ON THE PROTON MULTIPLICITY DISTRIBUTION OF HIGH ENERGY PION-NUCLEI COLLISIONS
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1. INTRODUCTION

During the past few years a great deal of our attention has been given to the experimental study in detail of the charged pion-xenon nuclei collisions at 2-10 GeV/c momentum. Among various characteristics of such reactions the multiplicity distributions of protons emitted have been obtained; these protons of kinetic energies 20-400 MeV we call fast protons later on. The studies of such proton multiplicity distributions are of wide interest up to the present time. The most complete and particularly interesting experimental information on the fast proton multiplicities has been obtained in a study of the pion-xenon nuclei collisions at 3.5 GeV/c momentum. a) the average proton multiplicity $\bar{n}_p$ in the cases with one and more secondary pions does not depend on the number of all the pions - neutral, positive, and negative - generated in the collision process, being $\bar{n}_p \approx 4$; b) some part, about 1.2%, of pion-xenon nuclei collision events exists in which the proton emission is not accompanied by any secondary pion - in such events the incident pion is absorbed by the target nucleus and the fast protons are emitted with the average multiplicity $\bar{n}_p \approx 8$, evidently larger than in the other events; c) some part, nearly 10%, of pion-xenon nuclei collisions exists in which only one secondary pion accompanied by any number of fast protons is emitted.
Fig. 1. Fast proton multiplicity distribution in pion-xenon nuclei collisions not accompanied by the multiparticle production - with zero and one secondary pion and any number of protons emitted. The experimental distribution is presented by the open circles /10/, the calculated distribution is presented by the solid black circles and by the dotted line.

Both kinds of events - without secondary pions and with single one secondary pion - were studied experimentally in more detail in our recent work /10/. The main purposes of those studies on the proton emission were to discover some empirical peculiarities in the experimental data, in hope that these peculiarities will be useful later on for a fundamental understanding of the fast proton emission process, and, consequently, for the further more deep understanding the particle production process in hadron-nuclei collisions. We believe, the expected peculiarities can appear in such simple cases of pion-nuclei collisions. As a result, among other features of the fast proton characteristics, the inmonotony in proton multiplicity distribution has been discovered, fig. 1, and reported in our previous work /10/.

In this paper I present a simple explanation and description of the fast proton multiplicity distribution in both types of events without multiparticle production - with zero and one secondary pion and any number of protons emitted, and I give the quantitative interpretation of the peak observed in the proton multiplicity distribution of the events without any secondary pions /10/. I show that the proton multiplicity distribution calculated here is in good agreement with the experimental one in all the region of multiplicity values $n_p$ in which the comparison can be performed, fig. 1 and 2.

Fig. 2. Differences between the calculated and the experimental points in the proton multiplicity distribution of the pion-xenon nuclei collisions not accompanied by the multiparticle production.
2. DERIVATION OF THE PROTON MULTIPLICITY DISTRIBUTION

We seek a simple model which will enable us to derive the proton multiplicity characteristic in a way making it intuitively obvious. Thus, we start with a simple working hypothesis: the number \( n_p \) of the fast protons emitted by the passage of high-energy pion through an atomic nucleus of the radius \( R \), at the distance \( d \) from its center, is equal to the number of protons met in the neighbourhood of close to the pion path \( r \) inside the nucleus:

\[
n_p = \pi D_0 \frac{Z}{A} \rho \cdot r - 2\pi D_0 \frac{Z}{A} \rho \sqrt{(R^2 - d^2)}.
\]

(1)

where \( D_0 \), the diameter of a nucleon; \( Z \), atomic number; \( \rho \), nucleon density along the path \( f \). We suppose that the ratio of the number of protons \( Z \) to the number of neutrons \( A-Z \) inside nucleus is constant, being

\[
\frac{Z}{A-Z} \sim 1.2, 1.3
\]

For convenience, the length \( f \) of the pion path inside nucleus we express further in diameters \( D_0 \) of nucleon, accepted as unity and defined by the formula

\[
\frac{R}{D_0} = \left( \frac{3A}{4\pi} \right)^{1/3}
\]

(2)

For the xenon nucleus \( A=131 \), \( Z=54 \), and \( R=3.15D_0=3.15 \). In accordance with the accepted working hypothesis the proton emission process goes monotonously along the total path \( f \), and the number \( n_p \) of the protons emitted is the function of the impact parameter \( d \) - for any definite value of \( n_p \) one definite value of the parameter \( d(n_p) \) exists for spherical atomic nucleus of radius \( R \) and definite nucleon density distribution \( \rho \).

