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Константы связи мюонов в слабом нейтральном токе

Рассматривается возможность определения констант связи мюонов в слабом нейтральном токе с помощью измерения асимметрии, возникающей в глубоконеупругом рассеянии мюонов на нуклонах при инверсии заряда и спиральности пучка. В частности, показано, что определение правого слабого заряда $I_3^R(\mu)$ и параметра $\sin^2\theta$ возможно без использования партонной модели.

Работа выполнена в Лаборатории высоких энергий ОИЯИ.

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Klein M., Riemann T., Savin I.A. **E1 - 12451**

Weak Neutral-Current Couplings of Muons

Inversion of both the charge and the helicity of muon beams is considered as a possibility of determining the weak neutral-current couplings of muons, in particular the right-handed weak charge $I_3^R(\mu)$ and $\sin^2\theta$ without using the parton model.

The investigation has been performed at the Laboratory of High Energies, JINR.

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Due to recently performed sensitive experiments, the weak neutral-current interaction of neutrinos, valence quarks and electrons is almost understood /1/. Different neutrino data have determined the vector and axial-vector couplings of u and d quarks /2/. The SLAC eD /3/ and the Novosibirsk Bi experiments /4/ have resolved the V-A ambiguity of elastic neutrino-electron scattering thereby measuring the electron couplings /5,6/. Nothing is known, however, about the weak neutral-current couplings of muons. In this note, a possibility is considered of extracting these couplings from deep inelastic polarized muon scattering at momentum transfers $Q^2 = O(100)$ $(GeV/c)^2$. The muon couplings are of fundamental interest for neutral-current μe universality, for the single Z-boson hypothesis /7/, for the existence of right-handed currents and of muon induced parity violation. The vector coupling, if interpreted, e.g., in the Weinberg-Salam theory (WS), fixes the mixing angle $\sin^2\theta$. Three relations for $\sin^2\theta$ are derived, two of them without using the quark parton model (QPM).

In deep inelastic muon scattering neutral currents are expected to be of the order of $k = Q^2 G/\sqrt{2}\,2\pi\alpha = 1.79 \cdot 10^{-4} \cdot Q^2$ (in $(GeV/c)^2$) resulting from the interference of one-photon exchange with Z-boson exchange. Muons couple to the Z field by

$$g_Z \cdot \bar{\mu}\gamma^m (v_\mu - a_\mu \gamma_5)\mu \cdot Z_m \qquad (1)$$

with strength $g_Z^2 / M_Z^2 = 2G/\sqrt{2}$. In SU(2)xU(1) gauge theories the couplings are

$$v_\mu = I_3^L + I_3^R + 2\sin^2\theta, \quad a_\mu = I_3^L - I_3^R, \qquad (2)$$

$I_3^{L(R)}$ being the left-handed (r.h.) weak μ^- charges. Neglecting radiative electromagnetic and weak corrections we can calculate the deep inelastic cross section of scattering muons, $d\sigma^\pm(\lambda)$, with charge ± and helicity λ off nucleons. Denoting the one-photon contribution by $d\sigma_0$ one gets /8,9/

$$\frac{d\sigma^{\pm}}{d\sigma_0} = 1 - k[v_\mu V \pm a_\mu A - \lambda(\pm a_\mu V - v_\mu A)]. \qquad (3)$$

Here $V(x,Q^2)$ and $A(x,Q^2) = A_0(x,Q^2)\cdot g(y)$ are ratios of interference to electromagnetic structure functions depending on the dynamics and on the structure of the hadronic neutral current with $g(y) = (1-(1-y)^2)/(1+(1-y)^2)$ and V and A_0 defined as in /9/.

For a given magnitude of beam helicity λ there exist three independent cross section asymmetries: two parity violation asymmetries of the type measured at SLAC /3/ and Serpukhov /10/

$$A^{\pm} = \frac{d\sigma^{\pm}(+\lambda) - d\sigma^{\pm}(-\lambda)}{d\sigma^{\pm}(+\lambda) + d\sigma^{\pm}(-\lambda)} = -k\lambda(\pm a_\mu V + v_\mu A) \qquad (4)$$

and a third asymmetry to be measured by conjugation of the muon beam

$$B = \frac{d\sigma^{+}(-\lambda) - d\sigma^{-}(+\lambda)}{d\sigma^{+}(-\lambda) + d\sigma^{-}(+\lambda)} = k(\lambda v_\mu - a_\mu)A. \qquad (5)$$

The measurement of these asymmetries is an obvious challenge for CERN SPS muon experiments reaching large Q^2 with high statistical accuracy *.

Further on we concentrate on the beam conjugation asymmetry B because i) the statistical accuracy of $B(\lambda)$ for positive λ is particularly high since it requires to utilize only the high intensity forward part of the $\pi(K)\to\mu\nu$ decay spectrum; ii) the only requirement for extracting a_μ and v_μ from B would be to control the axial-vector part of the hadronic current which is independent of $\sin^2\theta$; iii) available neutral-current data predict the largest effects just for B /6/. According to /9/ one estimates for $\sin\theta = 1/4$ in the WS theory $V = 4/5$, $A = -g(y)\cdot 9/5$, $a_\mu = -½$ and $v_\mu = 0$ giving at $Q^2 = 200\ (\mathrm{GeV}/c)^2$: $B(\lambda) = -3.2g(y)\%$ independently of λ and $A^{\pm} = \pm 1.8\lambda\%$ independently of y.

