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Ядерное рассеяние дейтронов при 4,3; 6,3 и 8,9 ГэВ/с

Под углом 103 мрад измерены импульсные спектры вторичных дейтронов от взаимодействий 4,3; 6,3 и 8,9 ГэВ/с-дейтронов с ядрами углерода, алюминия и висмута. Наблюденные импульсные спектры дейтронов от d−C -соударений удалось качественно воспроизвести, предположив, что в соударениях релятивистских дейтронов с ядрами часть нуклонов ядра, вовлеченных во взаимодействие с налетающим дейтроном, испускается в связанном состоянии.

Работа выполнена в Лаборатории вычислительной техники и автоматизации ОИЯИ.

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Azhgirey L.S. et al. **E1 - 12296**

Nuclear Scattering of Deuterons at 4.3, 6.3 and 8.9 GeV/c

The momentum spectra of secondary deuterons emitted at an angle of 103 mr in the interactions of 4.3, 6.3 and 8.9 GeV/c deuterons with C , Al and Bi nuclei have been measured. The analysis of the results in the framework of the multiple nucleon-nucleon scattering model is given. The observed momentum spectra of deuterons from d–C collisions have been qualitatively described on the assumption that in relativistic deuteron collisions the processes are of importance in which nuclear nucleons involved in interactions with incident deuterons should be emitted in the bound state.

The investigation has been performed at the Laboratory of Computing Techniques and Automation, JINR.

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1. INTRODUCTION

Recent d−d scattering experiments at 4.3, 6.3 and 8.9 GeV/c have revealed the existence of the double-peak structure in the high-momentum parts of the secondary deuteron spectra measured at an angle of 103 mr /1/. These data have been explained on the basis of the multiple nucleon-nucleon scattering model. The analysis performed in the framework of this model has shown that in d−d scattering at momentum transfers in the range of $|t| \sim 0.3\text{–}1.0\,(\mathrm{GeV/c})^2$, along with the quasielastic d−N scattering, such collisions are important when both the nucleons of the incident deuteron interact simultaneously with the nucleons of the target nucleus. As a result, the differential cross section of the elastic d−d scattering at these momentum transfers is comparable with the contribution of the quasielastic scattering (involving the target deuteron break-up), unlike p−d collisions where the relative contribution of the elastic p−d scattering is small under the same conditions /2,3/. Apparently, this peculiarity of deuteron interactions must be revealed in the nuclear scattering as well. These considerations have stimulated the study of deuteron-nucleus scattering.

Below we present the results of measurements and the analysis of the high-momentum parts of secondary deuteron spectra from deuteron-carbon scattering at 4.3, 6.3 and 8.9 GeV/c, as well as those from deuteron-aluminium and deuteron-bismuth scattering at 6.3 GeV/c. Some data obtained in the measurements with ^{6}Li and ^{7}Li nuclei at 6.3 GeV/c are given. Section 2 presents the experimental results. In Section 3 the approximations are treated which are used for deriving the expressions describing the quasi-elastic deuteron-nucleus scattering in the framework of the multiple scattering model, and the calculation results are compared with the experiment. The expressions used in the calculations are given in the Appendix. The derivation of these expressions will be presented elsewhere.

2. THE EXPERIMENTAL DATA

The experiments were carried out with the deuteron beam extracted from the JINR synchrophasotron. The secondary deuterons emitted in the reactions

$$d + A \to d + X \qquad (1)$$

were detected at an angle of 103 mr by means of a single-arm magnetic spectrometer with wire spark chambers on-line with the computer. The spectrometer and the experimental technique have been outlined in refs.[4,5].

The measured momentum spectra of secondary deuterons are shown in figs. 1 and 2. Fig. 1 shows the high momentum parts of the deuteron spectra from deuteron-carbon scattering at an angle of 103 mr and the initial deuteron momenta of 4.3, 6.3 and 8.9 GeV/c. The same figure shows the elastic d-p scattering peaks. They describe the spectrometer resolution. The arrows show the momentum values calculated from the elastic d-p , d-d and d-C kinematics.

It is seen that the shape of the d-C spectrum changes noticeably when passing from 4.3 to 8.9 GeV/c. The sharp right-hand slope of 4.3 GeV/c spectrum allows one to conclude that there is a certain contribution of deuterons from the diffraction scattering on carbon nuclei. With 6.3 and 8.9 GeV/c there is no visible contribution from the diffraction scattering. Nevertheless, the maxima of the spectra measured

