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FERMI MOTION CORRECTIONS FOR DEEP INELASTIC MUON SCATTERING ON NUCLEI AT LARGE Q^2

* On leave from Nuclear Center, Charles University, Prague.

Савин И.А., Жачек Й.

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Поправки к сечению глубоконеупругого рассеяния мюонов из-за ферми движения нуклонов в ядре

Исследованы фермиевские поправки к глубоконеупругому сечению на ядрах углерода при энергии первичных мюонов 300 ГэВ с целью получения количественных оценок в кинематической области НА-4 эксперимента.

Расчеты проведены с использованием некогерентного приближения в нерелятивистском подходе. Были получены соотношения между структурными функциями ядра и свободных нуклонов при $Q^2 > 10$

Расчеты показали, что ферми-движение дает средний сдвиг ~20% в кинематических переменных x и ν и поправки к сечению превышают 30% в области $x > 0.7$. Поэтому при интерпретации экспериментальных данных необходимо учитывать ферми-движение в ядрах.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1979

Savin I.A., Žáček J.

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Fermi Motion Corrections for Deep Inelastic Muon Scattering on Nuclei at Large Q^2

The effects due to nucleon Fermi motion were estimated for deep inelastic muon scattering on nuclei at CERN SPS energies. The corrections to the carbon structure function calculated in the nonrelativistic impulse approximation were found to exceed 30% for $x > 0.7$.

The investigation has been performed at the Laboratory of High Energy Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna 1979

1. INTRODUCTION

Deep inelastic charged lepton scattering on various targets has become a very important tool to study the structure of nucleons. Data analysis of scattering on nuclei has shown that, besides scattering on nucleons, processes in nuclei have to be taken into account. One of the important effects has been found [1] to be that due to Fermi motion of nucleons. The Fermi motion corrections have been calculated for deuterium in order to extract the neutron structure function [6] and to compare experimental data with model predictions [11].

This paper is devoted to quantitative estimates of the influence of nuclear Fermi motion on deep inelastic muon scattering on carbon. The kinematic range of $Q^2 \gtrsim 10\,(GeV/c)^2$ was chosen to cover the region of the NA-4 muon experiment being carried out at CERN energies. The calculations were performed in the nonrelativistic approximation similar to that described in ref. [1].

2. SMEARED NUCLEON STRUCTURE FUNCTIONS

The kinematics of lepton scattering on the target of mass M is illustrated in <u>Fig. 1.</u> The four momenta of the primary and scattered leptons and of the nucleon are denoted $\bar{p}_0(E_0, \vec{p}_0)$ $\bar{p}(E, \vec{p})$ and $\bar{P}(E_p, \vec{P})$ respectively.

It is convenient to introduce the scaling variables $x' = Q^2/2M\nu'$ and $\nu' = \bar{P}\bar{q}/M$, where $\bar{q} = \bar{p}_0 - \bar{p}$ and $Q^2 = 4E_0 E \sin^2\phi/2$. In the laboratory frame ($\vec{P} = 0$), x becomes to be the known quantity $x = Q^2/2M\nu$, where $\nu = E_0 - E$ is energy transfer. The relation between x' and x can be easily derived.

$$x' = \frac{xM}{E_p - P\sqrt{1 + Q^2/\nu^2}\cos\psi}, \quad P = |\vec{P}|, \qquad (1)$$

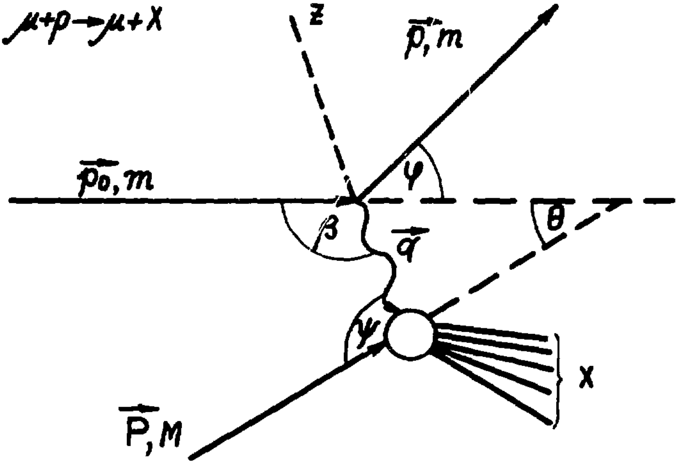

Fig.1. The kinematics of the lepton scattering on the target of mass M in the one-photon approximation.