We then write for the intensity \( W(n_p) \) of \( n_p \) proton emission:

\[
W(n_p) = \pi \frac{1}{4\pi} d(n_p) = \frac{1}{4\pi} f \cdot d(n_p)
\]

\[
\frac{1}{4\pi} \left[ d(n_p) + D_0 \right] d(n_p) - \frac{1}{4\pi} D_0 d(n_p) =
\]

\[
= 4\pi D_0 \cdot f \cdot d(n_p)
\]

(3)

This expression, being of simple geometrical meaning, gives the series of values \( W(1), W(2), W(3), \ldots \ldots \), which, after appropriate normalization, can be compared with the series of observed numbers \( N(n_p) \) of the collision events accompanied by the number \( n_p \) of emitted protons: \( N(1), N(2), N(3), \ldots \ldots \).

High energy pions traversing nuclear matter cause the monotonous emission of the fast protons along their total path inside nucleus only in some part \( k \) \% of all events. In the rest part \( (1-k) \) of the cases this monotonous emission takes place on the average path length \( \lambda \) only, corresponding to the average, almost constant for all collision events, number of protons emitted, being \( \bar{n}_p = 4 \) for pion-xenon nuclei collisions at 3.5 GeV/c. This path length \( \lambda \) fluctuates having, in fact, some unknown at yet distribution which can be although determined experimentally in principle. Such picture can be drawn in the light of the existing experimental data, mainly, taking into account the existence of the inmonotony of the proton multiplicity distribution, fig. 1, in the events sample without multiparticle production. This inmonotony can be caused by the appearance of some energetic secondary particle which, in one's turn, is able to cause emission of fast protons in passing in a nucleus. Such particles can be emitted in any directions and within wide energy interval. However, if only collision events with zero and one secondary pions are taken into account, according to the scanning conditions, the cases with energetic secondary particles being not stopped protons are automatically excluded.
At the first stage of approximation we accept too that the emission directions of such secondaries are strictly along the direction of the incident pion, and these particles cause only the monotonous emission of protons without tertiary fast particle production. In other words, we take into account only the onefold and onedimensional emission of the fast secondary pion inside the nucleus.

Thus, in some part of collision events in which the inmonotonous emission of protons takes place a shift of events with some number \( n_P \) of protons, corresponding to the monotonous emission, \( P \), to those class of events with larger \( n_P \) plays an important role, because of the additional proton emission process followed by the fast secondaries passage through the rest part of the nucleus. This shift process goes according to the scheme which we consider later on. The intensity \( W_1(n_P + i) \) of such events shifted is expressed by the formula:

\[
W_1(n_P + i) = (1 - k_1)W_1(n_P)k_1(n_P \to n_P + i),
\]

where \( k_1(n_P \to n_P + i) \) is the coefficient accounting the percentage of the events shifted from the sample with proton multiplicity \( n_P \) to the other one sample with new proton multiplicity \( n_P + i \), \( i \) being the natural numbers. The proton multiplicity distribution \( W'(n_P) \) of the total sample of collisions with zero and one secondary pion together is:

\[
W'(n_P) = k_1W_1(n_P) + (1 - k_1)W_1(n_P)k_1(n_P \to n_P + i).
\]

Using the expressions (3') and (5), and taking into account that \( \int|d(n_P)| = 2\sqrt{R^2 - d^2(n_P)} \) the values of \( W'(n_P) \) have been evaluated for the sample of experimental events of negative pion-xenon collisions at 3.5 GeV/c.

3. **Calculation of \( W'(n_P) \) for pion-xenon nuclei collisions**

We shall introduce the following approximations. We use the simplest model of spherical nucleus of nucleon density \( \rho \) distributed as shown in fig. 3 in accordance with the nuclear models being commonly in use /12,13/.

In order to facilitate the calculation, we divide the total nuclear sphere into three parts: the core part of the radius \( R_1 \) and the constant nucleon density \( \rho_1 \), and two shell-shape parts situated between the radii \( R_2 = R_1 \), and \( R_3 = R_2 \) of corresponding nucleon densities \( \rho_2 \) and \( \rho_3 \). The values \( R_1 = 2D_0, R_2 = 4D_0, R_3 = 9.5D_0 \) and \( \rho_1 = 1, \rho_2 = 0.28 \), and \( \rho_3 = 0.05 \) have been used. The value of \( k_1 = 0.56 \) is derived from experiment being the ratio of the number of collision events without secondary pions, corresponding to the proton multiplicity \( n_P = 8 \), to the number of such events with \( n_P \geq 8 \). The value \( n_P = 8 \), seen as a peak in the proton multiplicity distribution /10/, corresponds to the monotonous emission of protons in through passing the nucleus along its diameter by a high-energy pion. The mean value \( \bar{n} \) has been used, as corresponding to the experimentally estimated mean value \( \bar{n} = 4 \), instead of its unknown at yet distribution.