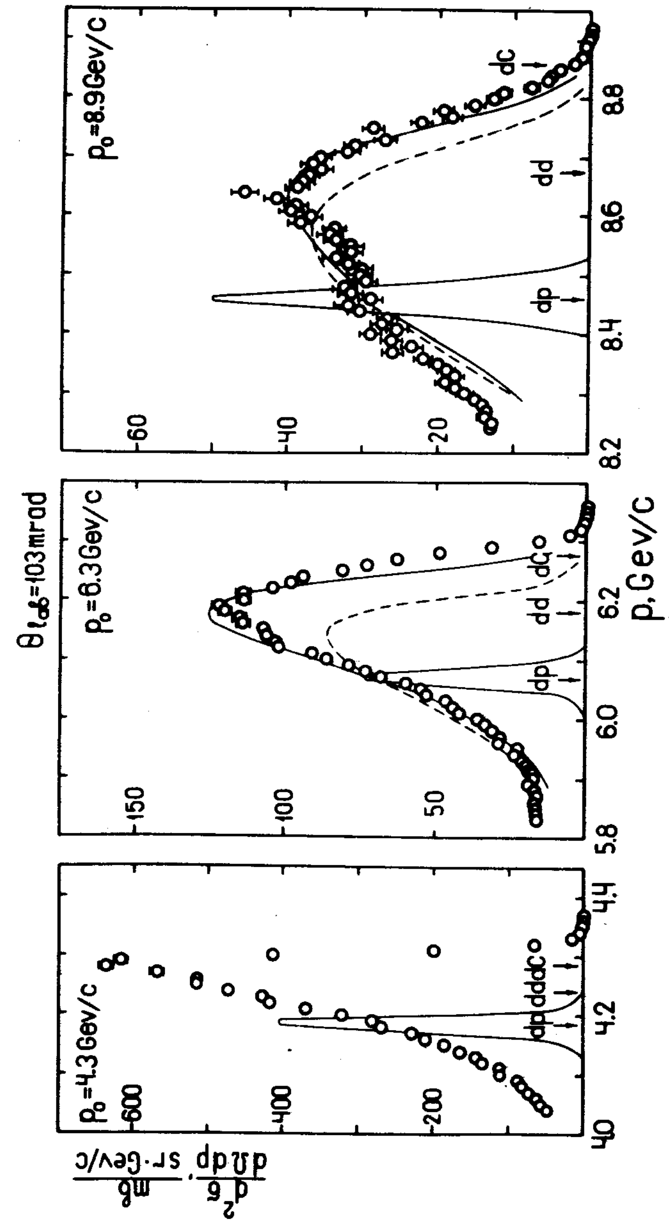

Fig. 1. High momentum spectra of deuterons emitted in the collisions of 4.3, 6.3 and 8.9 GeV/c deuterons with carbon nuclei at a lab. angle of 103 mr. The arrows show deuteron momentum values calculated from the elastic d – p , d–d and d–C kinematics. The elastic d – p scattering peaks characterize the spectrometer resolution. Dashed and solid curves are the results of calculations.

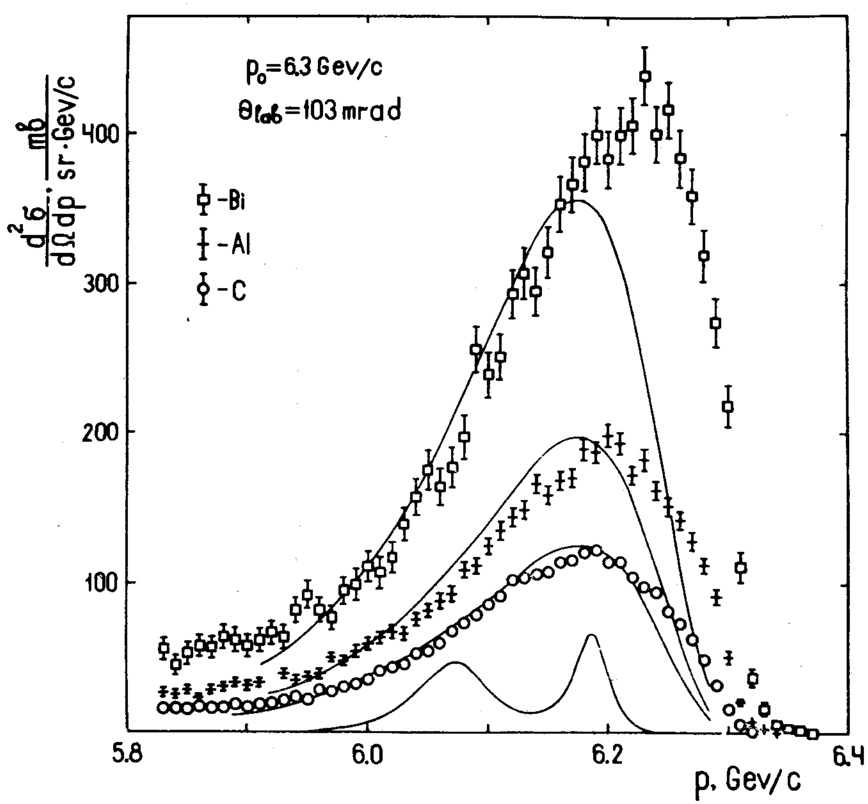

Fig. 2. High momentum spectra of deuterons emitted in the collisions of 6.3 GeV/c deuterons with C , Al and Bi nuclei at a lab. angle of 103 mr. Below the experimental points is shown the double-peak structure in the deuteron spectrum from d-d collisions (the data of ref. /1/). Solid curves are the results of calculations.

at 6.3 and 8.9 GeV/c are shifted to the right of the elastic d-p scattering peaks. Qualitatively, it can be regarded as an indication to the existence of deuteron scattering on more than one nucleon in the target nucleus.