where ψ is the angle between the virtual photon and nucleon momenta. Thus the nucleon motion causes a shift in the values of x depending on the nucleon momentum.

In terms of x and Q^2 the differential cross section of the lepton scattering on protons reads in the one photon approximation /2/

$$\frac{d^2\sigma_p}{dxdQ^2} = \frac{\pi\alpha^2}{Q^2E_0^2x^2}\cdot\frac{1}{\sqrt{(\bar{P}\bar{p}_0)^2}}F_2^p(x',Q^2)K_p(x',Q^2,\vec{P}), \qquad (2)$$

where $F_2^p(x', Q^2)$ is the proton structure function and

$$K(x',Q^2,\vec{P}) = M\frac{1-R_p}{1+R_p}x' + \frac{Q^2}{2(R_p+1)Mx'} + 4\frac{(\bar{P}\bar{p}_0)^2}{Q^2M}x' - 2\frac{\bar{P}\bar{p}_0}{M}. \qquad (3)$$

Here R_p is the ratio of the transverse and longitudinal cross-sections of the virtual photon scattering on the proton /3/.

The smeared cross section can be expressed as

$$\frac{d^2\sigma_p^s}{dx\,dQ^2} = \int f(\vec{P})d^3\vec{P}\;\frac{\sqrt{(\bar{P}\bar{p}_0)^2}}{ME_0}\cdot\frac{d^2\sigma_p}{dx\,dQ^2}, \tag{4}$$

where $f(\vec{P})d^3\vec{P}$ is the probability for finding the proton with the momentum $\vec{P}$.

Choosing the z - axis in the direction of the virtual photon momentum, using the spherical symmetry of $f(\vec{P})$ and expressing the azimuthal angle ψ of vector $\vec{P}$ as a function of x', one finds

$$\frac{d^2\sigma_p^s}{dx\,dQ^2} = \frac{\pi\alpha^2}{Q^2E_0^2x\sqrt{1+4M^2x^2/Q^2}}\int_0^\infty Pf(P)dP\int_0^{2\pi}d\gamma\int_{x'_-}^{x'_+}\frac{K_p(x',Q^2,\vec{P})}{x'^2}F_2^p(x',Q^2)\,dx', \tag{5}$$

where γ is the polar angle of the vector $\vec{P}$. The limits on the integration are given by eq. (1), where $\cos\psi = \pm 1$. In the integral (5) the product $\bar{P}\bar{p}_0 = E_0(E_p - P\cos\theta)$ must be expressed as a function of x' and γ using (see Fig.1) $\cos\theta = \cos\psi\cos\beta + \sin\psi\sin\beta\cos\gamma$, where

$$\cos\beta = \frac{\vec{p}\vec{q}}{|\vec{p}||\vec{q}|} = \frac{1 + Mx/E_0}{\sqrt{1 + (2Mx/Q)^2}}. \tag{6}$$

In the high energy region, where

$$M \ll E_0 \quad \text{and} \quad 4M^2x^2 \ll Q^2, \tag{7}$$

one has $\cos\beta \approx 1$, and eq. (5) takes the simpler form

$$\frac{d^2\sigma_p^s}{dx\,dQ^2} = \frac{\pi\alpha^2}{Q^2E_0^2Mx^2}K_p(x, Q^2, \vec{P} = 0)I_p(x, Q^2), \tag{8}$$

where

$$I_p(x, Q^2) = 2\pi Mx^2\int_0^\infty Pf(P)\,dP\int_{x'_-}^{x'_+}\frac{F_2^p(x',Q^2)}{x'^3}dx'. \tag{9}$$

The approximation (9) corresponds to that obtained in ref. /1/.