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*Note that all definitions and subsequent arguments are not only applicable to deep inelastic but also to elastic scattering if the ratios of structure functions V and A_0 are replaced by ratios of form factors.

The measurement of $B(\lambda)$ at two different helicities * is complete in the sense that it fixes the muon couplings. The vector coupling appears to be the slope of

$$\frac{B(\lambda)}{-kA} = a_\mu - \lambda v_\mu = \begin{cases} -2(I_3^R - \sin^2\theta), & \lambda = 1 \\ 2(I_3^L - \sin^2\theta), & \lambda = -1 \end{cases} \qquad (6)$$

whereas the axial coupling is the intercept at $\lambda = 0$. Eq. (6) makes clear that the experimentally preferred helicity $\lambda = 1$ implies sensitivity of B to the right-handed muon coupling and to $\sin^2\theta$. For illustration $(a_\mu - \lambda v_\mu)$ versus λ is given in the figure for standard $I_3^L = -½$ keeping I_3^R as a free parameter. Four different assignments of I_3^R are considered (-1, -1/2, 0, 1/2) corresponding to the r.h. multiplets /12,13/

$$\begin{pmatrix} M \\ M \\ \mu^- \end{pmatrix}_R ; \begin{pmatrix} M^0 \\ \mu^- \end{pmatrix}_R ; \quad \mu^-_R \quad \begin{bmatrix} \mu^- \\ M^- \end{bmatrix}_R \qquad (7)$$

containing heavy leptons M. The solid (dashed) curves in the figure belong to $\sin^2\theta = 0.2\ (0.3)$. It is of importance that the variations at λ near 1 due to I_3^R are dominant as compared to what is expected from $\sin^2\theta$. Thus, using very forward produced muons, i.e., the standard SPS muon beam, the r.h. weak charge can be measured. Heavy leptons of a few GeV mass may also give directly detectable signals /14/.

From the present status /1/ the WS theory would be expected to be confirmed ($I_3^L = -½$, $I_3^R = 0$). Then the next question would concern the details of this theory, i.e., the mixing angle and the Higgs multiplet structure which affects the ratio /1/ $\rho = M_W^2/M_Z^2\cos^2\theta$. The asymmetry $B(\lambda)$ at $\lambda = 0$ is independent of $\sin^2\theta$ (eq. 5). Thus ρ is fixed by

$$\rho = M_W^2/M_Z^2\cos^2\theta = 2B(0)/kA. \qquad (8)$$

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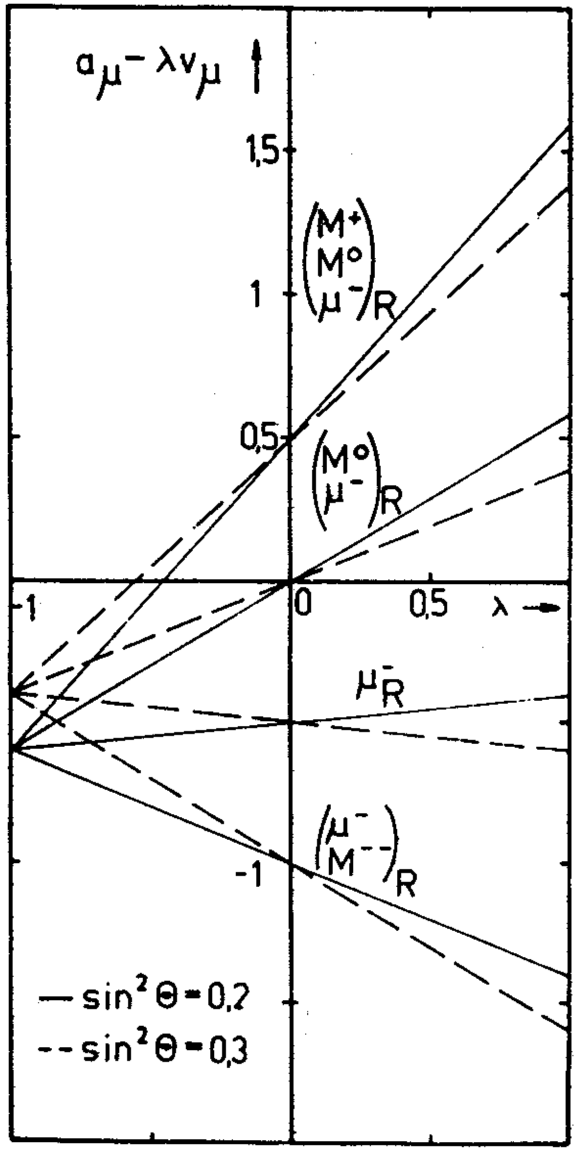
*Charge conjugation maintains the beam charge dependent part of radiative corrections. The resulting electromagnetic asymmetry B_{elm} has been calculated to be positive and smaller than 1% below $Q^2/s = 0.5$ /11/. One gets rid of B_{elm} by subtracting B asymmetries at two different energies $E_1 < E_2$ since B_{elm} is very likely to be scale invariant. This subtraction at fixed (x,y) decreases the weak asymmetry by the factor $(1 - E_1/E_2)$.