Figure 2 shows the momentum spectra of 6.3 GeV/c deuterons scattered on C , Al and Bi nuclei. The same figure shows, below the experimental points, the double-peak structure in the high-momentum part of the secondary deuteron spectrum from d-d collisions. The data have been taken from ref./1/. As can be seen, with increasing the atomic number of the target nucleus the centres of gravity of the deuteron momentum distributions are shifted towards larger momentum values. The values of the centres of gravity of the deuteron distributions calculated in the momentum interval from 5.92 to 6.40 GeV/c for ^1H, ^2H, ^6Li, ^7Li, C , Al and Bi nuclei are 6.073, 6.112, 6.133, 6.136, 6.140, 6.148 and 6.162 GeV/c, respectively. Both the increasing multiplicity of N-N collisions with increasing A of the target nucleus and the existence of deuteron scattering on nucleon clusters inside the nucleus can lead to this effect.

The differential cross sections of deuteron emission in the momentum intervals corresponding to the quasi-elastic scattering are given in the fifth column of Table 1. The third column of the same table presents the values of the differential cross sections in the momentum intervals corresponding to the "coherent" pion production by deuterons on nuclei, occurring without the incident deuteron break-up. The error quoted after each cross section is the point to point error for measurements at the initial deuteron momentum p_0 given, since it is composed of the contributions of the statistical error, short-time monitor fluctuations and the variation of the corrections for the absorption of secondary deuterons along the spectrometer. When comparing the differential cross sections for different p_0 as well as the results of other investigations, the possibility of the systematic error of ±20% due to the uncertainty in the absolute calibration of the primary beam intensity must be taken into account.

Table 1

Differential cross sections $d\sigma/d\Omega$ (in mb/sr) of deuteron emission at an angle of 103 mr in the reactions $d+A \to d+X$ for different momentum intervals Δp (in GeV/c) of secondary deuterons at 4.3, 6.3 and 8.9 GeV/c

A	Δp	$d\sigma/d\Omega$	Δp	$d\sigma/d\Omega$
		p_0 = 4.3 GeV/c		
^1H		3.8 ± 0.4	4.12 - 4.26	13.6 ± 0.6
^2H	3.44 - 4.05	5.5 ± 0.5	4.05 - 4.30	20.8 ± 1.0
C		21.4 ± 1.2	4.05 - 4.36	77.2 ± 3.5
		p_0 = 6.3 GeV/c		
^1H		2.4 ± 0.3	6.01 - 6.14	3.0 ± 0.3
^2H		3.8 ± 0.4	5.93 - 6.23	6.8 ± 0.4
^6Li		6.3 ± 0.5		14.3 ± 0.7
^7Li	5.02 - 5.90	7.4 ± 0.5		17.0 ± 0.8
C		12.4 ± 0.6	5.90 - 6.33	26.9 ± 1.4
Al		20.5 ± 1.2		44.1 ± 2.2
Bi		41.8 ± 3.3		97.1 ± 4.9
		p_0 = 8.9 GeV/c		
^1H		2.9 ± 0.3	8.38 - 8.55	2.8 ± 0.2
^2H	6.62 - 8.21	5.5 ± 0.4	8.30 - 8.73	5.6 ± 0.3
C		16.6 ± 0.9	8.21 - 8.91	16.5 ± 0.8

3. ANALYSIS OF EXPERIMENTAL DATA

One could hardly doubt about the applicability of the model of multiple N-N scattering [6,7] for analysing relativistic deuteron scattering on nuclei if at the moment of collisions with the nucleus a proton and neutron in the incident deuteron were always remoted from each other by a distance longer than the average one (> 3 fm) and interacted with nuclear nucleons nearly as free particles. However, the character of the space-time development of deuteron interaction with nuclei must be different in the case when at the moment of collision a proton and a neutron in the deuteron are at short distances (≲ 0.6 fm) and interact with nuclear nucleons as a whole.

At present there are no quantitative descriptions of the space-time development of such deuteron-nucleus interactions. Therefore, we have performed the analysis of high momentum parts of deuteron distributions in reactions (1) in the framework of the traditional model of multiple N-N scattering developed for the description of the momentum spectra of scattered protons [8-10]. All position correlations of target nucleons both due to the Pauli principle and nucleon clustering were neglected. Besides, we have limited ourselves to considering only such incoherent deuteron-nucleus interactions which result in knocking out not more than three nucleons from the nucleus. Since we analysed only the upper parts of momentum spectra corresponding to small momentum losses of incident deuterons, the production of new particles, pions, first of all, was not taken into account.

Consider, first, the angular distribution $d\sigma/d\Omega$ of the deuterons scattered quasi-elastically. For the uncorrelated nucleon distribution in the nucleus $d\sigma/d\Omega$ can be expressed in terms of the density distributions of colliding nuclei and the N-N scattering amplitude, and can be written as a sum of the differential cross sections corresponding to n nucleons knocked out from the nucleus:

$$\frac{d\sigma}{d\Omega} = \sum_n \frac{d\sigma^{(n)}}{d\Omega}, \qquad (2)$$

where

$$\frac{d\sigma^{(n)}}{d\Omega} = \frac{1}{n!} \int T(\vec{s}_1) \dots T(\vec{s}_n) |\mathcal{F}^{(n)}|^2 d\vec{s}_1 \dots d\vec{s}_n .$$

