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DIFFERENT FORMS OF SCALING VIOLATION AND DEEP INELASTIC EP SCATTERING DATA

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Разные формы нарушения скейлинга и эксперименты по глубоконеупругому e-p рассеянию

Предсказания квантовой хромодинамики и калибровочных масштабно-инвариантных моделей для нарушения скейлинга в глубоконеупругом ep - рассеянии сравниваются с имеющимися экспериментальными данными. Показано, что нельзя описать данные в рамках этих моделей, если для структурной функции νW_2 и отношения R используется только главное логарифмическое приближение без учета массы мишени. Удовлетворительное описание этих данных может быть достигнуто, если для R используются феноменологические параметризации типа $R = const$, $R = 4b/Q^2$. При этом допустимые значения Λ лежат в интервале $0,1 \leq \Lambda \leq 0,5$ ГэВ, $R = 0,23 \pm 0,02$ и $b = 0,18 \pm 0,02$ /ГэВ/^{2}.

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Different Forms of Scaling Violation and Deep Inelastic ep Scattering Data

The predictions for scaling violation given by QCD and some scale invariant models are compared with the deep inelastic ep scattering data. It is shown that these models are not in agreement with the presently available data if for νW_2 and R the leading logarithmic approximation without including the target mass corrections is used. It is shown also that these data are fitted well by the following phenomenological parametrization of R: $R = const$ and $R = 4b/Q^2$ (in this case for νW_2 parametrizations based on the considered models are used). The possible values of Λ are in the region $0.1 \leq \Lambda \leq 0.5$ GeV, $R = 0.23 \pm 0.02$ and $b = 0.18 \pm 0.02$ GeV^2.

The investigation has been performed at the Laboratory of Nuclear Problem, JINR.

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Introduction

The study of the deep inelastic lepton-nucleon scattering is one of the best methods to check our ideas on the nucleon structure. The last deep inelastic ep /1/ and μN /2/ scattering experiments indicate a significant violation of Bjorken scaling. The data presented in the papers /2/ show various deviations from scaling in different x regions: The structure functions increase with Q^2 in the region of small x ($x < 0.15$) and they decrease in the region $x > 0.15$. Such a behaviour of these functions is predicted by quantum field theory models. However, the form of scaling violation is different in different models: In quantum chromodynamics /3/ (QCD) and massive vector gauge models /4/ scaling for the moments of the structure functions is violated by powers of logarithms in Q^2; in the scale invariant models /5/ this violation has a degree in Q^2 behaviour.

The aim of our investigations in this paper is to answer the question: Which of the various quantum field theory models are in agreement with the experimentally observed deviations of scaling?

The quantum field theory models predict the Q^2 dependence of the moments of the structure functions at large Q^2. Therefore, the evaluation of these moments directly from the experimental data is the best method to test these models. The available deep inelastic ep scattering data, however, are at relatively low values of Q^2 ($Q^2 \leq 30(GeV/c)^2$) and therefore, it is impossible to determine these moments from the experiment without extrapolations. Then, in order to compare the theory with the data it is necessary to have explicit expressions for the structure functions $2MW_1$ and νW_2 (or νW_2 and $R \equiv \sigma_L/\sigma_T$) themselves. Such expressions can be obtained using the Mellin transform method for the moments. In fact, it is difficult to solve this problem as the n dependence of the moments is very complicated. So, in many papers (see for example ref. [6]) the Q^2 dependence of quark and gluon distribution is presented numerically. However, as has been pointed out in ref. [7], it is possible to find simple analytic expressions for the quark and gluon distributions which, with a very good accuracy, represent the Q^2 dependence for the moments predicted by quantum field theory models. We shall use these analytic expressions for our analysis of the experimental data.

In this paper we analyze the deep inelastic ep scattering data [1,8-10]. Unlike most of the authors, we compare the quantum field theory predictions for the scaling violations with the directly measured inelastic cross sections. For the structure function νW_2 and the ratio R , we use parametrizations based on QCD and various scale invariant models with four colour-triplet quarks. For R the phenomenological parametrizations are also used. The method of our analysis has been considered in detail in ref. [11]. Here, we note that all free parameters connected with νW_2 and R as well as the normalized factors (introduced to account the possible systematic errors) are determined by minimizing the functional χ^2 for the cross sections.

