mp}]}{\sigma[\Lambda(1520) \to \Lambda\pi^{+}\pi^{-}]} = \frac{\sigma[D03 \to \Sigma^{\pm}(1385)\pi^{\mp}]}{\sigma[D03 \to \Lambda\pi^{+}\pi^{-}]}.$$

The estimation of R can depend on the following factors:

1. The uncertainty of the model, i.e., a model with another set of waves can be chosen.
2. The structure of the DO3 wave.
3. The parametrization of the factor of the centrifugal barrier penetration.
4. The parametrization of the $L=I=0$ $\pi\pi$-scattering phase.
5. The parametrization of the $L=0, I=1$, $\pi\Lambda$-scattering phase.

We have made the analysis in which the dependence of the estimation of R on each of these factors are checked.

THE UNCERTAINTY OF THE MODEL

It is known that the main drawback of all partial-wave models is some arbitrariness of the choice of the states. In order to estimate the effect of such arbitrariness on the estimation of R we have limited ourselves to the analysis of one of the alternative models chosen from the following considerations. Among 17 waves of the initial model (Table I) there are four pairs of waves which interfere strongly with each other: they are PSO1 (σ) and PSO1 ($\Lambda\pi$), DS13 (ρ) and DP13 ($\Lambda\pi$), SP11 ($\Lambda\pi$) and SS11 (ρ), SPO1 (σ) and SPO1($\Lambda\pi$). If two waves interfere greatly with each other it means that they are functionally near and the neglection of one of them must not influence considerably the quality of the description of experimental data. So, in an alternative model we have rejected the following four waves; PSO1 ($\Lambda\pi$), SPO1(σ), SS11 (ρ), and DS13 (ρ) and made a fit simultaneously by the model with 17 waves (model I) and by the model without these four waves (model II).

THE STRUCTURE OF THE DO3 WAVE

The DO3 wave in model I has the following structure:

$$D03 \to DS03(Y^{*}) + DP03(\sigma) + DP03(\Lambda\pi).$$

The DSO3 (Y^{*}) wave is the basic wave in this state and two other waves are considerably smaller.

Naturally, we can put a question: can we neglect one of the two small waves from the point of view of statistics? So, simultaneously with main model I we used models in which we neglected one of the small waves in the DO3 state: either DPO3 (σ) (model III) or DPO3 ($\Lambda\pi$) (model IV).

THE PARAMETRIZATION OF CENTRIFUGAL BARRIERS

The dependence of the partial wave amplitudes of the model[1] on the energy of the corresponding pair of particles (c.m.i.) contains a factor of penetration of the centrifugal barrier which is written in the form $p^{2\ell}/(1+p^2r^2)^{\ell}$, where p is particle momentum in their c.m., ℓ is the angular momentum, r is a phenomenological parameter taken by analogy with the theory of resonance scattering in nuclear physics, where r has the meaning of the compound nuclear radius.

The authors of ref.[1] have chosen $r \approx 1$ fm arbitrarily. However, it is confirmed in the literature[13] that a smaller value r describes data better in the framework of isobar models. Therefore, in the analysis we have taken r as a free parameter and chosen its value taking into account the quality of the description of experimental data.

The uncertainties in the values of the mass M_R and the width Γ_0 of the resonance $\Sigma(1385)$ can influence the estimations of the values R and a_0.

The dependence of estimations of R and a_0 on these values has been checked as follows: the analysis was made in the framework of models I, II, III with two values of these parameters taken from ref.[1] and compilation[14]. The values R and a_0 obtained with these two sets of parameters coincide within error limits.

THE PARAMETRIZATION OF THE $L=I=0$ $\pi\pi$-SCATTERING PHASE

Up to now there are great uncertainties at the low energy $L=I=0$ $\pi\pi$-scattering phase (see, for example, ref.[15]). Taking into account this uncertainty we made the analysis in the framework of models I-IV with the three different parametrization of the phase $\pi\pi$: of ref.[1], of ref.[16] and of ref.[17].

THE PARAMETRIZATION OF THE $L=0, I=1$ $\pi\Lambda$-SCATTERING PHASE

There are two papers on the determination of the $\Lambda\pi$-phase-ref.[18] (the same phase has been used in the analysis[1]), and ref.[19]. The phase obtained in ref.[18] is positive but that in ref.[19] is negative. We have made the analysis with both parametrizations.