3 . NUCLEUS STRUCTURE FUNCTION

The cross section of the lepton scattering on nuclei at rest is written in the form similar to eq. (2)

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{\pi\alpha^2}{Q^2E_0^2Mx^2} K(x, Q^2, \vec{P}=0)\, F_2(x, Q^2), \qquad (10)$$

where $F_2(x, Q^2)$ stands for the nucleus structure function and $K(x, Q^2, \vec{P}=0)$ is given by eq. (3), where R_p is replaced by the corresponding ratio R for the nucleus.

On the one hand, the function F_2 can be extracted from the experimental data following eq. (10) and, on the other hand, it can be calculated in the framework of various models.

We have determined F_2 using the incoherent impulse approximation. In this case we have

$$\sigma = N\sigma_p^s + (A-N)\sigma_n^s , \qquad (11)$$

where A(N) is the number of nucleons (protons). Here σ_p^s is given by eq. (5) and σ_n^s is the smeared neutron cross section analogous to eq. (5). It follows from eqs. (10) and (5) that a relationship between F_2, F_2^p and F_2^n is complicated due to mixing the kinematic terms and integrals in eq. (5). However, in the kinematic region where the condition (7) is fulfilled one obtains

$$\frac{d^2\sigma}{dx\,dQ^2} \approx \frac{\pi\alpha^2}{Q^2E_0^2x^2M} [NK_p(x, Q^2, \vec{P}=0)I_p + (A-N)K_n(x, Q^2, \vec{P}=0)I_n]. \qquad (12)$$

The term I_n is the integral (9) over $F_2^n(x, Q^2)$. Assuming $R_p = R_n$, we obtain a simple relation between the structure functions of the nucleus and nucleons

$$F_2(x, Q^2) = 2\pi x^2 \int_0^\infty P f(P) dP \int_{x'_-}^{x'_+} \frac{NF_2^p(x', Q^2) + (A-N)F_2^n(x', Q^2)}{x'^3} dx'. \qquad (13)$$

4. CALCULATIONS

We have applied both eq. (11) and eq. (13) to estimate the corrections due to Fermi motion of nucleons in carbon nuclei for a primary beam energy of 280 GeV. The calculations have been performed by the numerical integration.

The function $f(P)$ has been determined using the nonrelativistic carbon wave function [4]

$$f(P)d^3\vec{P} = \frac{4}{\pi^{3/2}P_0^3 A}\left[1 + \frac{A-4}{6}\,\frac{P^2}{P_0^2}\right]e^{-P^2/P_0^2}\,d^3\vec{P}, \tag{14}$$

where P_0=127 MeV/c. The product $f(P)\,P^2$ is gaussian in shape with a mean value of $\langle P \rangle$ =187 MeV/c (see Fig. 2a).

The smearing of x and ν is illustrated in Fig. 2b, where $x'_{\pm}$ and $\nu'_{\pm}$ are plotted for $P=\langle P\rangle$. The relative maximum spreads are given by $\Delta\nu_{max}/\nu = \Delta x_{max}/x = P\sqrt{1+Q^2/\nu^2}/M$ with mean values of $\langle\Delta x_{max}\rangle/x = \langle\Delta\nu_{max}\rangle/\nu$ =0.19. The proton structure function was taken from ref. [5]

$$F_2^p(x, Q^2) = \sum_{i=3}^{5} c_i(1-x)^i \left(\frac{Q^2}{Q_0^2}\right)^{-a\ln 6x}, \tag{15}$$

where Q_0^2=3 $(\mathrm{GeV}/c)^2$, a=0.145, C_3=2.799, C_4=-4.048, and C_5=1.615. The neutron structure function was supposed to be of the simple form $F_2^n = (0.95-0.81\,x)\,F_2^p$ for $x>0.1$ which corresponds to the measurement of F_2^n/F_2^p carried out at SLAC [6].