Basic Formulae

The differential cross section of the process $e+p \to e+$ hadrons for unpolarized initial particles in the one-photon exchange approximation has the following form:

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2 \cos^2\theta/2}{4E^2 \sin^4\theta/2}\left(W_2 + 2\, tg^2\theta/2\, W_1 \right) , \qquad (1)$$

where E , E' and θ are the electron initial energy, final energy and scattering angle, respectively. The structure functions $F_1 = 2MW_1$ and $F_2 = \nu W_2$ are the functions of the scaling variable x ($x = Q^2/2M\nu$, where $Q^2 = 4EE'\sin^2\theta/2$, $\nu = E - E'$ and M is the proton mass) and Q^2 .

As usual, we introduce the ratio

$$R = \frac{\sigma_L}{\sigma_T} , \qquad (2)$$

where σ_L and σ_T are the total transverse and longitudinal absorption cross section of the virtual photon.

The structure functions F_1 , F_2 and the ratio R are related as follows:

$$F_1 = F_2 \frac{1 + Q^2/\nu^2}{x(1+R)} . \qquad (3)$$

In the framework of QCD and the models under consideration, the structure function F_2 in the leading logarithmic approximation can be written in the form:

$$F_2(x, Q^2) = x\left\{ \sum_{flavor} e_a^2 \left(q_a(x, Q^2) + \bar{q}_a(x, Q^2) \right) \right\} . \qquad (4)$$

Here e_a is the charge of the a -quark ($a = p, n$, etc.), q_a and $\bar{q}_a$ are quark and antiquark distributions in the proton at momentum transfer Q^2 .

In this approximation

$$F_1 = \frac{1}{x} F_2 . \qquad (4a)$$

Then, taking into account Eq. (3) for the quantity R we get

$$R = Q^2/\nu^2 . \qquad (5)$$

In the case of four colour-triplet quarks, Eq. (4) takes the form:

$$F_2(x, Q^2) = x\left\{ \frac{4}{9} V_8(x, Q^2) - \frac{1}{3} n_v(x, Q^2) + \frac{2}{9} S(x, Q^2) + \frac{4}{9} C(x, Q^2) \right\} , \qquad (6)$$

where

$$V_8 = p_v + n_v \,, \quad S = 6s \,, \quad C = 2c \,. \qquad (6a)$$

In Eqs. (6,6a) p_v , n_v , s and c are the valence p and n , strange and charm quark distributions in the proton. Note, that in order to obtain this expression for F_2 the SU (3)-symmetry for the sea quarks and Eq. $c = \bar{c}$ are assumed.

The moments of quark and gluon distributions are defined as follows:

$$\langle f(Q^2)\rangle_n = \int_0^1 dx\, x^{n-1} f(x, Q^2) \,, \qquad (7)$$

where $f = q, G$.

In the framework of QCD and scale invariant models, it is possible to calculate these quantities at large Q^2 ($Q^2 \gg M^2$) if their values at some fixed value of $Q^2 = Q_0^2$ ($M^2 \ll Q_0^2 \leq Q^2$)are known.

One can show (see ref./7/)*) that in the leading logarithmic approximation

$$\langle V_i(Q^2)\rangle_n = \langle V_i(Q_0^2)\rangle_n \exp\{-\gamma_n \bar{s}\}, \; i = 3, 8 \,, \qquad (8a)$$

$$\langle S(Q^2)\rangle_n = \tfrac{3}{4} \mathcal{D}_2^{(n)}(Q^2) + \tfrac{1}{4} \mathcal{D}_1(Q^2) \,, \qquad (8b)$$

$$\langle C(Q^2)\rangle_n = \tfrac{1}{4}\left(\mathcal{D}_2^{(n)}(Q^2) - \mathcal{D}_1^{(n)}(Q^2)\right) \,, \qquad (8c)$$

where

$$\mathcal{D}_1^{(n)}(Q^2) \equiv \langle S(Q_0^2)\rangle_n \exp\{-\gamma_n \bar{s}\} \,, \qquad (9a)$$

$$\mathcal{D}_2^{(n)}(Q^2) \equiv \{(1-\alpha_n)\langle q^s(Q_0^2)\rangle_n - \beta_n \langle G(Q_0^2)\rangle_n\}\exp(-\gamma_+^{(n)}\bar{s}) + \{\alpha_n \langle q^s(Q_0^2)\rangle_n + \beta_n \langle G(Q_0^2)\rangle_n\}\exp(-\gamma_-^{(n)}\bar{s}) - \langle V_8(Q_0^2)\rangle_n \exp\{-\gamma_n \bar{s}\} \,. \qquad (9b)$$

*) Note that in paper/7/ these equations were obtained in the framework of QCD. However, one can show that they are true also in the case of the scale invariant models.