The triangular graph amplitude in the commom case can be written with an accuracy to any analytical function of the $\pi\pi$ -system mass.

We have estimated the influence of the inaccuracy of theoretical description of the triangular graph on a_0 in the following way. We wrote the expression of the triangular graph amplitude with the subtraction constant and made the analysis with its three values. They have been chosen as follows:

a) The subtraction constant has been chosen so that the value of the triangular graph under $m^2_{\pi\pi}=(m^2_{\pi\pi})_{max}$ turned to zero. In other words

$$A = T[W, m^2_{\pi\pi}] - T[W, (m^2_{\pi\pi})_{max}]$$

b) The subtraction constant has been assumed to be equal to zero.

c) The subtraction constant has been assumed to be equal to the value of the triangular graph with $W=m_\Lambda$ (1520) and $m^2_{\pi\pi}=(m^2_{\pi\pi})_{max}$ with the same W.

III. RESULTS OF THE ANALYSIS

Model I with the parametrizations of $\pi\pi$ -and $\pi\Lambda$-scattering phase taken in ref.[1] and with $r = 1$ fm has been chosen as the main model. To estimate the values of other varied parameters in the framework of this model we have chosen initially 4 intervals by the momentum of the primary K^- meson from 370 MeV/c to 410 MeV/c, containing 7404 events (the total number of events in the experiment[1] is about 9200). Totally we have chosen about 80 random initial approximations in this four intervals and the minimization has been made for each of them. As a result, we have found a considerable number of minima of the likelihood functional. This situation is characteristic of the maximization problems of such kinds and the question always arises about the choice of the unique solution on the basis of only statistical criteria.

In our case the solution corresponding to the deepest minimum has been chosen as a unique solution. Such a choice has been as a result of the following considerations: first, the value of the likelihood functional in the highest minimum has been many orders smaller than in the neighbouring minima and, second, the corresponding solution had a continuity property in the passage from one energy interval to other one. Here we have used considerations developed by Tyapkin[20]. The solution obtained in the framework of main model I was chosen as the basic one in the following analysis of models I-IV with using different parametrizations $\pi\pi$- and $\pi\Lambda$-scattering phases, the centrifugal barrier and simulating the uncertainty of the triangular graph.

Figure 3 shows the contribution of each state to the cross section of $K^- p \to \Lambda\pi^+\pi^-$ at the momentum interval of primary K^--mesons (360-420 MeV/c) obtained in the framework of model I.

It is seen that the wave D03, i.e., the wave with the quantum numbers of the resonance $\Lambda(1520)$ is dominant.

Consider now the influence of different uncertainties on the estimation of parameters for models I-IV.

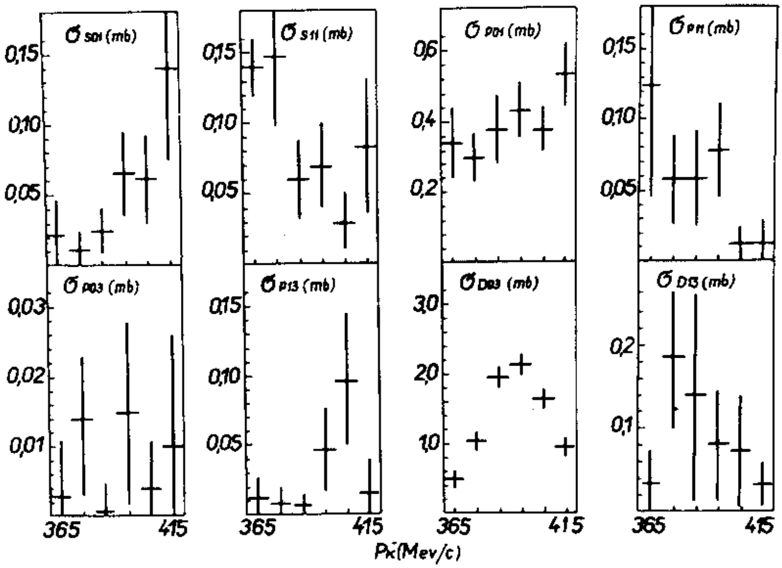


Fig. 3. Contributions of different states to the cross section of the $K^- p \to \Lambda\pi^+\pi^-$ reaction.