Figure

$(a_\mu - \lambda v_\mu)$, eq. (6), as a function of the μ^- beam helicity λ for $I_3^L = -½$ and $I_3^R = (-1, -½, 0, +½)$. Solid (dashed) curves belong to $\sin^2\theta = 0.2\ (0.3)$.

Neutral-current neutrino data giving $\rho = 0.98 \pm 0.05$ indicate a minimal Higgs structure [2]. Eq. (5) can be rewritten to determine $\sin^2\theta$ from the leptonic current as

$$\sin^2\theta = 1/4 - [2B(\lambda)/kA - 1]/4\lambda. \tag{9}$$

A similar relation using both parity violation asymmetries (eq. 4) has been derived in [15]. One can avoid the QPM calculation of A having measured $B(\lambda)$ at two different helicities. Calculating $B_1 = B(\lambda_1)$ and $B_2 = B(\lambda_2)$ at the same (Q^2, x) one gets independently of ρ

$$\sin^2\theta = 1/4 - [(B_2 - B_1)/(B_1\lambda_2 - B_2\lambda_1)]/4. \tag{10}$$

This relation expresses $\sin^2\theta$ in terms of measurable quantities only and is free of any dynamical assumption.

Recently it has been shown by several authors that the hadronic axial-vector current can be related by isospin invariance to the difference between antineutrino and neutrino charged current cross sections [15-17]. This allows one to introduce a neutrino beam conjugation asymmetry, B_ν, being completely analogous to B (eq. 5)

$$B_\nu = \frac{d\sigma^{\bar\nu} - d\sigma^{\nu}}{d\sigma^{\bar\nu} + d\sigma^{\nu}}. \tag{11}$$

B_ν is approximately [16] equal to $A \cdot 5/9$ giving

$$B(\lambda) = k \cdot (\lambda v_\mu - a_\mu) \cdot B_\nu \cdot 9/5. \tag{12}$$

Therefore, the muon couplings and the parameters of the WS theory are given by combining deep inelastic muon and neutrino scattering data at the same (Q^2, x). It follows a third possibility to calculate $\sin^2\theta$

$$\sin^2\theta = 1/4 - [10B(\lambda)/9kB_\nu - 1]/4\lambda. \tag{13}$$

The present world average for $\sin^2\theta$ is 0.23 ± 0.02 [1]. Thus almost equal beam conjugations are expected in muon and neutrino scattering which differ only by the corresponding constants and propagators, respectively,

$$B \cdot 2\pi\alpha/Q^2 = B_\nu \cdot G/\sqrt{2}. \tag{14}$$

A fundamental problem to be investigated with charged lepton beams is parity violation. The natural way to search for parity violation would be to measure the asymmetries $A^\pm$

(eq. 4) containing only V-A combinations. Nevertheless, one can ask for how to study parity violation when measuring B. The answer is obvious after rewriting B for different $\mu^\pm$ helicities as

$$B(\lambda_1,\lambda_2) = \frac{d\sigma^+(\lambda_1) - d\sigma^-(\lambda_2)}{d\sigma^+(\lambda_1) + d\sigma^-(\lambda_2)} =$$

$$= k[a_\mu A + v_\mu A\cdot(\lambda_1 - \lambda_2)/2 - a_\mu V\cdot(\lambda_1 + \lambda_2)/2]. \qquad (15)$$

For large $(\lambda_1-\lambda_2)$, as considered above, the measurement is sensitive to $v_\mu A$. For electrons, this combination is suppressed in the heavy atom experiments. In the WS theory it is expected to be small. For large $(\lambda_1+\lambda_2)$ the measurement is sensitive to $a_\mu V$. This combination has been essentially observed at SLAC and Novosibirsk. In the WS theory at $\sin^2\theta = 1/4$ one estimates $a_\mu V = -0.4$ to be compared with the parity conserving contribution to B, $a_\mu A = 0.9\cdot g(y)$. Note that only $a_\mu V$ should survive if $B(\lambda_1,\lambda_2)$ is calculated for y tending to zero.

To summarize, the muon beam conjugation asymmetry B, eq. (5), is of particular interest since it is measurable rather accurately and promises to determine the weak neutral-current couplings of muons. Helicity λ near 1 (forward produced muons) implies particular sensitivity of B to the right-handed weak charge $I_3^R(\mu)$. Several relations for $\sin^2\theta$ have been derived which are based either on the parton model or, independently of it, on two measurements of $B(\lambda)$ and on a neutrino beam conjugation asymmetry, respectively. The helicity and y dependence of B give insight into the question of muon induced parity violation in a new range of momentum transfers.

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