Figures 3a and 3b show the behaviour of the carbon structure function (13) per nucleon and of the mean value of $(F_2^n + F_2^p)/2$ for Q^2=100 $(\mathrm{GeV}/c)^2$ and x=0.5, respectively.

The deviations of these functions become sizeable for $x > 0.7$. The effect of Fermi motion can be expressed by the factor:

$$S_1 = 1 - \frac{(\sigma_p + \sigma_n)/2}{\sigma\ \mathrm{nucleon}}. \tag{16}$$

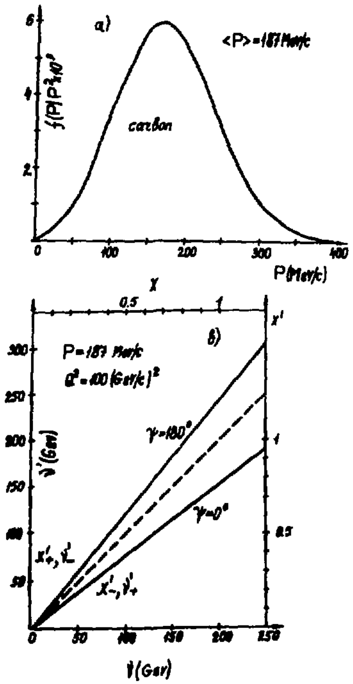

Fig.2. The distribution of the probability of finding the nucleon with momentum P(a). The dependence of the limits $x'_{\pm}$ and $\nu'_{\pm}$ on x and ν(b).

The dependence of S_1 on x and Q^2 is shown in Fig. 3a and 3b. The factor S_1 rises rapidly for $x > 0.7$ and varies slowly with Q^2. This fact can be explained by shifting the values of x and by the sharply falling function F_2^p with x. (For instance, for $x = 0.8$ the limits are $x'_{-} = 0.665$ and $x'_{+} = 0.95$ for $P = \langle P \rangle$; in this interval F_2^p decreases by a factor of $\sim 10^{-2}$). The constant S_1 - contours for $S_1 = 5\%$, 10% and 50% are given in Fig. 3.

The difference between the values of S_1 calculated according to eqs. (11) and (13) has been found to be 10% and 1.8% at $Q^2 = 10$ $(\mathrm{GeV/c})^2$, and $Q^2 = 100$ $(\mathrm{GeV/c})^2$, respectively, for $x = 0.9$. This difference vanishes for $x < 0.7$ in the low Q^2 region and for $x < 0.9$ in the large Q^2 range.

The calculation was performed assuming $R_p = R_n = 0.2$. The values of R_p have been found to be 0.25 ± 0.10 at SLAC