The optimal value r in the parametrization of the penetration factor of the centrifugal barrier is chosen by comparing the summed values of the functional ϕ in 5 energy intervals (370 MeV/c $\le p_{K^-} \le$ 420 MeV/c), the total number of events being 8020) in the models I-IV with parametrization of $\pi\pi$- and $\pi\Lambda$-scattering phases as in ref. /1/ and with three values r = o; 0.5 and 1 fm.

Then we have made calculations with the $\pi\Lambda$-scattering phase the same as in ref. [1] and with the $\pi\pi$-scattering phase the same as in ref. [16] and ref. [17]. These calculations were made only in the framework of model I. The value r = 0 was the best one in these two cases. Besides, the calculations have been made in the framework of model I and when parametrization of ref. [1] for the $\pi\pi$-scattering phase was taken and parametrization of ref. [19] for $\pi\Lambda$-scattering was taken. The value r = 0.5 fm was the best one.

In accordance with these results we chose r = 0 for all the models when using the $\pi\Lambda$-scattering phase from ref. [18] and r = 0.5 fm for the models applying the $\pi\Lambda$-scattering phase of ref. [19].

All the considered models with these values were permissible from point of view of statistical criteria.

Table II presents the values $\bar{R}$ obtained with different models. The value $\bar{R}$ and its error are calculated as follows:

$$\bar{R} = \sum_i \frac{R_i}{(\Delta R_i)^2} \Big/ \sum_i \frac{1}{(\Delta R_i)^2}, \quad (\Delta\bar{R})^2 = 1/\sum_i \frac{1}{(\Delta R_i)^2}.$$

Here R_i, ΔR_i are the values of the ratio R and its error in the corresponding energy interval. Summation is made over all energy intervals.

The values $\bar{a}_0$ and $\Delta\bar{a}_0$ obtained in different models and calculated by the formulas as $\bar{R}$ and $\Delta\bar{R}$ are presented in Table III. The uncertainties in the subtracting constant of the triangular graph mentioned above, give only a change of the value $\Delta\bar{a}_0$ but not $\bar{a}_0$. Table III presents the values corresponding to the case when the amplitude of the triangular graph is written in the form:

$$A = T(s, m^2_{\pi\pi}) - T(s, (m^2_{\pi\pi})_{max}).$$

In this case the error $\Delta\bar{a}_0$ is the largest one.

Figs. 4,5 show the angular and mass distributions demonstrating the quality of description of experimental data in the framework of one of the models. One mass and one angular distributions are given as an example. Totally we have considered 13 different distributions containing 330 experimental points. Taking into account considerations of ref. [21] it is possible to state that the distribution of the sum of χ^2 over all the spectra must have in our case the distribution which is intermediate between $\chi^2(317)$ and $\chi^2(284)$. Our values of χ^2 are in complete agreement with those distributions.

Table II

Values of R obtained with different models and various parametrization of the phases $\pi\pi$ and $\pi\Lambda$

Parameterization of the phases $\pi\pi$ and $\pi\Lambda$	Model	R
Phases $\pi\pi$ and $\pi\Lambda$, ref. /1/	Model I	0.339 ± 0.046
	Model III	0.522 ± 0.042
	Model IV	0.272 ± 0.034
	Model II	0.297 ± 0.038
Phase $\pi\pi$, ref. /16/, Phase $\pi\Lambda$, ref. /18/	Model I	0.399 ± 0.053
Phase $\pi\pi$, ref. /17/ Phase $\pi\Lambda$, ref. /18/	Model I	0.224 ± 0.048
Phase $\pi\pi$, ref. /1/ Phase $\pi\Lambda$, ref. /19/	Model I	0.295 ± 0.049

IV. CONCLUSION

We have made the partial-wave analysis of the $K^- p \to \Lambda\pi^+\pi^-$ reaction near the $\Lambda(1520)$ resonance in the framework of the isobar model taking into account the influence of the logarithmic singularity of the triangular graph.