The amplitude of the incoherent interaction of the incident deuteron with target nucleons $\mathcal{F}^{(n)}$ is given by

$$\mathcal{F}^{(n)}(\vec{q}, \vec{s}_1, \dots, \vec{s}_n) = \frac{ip_0}{2\pi} \int \exp(i\vec{q}\vec{b}) \prod_{i=1}^{n} \Gamma(\vec{b}-\vec{s}_i, \vec{s}_D) \times$$

$$\times \exp[-\int \Gamma(\vec{b}-\vec{s}', \vec{s}_D) T(\vec{s}') d\vec{s}'] \, r(\vec{s}_D) \, d^2b \, d^2s_D ,$$

$$\Gamma(\vec{b}-\vec{s}, \vec{s}_D) = \gamma(\vec{b}-\vec{s}-\frac{\vec{s}_D}{2}) + \gamma(\vec{b}-\vec{s}+\frac{\vec{s}_D}{2}) - \gamma(\vec{b}-\vec{s}-\frac{\vec{s}_D}{2}) \times \qquad (3)$$

$$\times \gamma(\vec{b}-\vec{s}+\frac{\vec{s}_D}{2}),$$

$$\gamma(\vec{b}) = \frac{2}{i\pi p_0} \int f_{NN}(q) \exp(-i\vec{q}\vec{b}) d^2q ,$$

$$T(\vec{s}) = \int \rho(\sqrt{s^2 + z^2}) dz ,$$

$$r(\vec{s}_D) = \int |\psi_D(\sqrt{s_D^2 + z^2})|^2 dz ,$$

where p_0 is the momentum of the incident deuteron, $\psi_D(r)$ is the deuteron wave function, $\rho(r)$ is the nuclear density distribution, and $f_{NN}(q)$ is the nucleon-nucleon scattering amplitude at $(1/2)p_0$.

Each of the amplitudes $\mathcal{F}^{(n)}$ can be represented as a sum of the terms corresponding to the certain combination of collisions between deuteron nucleons and the target ones. The number of terms in $\mathcal{F}^{(n)}$ is increased rapidly with increasing n. For example, the amplitude $\mathcal{F}^{(3)}$ has 14 different terms describing various types of multiple nucleon-nucleon collisions, the multiplicity being varied from 3 to 6. In the general case, all the terms in $\mathcal{F}^{(n)}$ have the same orders of magnitude and must be taken into account in the calculations. That is why we have confined ourselves to $n_{max} = 3$.

If the momentum transfer q is sufficiently large, so that $q \gg 1/R_D$, one can neglect the terms describing the collisions of only one of the deuteron nucleons with the target nucleons which are knocked out. In the Born approximation the contributions of such terms are proportional to $\exp(-q^2R_D^2/16)$, and for θ=103 mr, p_0=6.3 and 8.9 GeV/c they are small in comparison with the other terms of the amplitude. Moreover, for $n > 1$ the terms have been omitted which correspond to the collisions with the maximum $2n$ multiplicity since, according to our estimations, their contribution does not exceed 5%. Finally, 1, 3 and 12 different terms have been taken into account in the amplitudes $\mathcal{F}^{(1)}$, $\mathcal{F}^{(2)}$ and $\mathcal{F}^{(3)}$, respectively. Thus, the differential cross section $d\sigma^{(n)}/d\Omega$, in turn, has the form of a sum with the terms representing the contributions of various types of the multiple collisions and their interferences, and it can be written as

$$\frac{d\sigma^{(n)}}{d\Omega} = \sum_{\ell=1}^{\ell_{max}} \frac{d\sigma_\ell^{(n)}}{d\Omega}, \qquad (4)$$

where ℓ_{max} = 1,4 and 20 for n = 1,2 and 3, respectively.

In deriving explicit expressions for $d\sigma_\ell^{(n)}/d\Omega$ we have used the usual approximation that the N-N interaction range determined by the slope parameter b for differential cross section of elastic N-N scattering is small compared with the nuclear radius, and the corrections proportional to b/R_A^2 have not been taken into account. Only the most essential correction terms of those proportional to b/R_D^2 have been considered. They appear as $\Delta_{\ell R}^{(n)}$ in the formulas for $d\sigma_\ell^{(n)}/d\Omega$ given in the Appendix. Similarly, only the main corrections of the coherent eclipse ones resulting from the expansion of the exponent in eq. (3) in powers of

$$\int \gamma(\vec{b}-\vec{s}'-\frac{\vec{s}_D}{2}) \gamma(\vec{b}-\vec{s}'+\frac{\vec{s}_D}{2}) T(\vec{s}') d\vec{s}'$$

have been taken into consideration. They are referred to as $\Delta_{\ell E}^{(n)}$ in the Appendix.