The quantity q^s is a singlet with respect to the flavour SU (4) group. Eqs. (9) have been obtained assuming that $\langle C(Q_0^2)\rangle_n = 0$. In this case

$$\langle q^s(Q_0^2)\rangle_n = \langle S(Q_0^2)\rangle_n + \langle V_8(Q_0^2)\rangle_n \,. \qquad (10)$$

In Eqs. (8,9) $\alpha_n, \beta_n, \gamma_n, \gamma_\pm^{(n)}$ are the well known/3,7,12/ model dependent quantities. They are connected with the anomalous dimensions of the leading operators involved in the expansion of the product of two electromagnetic currents near the light cone.

As an example, the expression for the quantities γ_n is given bellow:

$$\gamma_n = G\left(1 - \frac{2}{n(n+1)} + 4\sum_{j=1}^{n} \frac{1}{j}\right). \qquad (11)$$

In different quark-gluon models the value of G is different. For instance, in QCD with four colour-triplet quarks

$$G = \frac{4}{25} \,. \qquad (12a)$$

In the case of a scale invariant QCD (the assumption that the Gell-Mann-Low function has an ultraviolet stable zero at $\alpha = \alpha_0 \ll 1$ is made)

$$G = \frac{\alpha_0}{3\pi} \quad (\alpha_0 = g_0^2/4\pi) \,. \qquad (12b)$$

In the scale invariant Abelian vector gluon model

$$G = \frac{\alpha_0'}{4\pi} \,. \qquad (12c)$$

Note, that in our analysis of the experimental data the quantities α_0 and α_0' are free parameters.

Finally, in Eqs. (8,9)

$$\bar{s} = \ln \frac{\ln Q^2/\Lambda^2}{\ln Q_0^2/\Lambda^2} \qquad (13)$$

for the case of QCD, and

$$\bar{s} = \ln Q^2/Q_0^2 \tag{14}$$

for the scale invariant models.

In our analysis of the deep inelastic ep scattering data the following parametrization[7] for the valence quark distributions has been used:

$$x V_8(x,Q^2) = \frac{3\,\Gamma(\eta_1(\bar{s}) + \eta_2(\bar{s}) + 1)}{\Gamma(\eta_1(\bar{s}))\,\Gamma(\eta_2(\bar{s})+1)}\, x^{\eta_1(\bar{s})} (1-x)^{\eta_2(\bar{s})}, \tag{15}$$

$$x\, n_v(x,Q^2) = \frac{\Gamma(\eta_3(\bar{s}) + \eta_4(\bar{s}) + 1)}{\Gamma(\eta_3(\bar{s}))\,\Gamma(\eta_4(\bar{s})+1)}\, x^{\eta_3(\bar{s})} (1-x)^{\eta_4(\bar{s})}, \tag{16}$$

where

$$\eta_i(\bar{s}) = \eta_i + \eta_i' G \bar{s}, \quad i = 1, \dots 4. \tag{17}$$

As for the distributions of strange and charmed quarks they have been taken in the form[7]:

$$x\, S(x,Q^2) = P_s\left(\frac{1}{\langle x\rangle_s} - 1\right)(1-x)^{\left(\frac{1}{\langle x\rangle_s} - 2\right)}, \tag{18}$$

$$x\, C(x,Q^2) = P_c\left(\frac{1}{\langle x\rangle_c} - 1\right)(1-x)^{\left(\frac{1}{\langle x\rangle_c} - 2\right)}, \tag{19}$$

where

$$P_s \equiv \langle S(Q^2)\rangle_2, \quad P_c \equiv \langle C(Q^2)\rangle_2, \tag{20a}$$

$$\langle x\rangle_s \equiv \frac{\langle S(Q^2)\rangle_3}{P_s}, \quad \langle x\rangle_c \equiv \frac{\langle C(Q^2)\rangle_3}{P_c}. \tag{20b}$$