Our results differ considerably from those of ref. /1/ with the same set of experimental data. The main difference is that the contribution of the DSO3 (Y^*) amplitude to the $K^- p \to \Lambda\pi^+\pi^-$ reaction cross section turned to be considerably smaller than in ref. /1/: the value of the ratio

$$R = \frac{\Gamma[\Lambda(1520) \to \Sigma(1385)\pi]}{\Gamma[\Lambda(1520) \to \Lambda\pi\pi]}$$

Table III

Values of a_0 obtained with different models and various parametrizations of the phase $\pi\pi$ and $\pi\Lambda$

Parametrization of the phases $\pi\pi$ and $\pi\Lambda$	Model	a_0
Phases $\pi\pi$ and $\pi\Lambda$, ref.[1]	Model I	-0.005 ± 0.043
	Model III	-0.018 ± 0.038
	Model IV	0.053 ± 0.076
	Model II	0.000 ± 0.050
Phase $\pi\pi$, ref.[16] Phase $\pi\Lambda$, ref.[1]	Model I	-0.005 ± 0.045
Phase $\pi\pi$, ref.[17] Phase $\pi\Lambda$, ref.[1]	Model I	0.011 ± 0.062
Phase $\pi\pi$, ref.[1] Phase $\pi\Lambda$, ref.[19]	Model I	0.009 ± 0.057

varies according to our estimations from (0.22 ± 0.05) to (0.52 ± 0.04) depending on the model version, while in ref.[1] $R = 0.82\pm0.10$. We also have determined that the main inaccuracy in the estimation of the value R results from the model uncertainty (i.e., some models with different sets of waves turn out permissible from the point of view of statistics). In previous refs.[22,25] model inaccuracy has not been considered.

Our analysis permits also the estimation of partial widths of the $\Lambda(1520)$ resonance decays by the following channels:

$$\Lambda(1520) \to \Lambda\pi\pi \qquad \text{(all charge states)} \qquad (6)$$

$$\Lambda(1520) \to \Sigma(1385)\pi \qquad \text{(all charge states)} \qquad (7)$$

We took the elasticity of $\Lambda(1520)$ equal to (0.46 ± 0.01) [14] under the calculation of the resonance widths by channels (6) and (7). Our estimations show that the width of the $\Lambda(1520)$ decay by channel (6) varies from (1.22 ± 0.08) MeV to (1.52 ± 0.06) MeV. Such a change of the decay width corresponds to the deviation of the ratio of the $\Lambda(1520)$ decay by channel (6) from $(8.2\pm0.5)\%$ to $(10.2\pm0.4)\%$.

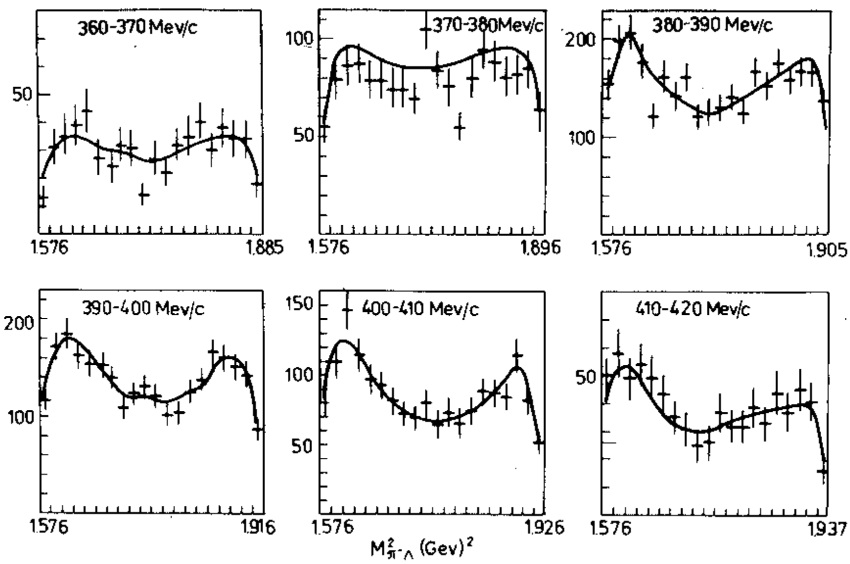


Fig. 4. Distribution of events over $M^2_{\pi^-\Lambda} = (p_{\pi^-} + p_\Lambda)^2$. Full curves correspond to model I of this paper.

The decay width of $\Lambda(1520)$ by channel (7) varies from (0.26 ± 0.10) MeV to (0.65 ± 0.08) MeV. Such a deviation at the widths of the $\Lambda(1520)$ decay is caused by the model uncertainty and the uncertainties in the knowledge of $\pi\pi$- and $\pi\Lambda$-scattering phases.