Certain additional information about the structure of the target nucleus is needed to derive the momentum distributions $d^2\sigma/d\Omega\, dp$ of deuterons scattered quasi-elastically. First of all, one must know the function $R(\vec r, \vec k_F)$ of the target nucleon distribution in the momentum $\vec k_F$ at the point with the coordinate $\vec r$. In the independent particle model the function $R(\vec r, \vec k_F)$ is expressed in terms of the density matrix $\hat\rho(\vec r_1, \vec r_2)$ for nucleons in the nucleus as

$$R(\vec r, \vec k_F) = \int \hat\rho\left(\vec r - \frac{\vec r'}{2}, \vec r + \frac{\vec r'}{2}\right) \exp(i\vec k_F \vec r')\, d\vec r'.$$

Similarly to ref. [10] the function $R(\vec r, \vec k_F)$ is assumed to be factorized approximately:

$$R(\vec r, \vec k_F) \approx \rho(\vec r)\rho_1(\vec k_F),$$

where $\rho(\vec r)$ is the density distribution of nucleons in the target nucleus, and $\rho_1(k_F)$ is the momentum distribution of the target nucleons. To carry out the major part of calculations analytically the function $\rho_1(\vec k_F)$ is assumed to be

$$\rho_1(\vec k_F) = (\alpha\pi)^{-3/2} \exp(-k_F^2/\alpha),$$

where

$$\alpha = \frac{2}{3} \langle k_F^2 \rangle .$$

Other necessary information is related to the wave functions of the knocked out nucleons in the final state and, in particular, to the possibility of their forming the composite systems which are fragments of the target nucleus. These problems have not been studied sufficiently, and additional simplifying assumptions must be made in carrying out practical calculations.

The general expressions for the momentum spectra of deuterons scattered quasi-elastically can be written as

$$\frac{d^2\sigma^{(n)}}{d\Omega\, dp} = \sum_\ell \frac{d\sigma_\ell^{(n)}}{d\Omega} G_\ell^{(n)}(p_0, p, q), \qquad (5)$$

where the spectral functions $G_\ell^{(n)}$ are normalized to

$$\int_0^{p_0} G_\ell^{(n)}(p_0, p, q)\, dp = 1. \qquad (6)$$

The explicit form of the function $G_\ell^{(n)}$ describing the shape of deuteron momentum distributions resulting from different combinations of multiple collisions between the deuteron nucleons and the target ones seems to depend considerably on the peculiarities of the interaction of outgoing nucleons.

The functions $G_\ell^{(n)}$ used at the first stage of calculations were derived under the assumption that the plane waves should be the wave functions of the outgoing nucleons. This implies the full neglect of nucleon interaction in the final state. The deuteron momentum spectra in the reaction $d + C \to d + X$ at 6.3 and 8.9 GeV/c, computed in this approximation, are shown by dashed curves in fig. 1. It is seen that there is a great discrepancy with the experimental data especially in the range of the small momentum losses.

Two possibilities were applied for improving the agreement of the calculations with the experiment at the second stage. The first one was to take into account the terms with $n \geq 4$ in the amplitude of the deuteron-nucleus scattering. However, the contribution of $d\sigma^{(4)}/d\Omega$ to the calculated differential cross section was estimated to be small in comparison with $\sum_{n=1}^{3} d\sigma^{(n)}/d\Omega$ at least for the reaction $d + C \to d + X$.

The second possibility to obtain harder calculated deuteron momentum distributions is connected with the assumption that a part of nucleons which have interacted with incident deuterons should be emitted in the bound states. In this case the form of $G_\ell^{(n)}$ must be changed, normalization condition (6) being conserved.

As previously mentioned, the strict consideration of these effects is rather complicated and in the present investigation we have used the phenomenological approach implying the replacement of $G_\ell^{(n)}$ by the functions

$$(1-x^{(n)})G_\ell^{(n)} + x^{(n)}\tilde{G}^{(n)},$$

where $\tilde{G}^{(n)}$ is a spectral function corresponding to the emission of n knocked out nucleons in the bound state, namely, deuterons ($n=2$) or ^3H and ^3He nuclei ($n=3$), etc. The coefficient $x^{(n)}$ means the probability that n outgoing nucleons should form the bound state.

In deriving the expressions for $G_\ell^{(n)}$, similarly to ref. /10/, the intranuclear motion of target nucleons in the longitudinal direction was neglected. Therefore, generally speaking, the functions $G_\ell^{(n)}$ have incorrect behaviour when $p \to p_0$. However, this has a small effect on the calculation results since the centres of gravity of the momentum distributions calculated are far away from p_0. The effect of the longitudinal motion of the intranuclear nucleons on the functions $\tilde{G}^{(n)}$ is more significant. This effect was taken into account in deriving the expressions for $\tilde{G}^{(n)}$ given in the Appendix.

The results of calculations made under the assumption that a fraction of the nucleons knocked out from the nucleus by deuterons should emerge in the bound state are shown in fig. 1 by solid curves. The calculations were carried out with $x^{(2)} = x^{(3)} = 0.4$ for 6.3 GeV/c and $x^{(2)} = x^{(3)} = 0.2$ for 8.9 GeV/c. It is seen that with comparatively small values of the parameters $x^{(n)}$ - which do not contradict the simple estimations of their values - a considerably better description for the right-hand slopes of the momentum spectra of deuterons in the reaction $d + C \to d + X$ can be obtained than at the first stage of the calculations with $x^{(2)} = x^{(3)} = 0$, when the possibility of the fragment formation was neglected. The calculation of the deuteron momentum spectrum at 4.3 GeV/c requires some approximations used in the present calculations to be reconsidered, and it will be presented elsewhere.