In the analysis of the data we have chosen the parameters η_i and the values of the moments $\langle S(Q_0^2)\rangle_{2,3}$ and $\langle C(Q_0^2)\rangle_{2,3}$ as follows:

$$\eta_1 = 0.70, \quad \eta_2 = 2.60, \quad \eta_3 = 0.85, \quad \eta_4 = 3.35;$$
$$\langle S(Q_0^2)\rangle_2 = 0.110, \quad \langle S(Q_0^2)\rangle_3 = 0.9167\times 10^{-2}; \tag{21}$$
$$\langle G(Q_0^2)\rangle_2 = 0.402, \quad \langle G(Q_0^2)\rangle_3 = 0.335\times 10^{-1}.$$

These values have been found in paper[7] from the data analysis at the fixed value of Q^2 : $Q_0^2 = 1.8\,(GeV/c)^2$ *). The parameters η_i', Λ, α_0 and α_0' are determined from the experimental data at all available Q^2 larger than $2(GeV/c)^2$ by minimizing the functional χ^2.

Results of the Analysis

In this section we give the results of analysis of the SLAC data[1,8-10]. We only use the data points that satisfy $2 \leq Q^2 \leq 30\,(GeV/c)^2$, $x \leq 0.8$ and hadron mass $W \geq 2$ GeV (330 points).

1. Our analysis has shown that no satisfactory description of the available data can be obtained $\left(\chi^2/\bar{\chi}^2 = \frac{500}{323}\right)$ if the leading logarithmic approximation for F_2 and R is used in QCD and the considered scale invariant models. Therefore, in order to verify these models one has to take into account the effects of target mass and next order effective coupling constant corrections to F_2 and R. First of all the target mass corrections should be taken into account since the available data are at relatively low values of Q^2. Such attempts have been undertaken in papers [6,13] but we think them to be inconsistent.

2. We have further analyzed the data when the structure function F_2 is given by Eqs. (6,12-20) but R is a constant

$$R = a. \tag{22}$$

Here a is a free parameter.

It turned out that for all considered parametrizations of the structure function F_2, one can obtain a satisfactory description of the data. The results of the analysis are presented in table I.

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*) We have analyzed also the data at fixed value of Q^2: $Q_0^2 = 2(GeV/c)^2$. For the parameters η_i in this case we have received the values which coincide with (23) within errors ($\eta_1 = 0.60 \pm 0.20$, $\eta_2 = 2.86 \pm 1.05$, etc.).

It is seen from this table that for all values of Λ in the region $0.1 \leq \Lambda \leq 0.5$ GeV/c the data are fitted very well.*) In spite of the fact that with decreasing Λ we get a better fitting of the data, one cannot conclude that the value of $\Lambda = 0.1$ GeV/c is more preferable than $\Lambda = 0.5$ GeV/c. The values for Λ which we have got are in agreement with the estimates of this quantity obtained by other authors[14]. It is also seen from table I that a satisfactory description of the data can be obtained for the values of the parameters α_0 (see Eq. (12b)) and α_0' (see Eq. (12c)) in the following regions: $0.10 \leq \alpha_0 \leq 0.40$; $0.01 \leq \alpha_0' \leq 0.03$.

We want to emphasize that the values for the quantity R we have received are large. For instance, when $\Lambda = 0.5 \frac{GeV}{c}$ (Eq. (13) for $\bar{S}$)

$$R = 0.23 \pm 0.02 . \tag{23}$$

Practically, for all other parametrizations of F_2 (Eqs.(12b,c) for G and Eq.(14) for $\bar{S}$) the value of R coincides with (23). This value of R is in agreement with the values of this quantity obtained in refs.[15]

3. The data have been analyzed also using for R the following parametrization:

$$R = 4b/Q^2 . \tag{24}$$

Such an expression for R can be obtained in the framework of the parton model[16]. In this model the parameter b is the mean value of the square parton transverse momentum transfer. The results of analysis are presented in table II. As one can see from this table the experimental data are well fitted by means of all considered parametrizations of the structure function F_2 . In the case $\Lambda = 0.5 \frac{GeV}{c}$, for the parameter b we get

$$b = 0.18 \pm 0.02 \left(\frac{GeV}{c}\right)^2 . \tag{25}$$

*) At the values of Λ smaller than 0.1, the errors of the parameters η_i' are larger than their mean values.

Note, that in table II we give also the obtained values of the normalized factors. The deviations of the latter from unity are not larger than 4-5%.