The estimation of the value R is important for calculating the parameter of the mixing SU(3) singlet of $J^P=(3/2)^-$ with the isoscalar component of $(3/2)^-$ octet. The resonance $\Lambda(1960)$ is a mixing partner, for $\Lambda(1520)$ in the framework of SU(3), i.e.:

$$\Lambda(1520) = \cos\theta|1\rangle - \sin\theta|8\rangle, \quad \Lambda(1690) = \sin\theta|1\rangle + \cos\theta|8\rangle.$$

Since the singlet state cannot decay into $\Sigma(1385)\pi$ we write

$$tg^2\theta = \frac{\Gamma(1520)}{\Gamma(1690)}\rho.$$

where $\Gamma(1520)$ and $\Gamma(1690)$ are the partial widths of $\Lambda(1520)$ and $\Lambda(1690)$ decays into $\Sigma(1385)\pi$, respectively, and ρ is the ratio of phase spaces. There is no unique opinion on how to calculate ρ and this value changes from 7.1 [22] to 9.5 [1] in different estimations.

Further, the full width of the $\Lambda(1690)$ resonance is not known accurately either. The values from 30 to 80 MeV are given for this width in the compilation [14]. We have taken the value 0.41 for the ratio of the decay $\Lambda(1690)\to\Sigma(1385)\pi$ [23].

Then:

$$(tg^2\theta)_{min} = 0.056,$$

$$(tg^2\theta)_{max} = 0.500.$$

It corresponds to the angular range:

$$13^\circ \le |\theta| \le 35^\circ$$

This is in good agreement both with the estimations obtained from the D-wave decay analysis [13] and with the predictions of theories with higher symmetries [24].

The interpretation of the obtained estimations a_0 is a bit more complicated. The most of them are in agreement with the predictions of the current algebra but the uncertainty in the estimation is quite considerable: a_0 varies from $-.02\pm0.04$ to 0.05 ± 0.08. It is possible to state that $a_0 \le 0.21$ at a 95% confidence level. It is possible to say also that the logarithmic singularity contribution to the reactions, considered, occurred to be less favourable than it could be expected from the results of ref. [1]. The main reason is that the amplitude with the production of the isobar $\Sigma(1385)$ is considerably smaller than in ref. [1]. Further more, as has been mentioned above, the most favourable kinematical conditions to estimating a_0 in this reaction occurrs at higher primary energies. All this means that the estimation of a_0 obtained by us has a considerable uncertainty. It is possible to await that a similar analysis in the region of K^- meson momenta p_{K^-} = 500÷550 MeV/c will give more reliable estimations for a_0 when using detailed experimental data.

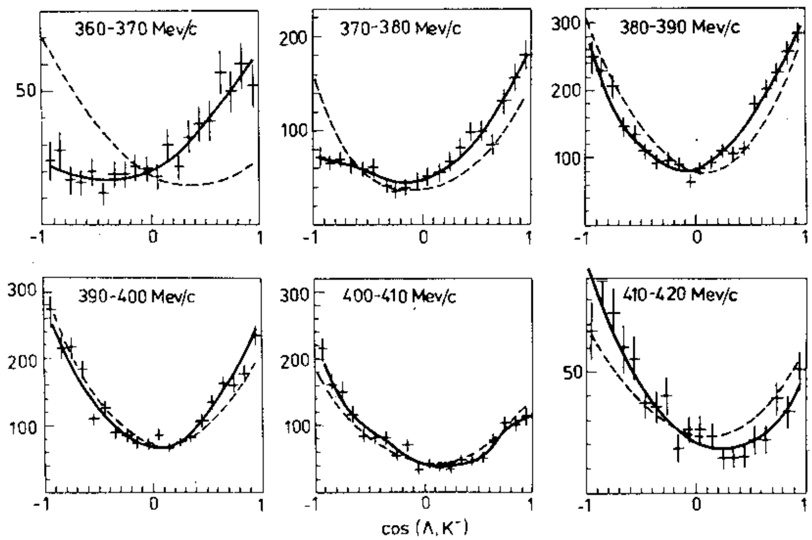


Fig. 5. Distribution of events over $\cos(\Lambda.K^-)$ $\frac{(\vec{p}_{K^-} \cdot \vec{p}_\Lambda)}{|\vec{p}_{K^-}||\vec{p}_\Lambda|}$.
Full curves correspond to model I of this paper, dotted curves correspond to the model of ref. [1] with theor parameters.

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