Figure 2 shows the calculated results for $d-C$, $d-Al$ and $d-Bi$ collisions at 6.3 GeV/c under the assumption that $x^{(2)} = x^{(3)} = 0.4$. It is seen that for $d-C$ collisions one can obtain some reasonable agreement with the experimental data, but no such agreement is obtained for $d-Al$ and $d-Bi$ collisions. A better agreement of calculations with experiment for medium and heavy nuclei can be obtained by taking into consideration, first, still higher order multiple interactions of the incident deuteron nucleons with target nucleons, second, the possibility of knocking out the fragments which are heavier than ^2H, ^3H and ^3He. These processes become more probable with increasing the atomic number of the target nucleus. One cannot reject the existence of other important effects not considered in the analysis.

The fact that the observed momentum distributions of scattered deuterons are better reproduced on the assumption that nuclear scattering of relativistic deuterons at the momentum transfers $|t| \sim 0.4\text{-}0.8\,(GeV/c)^2$ must be accompanied by knocking out the groups of bound nucleons may be a simple phenomenological description of the peculiarities of deuteron-nucleus interactions. The successive quantitative description of deuteron-nucleus interactions is rather a complicated problem. Further experimental and theoretical investigations are required to solve it.

On the whole the situation for deuteron-nucleus scattering is quite different from that for nucleon-nucleus scattering when within the framework of the model of multiple nucleon scattering without assuming the knock out of nucleon groups from nuclei it is possible to describe satisfactorily the shape of the high momentum part of the distribution of scattered nucleons (see, e.g., ref. /3/).

APPENDIX

The formulas used in the calculations of the momentum distributions of deuterons from quasi-elastic deuteron nucleus scattering

The elastic N-N scattering amplitude is parametrized in the form

$$f_{NN}(t) = (p'\sigma/4\pi)(i+\rho)\exp(\tfrac{1}{2}bt),$$

where p' is the momentum of the incident nucleon, $p' = \frac{1}{2}p_0$, t is the four-momentum transfer squared, $\rho = \mathrm{Re}\, f_{NN}(0)/\mathrm{Im}\, f_{NN}(0)$, σ is the total N-N cross section, and b is the slope parameter for the differential cross section of elastic N-N scattering. The values of the parameters σ, ρ and b are the same as in ref. /1/.

The deuteron wave function is taken in the following form:

$$\psi_D(r) = (\pi R_D^2)^{-3/4}\exp(-r^2/2R_D^2), \quad R_D = 2.28 \text{ fm}.$$

The momentum distribution of quasi-elastically scattered deuterons can be expressed as follows:

$$\frac{d^3\sigma}{d\Omega\, dp} = \sum_{n=1}^{3} \frac{d^2\sigma^{(n)}}{d\Omega\, dp},$$

where

$$\frac{d^2\sigma^{(1)}}{d\Omega\, dp} = 2m_N N_1 \left(\frac{d\sigma}{d\Omega}\right)_{dN} C^{(1)} G^{(1)},$$

$$\frac{d^2\sigma^{(2)}}{d\Omega\, dp} = \frac{1}{2\lambda} K N_2 (1+\rho^2)^2 \sum_{\ell=1}^{4} C_\ell^{(2)} G_\ell^{(2)},$$

$$\frac{d^2\sigma^{(3)}}{d\Omega\, dp} = \frac{3}{16} K N_3 (1+\rho^2)^3 \sum_{\ell=1}^{20} C_\ell^{(3)} G_\ell^{(3)},$$

$$K = (2m_N p/E)(p_0\sigma/4\pi)^2 \left(\sigma/4\pi\left(\tfrac{1}{2}R_D^2 + 2b\right)\right)^2 \exp(-bq^2/2),$$

$\lambda = \sigma/4\pi b$, m_N is the nucleon mass, p_0 is the incident deuteron momentum, E_0 and E are the energies of deuterons with the momenta p_0 and p, respectively, $q^2 = p_0 p\,\theta^2$, q being the momentum transfer, $(d\sigma/d\Omega)_{dN}$ is the differential cross section of the elastic deuteron-nucleon scattering (we have used the values of the differential cross sections for the elastic d-p scattering measured in the present experiment).

1) For the single scattering we have:

$$C^{(1)} = 1 + \frac{\lambda}{4}\frac{N_2}{N_1}(1-\rho^2),$$

$$G^{(1)} = \frac{1}{\alpha}\exp[-(q^2+k^2)/\alpha]\, I_0(2qk/\alpha),$$

$$\alpha = \frac{2}{3}\langle k_F^2\rangle \approx 0.028\,(\mathrm{GeV/c})^2, \quad k^2 = 2m_N(E_0 - E),$$

where $k^2/2m_N$ is the sum of the kinetic energies of nucleons knocked out from the nucleus by the incident deuteron, I_0 is the modified Bessel function of the zeroth order.