The parameters η_i' have been also determined from the equations for the moments (8a). These equations have been solved for $n = 2, .. 12$ and $0.02 \leq \bar{S} \leq 0.22$. Note, that the values of the parameters η_i' we have got this way ($\eta_1' = -2.19 \pm 0.04$;

$\eta_2' = 5.33 \pm 0.19$; $\eta_3' = -2.63 \pm 0.08$; $\eta_4' = 5.55 \pm 0.48$)

disagree with the values of these parameters presented in tables I and II.

As a result of the analysis we have made we come to the following conclusions:

1. QCD and the considered scale invariant models are not in agreement with the deep inelastic ep scattering data if for F_2 and R the leading logarithmic approximation is used without including target mass corrections.

2. These data are well fitted if for R the following phenomenological parametrizations: $R = const$ and $R = 4b/Q^2$ are used. The available experimental data, however, are not able to distinguish between the parametrization of F_2 based on QCD and the corresponding expressions for F_2 based on the considered scale invariant models.

Table I

The Results of Analysis of the Data[1,8-10]

$R = const$

	Parametrization (6),(12a), (13), (15) - (21)			Parametrization (6),(12b),(14),(15-21)			Parametrization (6),(12b),(14-21)	
	Λ^2			α_0			α'_c	
	0.01	0.11	0.25	0.10	0.20	0.40	0.01	0.03
η'_1	-3.66±0.33	-2.15±0.16	-1.63±0.11	-8.97±1.07	-4.45±0.53	-2.19±0.27	-119.41±15.01	-39.13±5.61
η'_2	6.36±0.80	4.08±0.21	3.29±0.31	15.03±1.96	7.59±1.00	3.87±0.49	200.64±28.34	67.96±9.68
η'_3	-15.07±1.86	-8.69±0.99	-6.47±0.74	-37.61±5.22	-18.87±2.62	-9.52±1.33	-498.28±72.37	-164.93±26.53
η'_4	6.62±6.12	10.24±4.17	11.53±3.57	-12.55±10.55	-6.22±5.47	3.10±2.80	-164.41±147.70	-50.82±50.18
R	0.21±0.02	0.22±0.02	0.23±0.02	0.21±0.02	0.22±0.02	0.23±0.02	0.20±0.02	0.21±0.02
$\chi^2/\bar{\chi}^2$	361/322	373/322	376/321	355/322	359/322	368/322	355/322	358/322

Table II

The Results of the Analysis of the Data[1,8-10]

$R = \frac{4b}{a^2}$

	Parametrization (6),(12a),(13),(15-21)			Parametrization (6),(12b),(14),(15)-(21)			Parametrization (6),(12b),(14),(15-21)	
	Λ^2			α_0			α'_c	
	0.01	0.11	0.25	0.10	0.20	0.40	0.01	0.03
$N_1^{[9,10]}$	1.007±0.010	1.005±0.010	1.004±0.010	1.010±0.009	1.012±0.009	1.015±0.009	1.009±0.009	1.012±0.009
$N_2^{[1]}$	0.962±0.014	0.969±0.015	0.957±0.015	0.966±0.014	0.969±0.014	0.975±0.014	0.965±0.014	0.969±0.014
$N_3^{[8]}$	0.980±0.007	0.976±0.007	0.974±0.007	0.984±0.007	0.983±0.007	0.982±0.007	0.984±0.007	0.983±0.007
η'_1	-4.31±0.27	-2.66±0.13	-2.08±0.09	-9.66±1.16	-4.86±0.56	-2.46±0.28	-127.93±15.73	-43.06±5.11
η'_2	6.13±0.74	3.76±0.38	2.94±0.27	15.45±2.62	7.71±1.28	3.84±0.63	207.37±35.17	68.67±11.74
η'_3	-11.35±1.56	-6.56±0.83	-4.83±0.61	-26.62±5.60	-13.37±2.74	-6.81±1.36	-350.67±75.78	-117.37±28.85
η'_4	21.62±5.67	21.33±3.70	21.61±3.14	13.51±13.34	7.28±6.65	4.01±3.32	186.86±181.70	67.54±61.11
b	0.18±0.02	0.18±0.02	0.18±0.02	0.18±0.02	0.19±0.02	0.20±0.02	0.18±0.02	0.19±0.02
$\chi^2/\bar{\chi}^2$	362/322	377/322	381/321	365/322	358/322	370/322	352/322	357/322

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