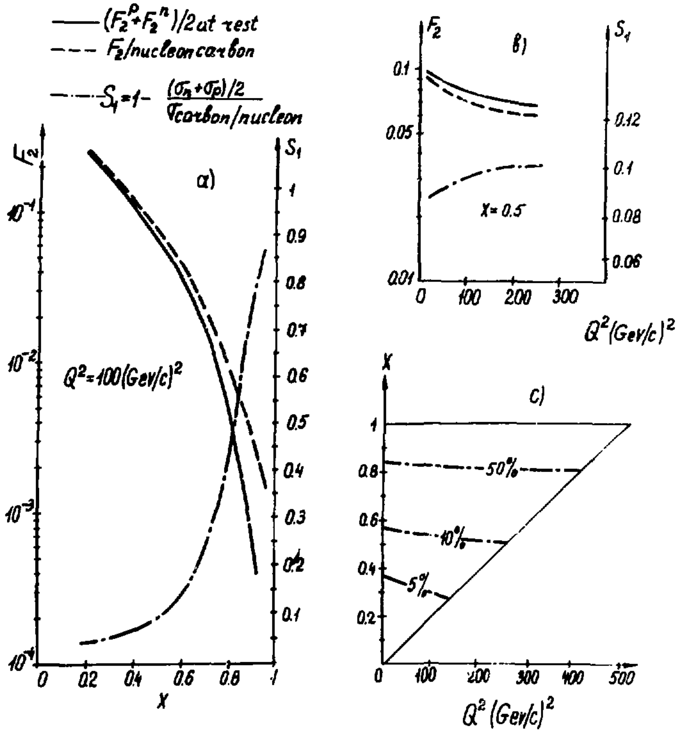

Fig.3. The structure function of carbon per nucleon (dashed line), the mean nucleon structure function (full line) and the ratio S_1 (dashed dotted line) for $Q^2=100\ (GeV/c)^2$ (a) and for x =0.5 (b). Constant S_1 - contour lines (c).

and 0.44 ± 0.25 at Fermilab /8/. The role of R_p, discussed in ref. /9/, is demonstrated in fig. 4 where the constant contours of the ratio $S_2 = 1 - \sigma_p(R_p = 0.75)/\sigma_p(R_p = 0.15)$ are drawn. Thus the correction factor S_1 can be affected by the uncertainties in R_p and R_n in the region of $x \sim Q^2/2ME_0$ especially.

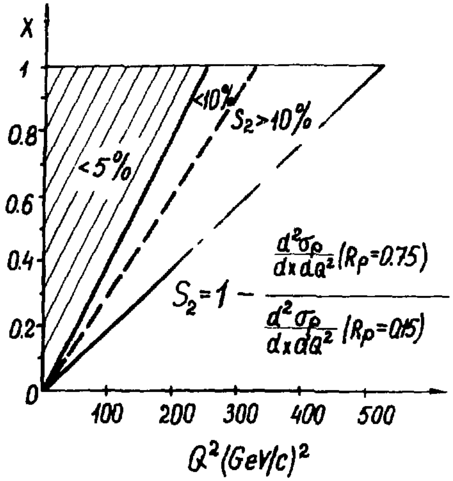

Fig.4. The relative deviation of the proton cross section for $R_p = 0.15$ and $R_p = 0.75$.

5. CONCLUSIONS AND DISCUSSION

We have estimated the corrections due to Fermi motion of nucleons in carbon nuclei in the kinematic region (7). The results of our calculations can be summarized as follows:

i) In the kinematic region (7), which is the kinematic acceptance of the NA 4 experiment, formula (8) approximates well the cross section (5).

ii) The mean values of relative smearing of ν and x are constant and $\langle\Delta\nu_{max}\rangle/\nu = \langle\Delta x_{max}\rangle/x = 20\%$. Therefore a study of any scattering process on nuclei depending on x and ν should include systematic uncertainties of the determination of x and ν.

iii) The correction factor S_1 exceeds 0.3 for $x > 0.7$, where it rises sharply when x approaches 1. Thus the interpretation of the experimental data in terms of scattering on nucleons should be made carefully, particularly in the region of $x > 0.7$.

In conclusion we discuss briefly the validity of the assumptions connected with eqs. (14) and (15). It is reasonable to suppose that the structure function (15), extracted /5/ from μ-data at Q^2 up to 50 $(GeV/c)^2$, can be interpolated into the large Q^2 region. On the other hand, the neutron structure function has not been yet obtained, except for very low Q^2 data /6/. Similarly, the values of R_p obtained up to now are in disagreement with each other /7,8/. Various values of R_p were found to affect the calculations of S_1 by 20% for $x \sim Q^2/2ME_0$. The assumption of using the nonrelativistic wave function (14) has been discussed in detail in ref. /10/. In that paper it has been shown that the usual nonrelativistic estimates of S_1 are very likely to be unreliable for $x \to 1$. However, relativistic corrections cannot be calculated because of lack of knowledge of relativistic vertex functions.

We wish to acknowledge the discussion we had with R.Mach.

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