2) For the deuteron-nucleus scattering in which two nucleons are knocked out we have:

$$C_1^{(2)} = \frac{\tilde{N}_2}{N_1}\left(\frac{R_D^2}{2b} + 2\right)(1+\Delta_{1E}^{(2)})(1+\Delta_{1R}^{(2)}),$$

$$C_2^{(2)} = \lambda^2(1+\rho^2)(1+\Delta_{2E}^{(2)})(1+\Delta_{2R}^{(2)})\exp(bq^2/8),$$

$$C_3^{(2)} = \lambda^2(1+\rho^2)(1+\Delta_{3E}^{(2)})(1+\Delta_{3R}^{(2)}),$$

$$C_4^{(2)} = -8\lambda(1+\Delta_{4E}^{(2)})(1+\Delta_{4R}^{(2)}),$$

$$\Delta^{(2)}_{1E} = \frac{8b}{R^2_D + 12b} \frac{N_2}{\tilde{N}_2} \kappa (1-\rho^2)\Delta^{(2)}, \Delta^{(2)} = \frac{\lambda}{4} \frac{3N_3}{N_2}, \ \kappa = 0.91,$$

$$\Delta^{(2)}_{2E} = \Delta^{(2)}_{3E} = \frac{8}{15}(1-\rho^2)\Delta^{(2)}, \ \Delta^{(2)}_{4E} = [\frac{1}{6}(1+\rho^2) + \frac{1}{4}(1-3\rho^2)]\Delta^{(2)},$$

$$\Delta^{(2)}_{4R} = -4b/(R^2_D + 4b),$$

other $\Delta^{(2)}_{\ell R}$ have been taken to be zero.

Spectral functions $G^{(2)}_\ell$ have the following form:

$$G^{(2)}_\ell = \frac{k^2}{\alpha \alpha_\ell} \exp(-\frac{q^2}{2\alpha} - \frac{\beta_\ell q^2}{\alpha_\ell}) \int_0^1 \exp\{-\frac{k^2 x}{\alpha} - \frac{k^2(1-x)}{\alpha_\ell}\} \times$$

$$\times I_0(\frac{2qk}{\alpha}\sqrt{\frac{x}{2}}) I_0(\frac{2qk}{\alpha_\ell}\sqrt{\beta_\ell(1-x)})\,dx,$$

where

$$\alpha_1 = \alpha + 4/(R^2_D + 4b), \quad \alpha_2 = \alpha_3 = \alpha + 1/b,$$

$$\alpha_4 = \alpha + 8/(R^2_D + 4b), \quad \beta_1 = \beta_3 = \beta_4 = 0, \ \beta_2 = 1/8.$$

If the neutron and the proton knocked out from the nucleus by the incident deuteron emerge as the deuteron, the functions $G^{(2)}_\ell$ are replaced by the functions

$$(1-x^{(2)})G^{(2)}_\ell + x^{(2)}\tilde{G}^{(2)},$$

where

$$\tilde{G}^{(2)} = \frac{1}{\sqrt{2\alpha q^2}}\{\exp[-\frac{(k-q_2)^2}{\alpha}] - \exp[-\frac{(k+q_2)^2}{\alpha}]\}, \ q_2 = \sqrt{\frac{q^2}{2}}.$$

3) For the deuteron-nucleus scattering in which three nucleons are knocked out we have

$$C^{(3)}_\ell = \tilde{C}^{(3)}_\ell(1 + \Delta^{(3)}_{\ell E})(1 + \Delta^{(3)}_{\ell R}),$$

$$\Delta^{(3)}_{1E} = \frac{N_3}{\tilde{N}_3} \frac{8b}{R^2_D + 10b}(1-\rho^2)\Delta^{(3)}, \ \Delta^{(3)} = \lambda\frac{N_4}{N_3}, \ \Delta^{(3)}_{2E} = \frac{1}{3}(1-\rho^2)\Delta^{(3)},$$

$$\Delta^{(3)}_{3E} = \frac{4}{7}(1-\rho^2)\Delta^{(3)}, \ \Delta^{(3)}_{4E} = \frac{5}{7}(1-\rho^2)\Delta^{(3)}, \Delta^{(3)}_{9E} = [\frac{2}{9}(1+\rho^2) +$$

$$+ \frac{1}{2}(1-3\rho^2)]\Delta^{(3)},$$

$$\Delta^{(3)}_{10E} = [\frac{4}{23}(1+\rho^2) + \frac{3}{11}(1-3\rho^2)]\Delta^{(3)},$$

other $\Delta^{(3)}_{\ell E}$ are taken to be zero,

$$\Delta^{(3)}_{2R} = -9b\delta_2, \ \Delta^{(3)}_{9R} = -8b\delta_2/3, \ \Delta^{(3)}_{10R} = -6b\delta_2, \ \delta_2 = 1/(R^2_D + 3b),$$

other $\Delta^{(3)}_{\ell R}$ are taken to be zero, and the values of $\tilde{C}^{(3)}_\ell$ are given in table 2.

The functions $G^{(3)}_\ell$ have the form:

$$G^{(3)}_\ell = \frac{k^4}{\alpha\omega_\ell\tau_\ell}\exp(-\frac{q^2}{3\alpha} - \frac{\beta_\ell q^2}{\omega_\ell} - \frac{\gamma_\ell q^2}{\tau_\ell}) \times$$

$$\times \int_0^1\int_0^1 x \exp(-\frac{k^2(1-x)}{\alpha} - \frac{k^2 xy}{\omega_\ell} - \frac{k^2 x(1-y)}{\tau_\ell}) \times$$

$$\times I_0(\frac{2qk}{\alpha}\sqrt{\frac{1-x}{3}}) I_0(\frac{2qk}{\omega_\ell}\sqrt{\beta_\ell xy}) I_0(\frac{2qk}{\tau_\ell}\sqrt{\gamma_\ell x(1-y)})\,dx\,dy,$$

where $\omega_\ell = \alpha + \xi_\ell$, $\tau_\ell = \alpha + \zeta_\ell$ and the values of β_ℓ, γ_ℓ, ξ_ℓ and ζ_ℓ are given in table 2 where the definitions are as follows:

$$\delta_1 = 1/b, \quad z = 1+\rho^2, \quad u = 1-\rho^2, \quad H = bq^2.$$

If three nucleons knocked out from the target nucleus form the bound system, such as ^3H or ^3He, the functions $G^{(3)}_\ell$ are replaced by the functions

$$(1-x^{(3)})G^{(3)}_\ell + x^{(3)}\tilde{G}^{(3)},$$

Table 2

Expressions for $\tilde{C}_\ell^{(3)}$, β_ℓ , γ_ℓ , ξ_ℓ and ζ_ℓ

ℓ	$\tilde{C}_\ell^{(3)}$	β_ℓ	γ_ℓ	ξ_ℓ	ζ_ℓ
1	$(\tilde{N}_3/4N_3 b\delta_2)\exp(H/8)$	0	1/24	δ_1	$3\delta_2$
2	8	0	1/6	$4\delta_2$	$3\delta_2$
3	$(1/4)\lambda^2 z\exp(H/4)$	0	1/24	$\delta_1/2$	$3\delta_1/2$
4	$(2/5)\lambda^2 z\exp(9H/40)$	0	1/600	δ_1	$3\delta_1/5$
5	$(1/3)\lambda^2 z\exp(H/6)$	0	1/6	δ_1	δ_1
6	$(2/3)\lambda^2 z\exp(H/24)$	0	1/24	δ_1	δ_1
7	$(1/24)\lambda^4 z^2\exp(7H/24)$	0	1/24	$2\delta_1$	δ_1
8	$(8/105)\lambda^4 z^2\exp(4H/15)$	0	2/75	$8\delta_1/7$	$3\delta_1/5$
9	$-(4/3)\lambda\exp(3H/16)$	0	1/24	$2\delta_1/3$	$6\delta_2$
10	$-4\lambda\exp(3H/32)$	0	1/24	δ_1	$6\delta_2$
11	$-2\lambda\exp(H/8)$	0	1/24	δ_1	$6\delta_2$
12	$-4\lambda\exp(H/32)$	1/32	1/24	δ_1	$6\delta_2$
13	$(16/15)\lambda^2 u\exp(H/5)$	1/200	1/24	$16\delta_1/15$	$6\delta_2$
14	$(2/3)\lambda^2 u\exp(H/8)$	0	1/24	$4\delta_1/3$	$6\delta_2$
15	$(8/15)\lambda^2 z\exp(H/5)$	0	8/75	$2\delta_1/3$	$6\delta_1/5$
16	$(16/15)\lambda^2 z\exp(2H/15)$	1/72	1/600	$\delta_1/2$	$6\delta_1/5$
17	$-(4/11)\lambda^3 z\exp(H/4)$	1/8	0	δ_1	δ_1
18	$-(4/25)\lambda^3 z\exp(H/4)$	0	1/600	$4\delta_1/5$	$6\delta_1/5$
19	$-(4/9)\lambda^3 z\exp(5H/24)$	1/18	1/24	$4\delta_1/3$	δ_1
20	$-(2/9)\lambda^3 z\exp(H/6)$	0	1/96	$4\delta_1/3$	δ_1

where

$$\tilde{G}^{(3)} = \sqrt{\frac{3}{4\alpha q^2}}\{\exp[-\frac{(k-q_3)^2}{\alpha}] - \exp[-\frac{(k+q_3)^2}{\alpha}]\}, \quad q_3 = \sqrt{\frac{q^2}{3}}.$$

In the above-mentioned formulas the effective numbers N_n , $\tilde{N}_2$ and $\tilde{N}_3$ are used which are expressed as follows:

$$N_n = \frac{\pi}{\sigma n!}\int_0^\infty \exp\{-2\sigma T(s)\}[2\sigma T(s)]^n s\,ds,$$

$$\tilde{N}_2 = \frac{\pi}{2\sigma}\int_0^\infty |\mathcal{F}_1(s)|^2 s\,ds, \quad \tilde{N}_3 = \frac{\pi}{6\sigma}\int_0^\infty \mathcal{F}_1(s)\mathcal{F}_2(s)s\,ds,$$

where

$$\mathcal{F}_k(s) = \frac{4}{R_b^2}\int_0^\infty \exp\{-\frac{2(s^2+s'^2)}{R_b^2}\}I_0(\frac{4ss'}{R_b^2})\exp\{-\sigma T(s')\}\times$$
$$\times[2\sigma T(s')]^k s'ds', \qquad R_b^2 = \frac{1}{2}R_D^2 + 2b,$$

and the thickness function of the nucleus $T(s)$ is defined by

$$T(s) = \int_{-\infty}^{\infty}\rho(\sqrt{s^2+z^2})dz.$$

The nuclear density distribution $\rho(r)$ is taken in the Woods-Saxon form:

$$\rho(r) = \frac{A\rho_0}{1+\exp[(r-c)/d]}$$

with the parameters c and d given in table 3.

Table 3

Values of the parameters c and d used in the calculations

Nucleus	C	Al	Bi
c, fm	2.214	3.08	6.41
d, fm	0.488	0.563	0.637

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