Under such simplification the events which could appear belonging to ones with \( n_P = 5 \), if monotonous emission of protons takes place only, are shifted to the class of events with \( n_P = 6 \), with the probability \( W(5) \approx 0.44 \).

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**Fig. 3.** Nucleon density distribution in the xenon nucleus used in calculation of the proton multiplicity distribution.
Similarly, the events with \( n_P = 6 \) are shifted to those with \( n_P = 8 \), with the probability \( W(6) = 0.44 \), those with \( n_P = 7 \) to those with \( n_P = 10 \), with the probability \( W(7) = 0.44 \); and, at least, those with \( n_P = 8 \) go to those with \( n_P = 12 \), with the probability \( W(8) = 0.44 \).

Thus on the picture we have developed the formula (1) expressing the dependence of the impact parameter \( d \) on the proton multiplicity \( n_P \) is presented in the following form being more convenient for our approximate calculation:

\[
\begin{align*}
    n_P &= 2\pi D_0^2 \frac{Z}{A} \left[ \rho_1 \sqrt{R_1^2 - d^2(n_P)} \right] + \\
    &+ \rho_2 \left( \sqrt{R_2^2 - d^2(n_P)} - \sqrt{R_1^2 - d^2(n_P)} \right) + \\
    &+ \rho_3 \left( \sqrt{R_3^2 - d^2(n_P)} - \sqrt{R_2^2 - d^2(n_P)} \right) \tag{1'}
\end{align*}
\]

This calculation procedure is essential as a guide for the future more accurate one.

Using this formula the set of values \( W(n_P) \) has been calculated in assumption that only the monotonous emission of protons takes place. Results are given in the column \( W \) of the table. These data are the basis for evaluation of the proton multiplicity distribution \( W'(n_P) \) in the more general case of collisions in which the monotonous emission of protons exists. The final results are shown in fig. 1 - the dotted line and the black solid circles.

4. COMPARISON OF THE CALCULATED PROTON MULTIPLICITY DISTRIBUTION WITH THE EXPERIMENTAL ONE

Both the calculated distribution \( W'(n_P) \) and the experimental one \( N(n_P) \) are confronted for comparison, fig. 1. Normalization has been performed to the total number of experimentally estimated events at \( n_P = 2 \) and \( n_P = 3 \) together; at this region of the proton multiplicity values the shifting effect is mostly negligible. Two consequences from this comparison are evidently seen: a) Both distributions, that from experiment and the evaluated in calculation one, agree well within the proton multiplicity diapason \( n_P = 1-5 \); b) Both have very similar shapes. The percentage difference \( \Lambda \) between the experimental and the calculated values of \( n_P \), within the proton multiplicity region \( n_P = 1-4 \), characterized by the expression

\[
\Lambda = \frac{W'(n_P) - N(n_P)}{N(n_P)} \times 100 \%
\]

is shown in fig. 2.

Disagreement of \( W'(n_P) \) and \( N(n_P) \) at \( n_P > 6 \), fig. 1, is caused by the approximations applied; the monotonous emission of the fast protons has been considered as an unidimensional process, and the mean value \( \Lambda \) instead of its distribution has been used.

5. RESULTS

It has been shown above that the fast proton multiplicity distribution in the class of the pion-nuclei collisions without the multiparticle production can be described

<table>
<thead>
<tr>
<th>( n_P )</th>
<th>( W )</th>
<th>( W'(n_P) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.72</td>
<td>15.45</td>
</tr>
<tr>
<td>2</td>
<td>3.86</td>
<td>16.26</td>
</tr>
<tr>
<td>3</td>
<td>2.47</td>
<td>17.30</td>
</tr>
<tr>
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<td>2.32</td>
<td>17.95</td>
</tr>
<tr>
<td>5</td>
<td>1.68</td>
<td>17.70</td>
</tr>
<tr>
<td>6</td>
<td>0.85</td>
<td>17.91</td>
</tr>
<tr>
<td>7</td>
<td>0.27</td>
<td>17.99</td>
</tr>
<tr>
<td>8</td>
<td>0.10</td>
<td>18.00</td>
</tr>
</tbody>
</table>
using a simple picture of the monotonous emission along pion path in traversing of it through the nuclear matter. The number of the protons ejected along some path length equals to the number of the protons met.

These protons are comparatively energetic, being, in average, of nearly 80 MeV of their kinetic energy for any proton multiplicity /7,10/. Thus, it must be postulated that high-energy pion traversing the nuclear matter undergoes the monotonous energy loss, like the charged particle by its passage through materials. Obviously, this energy loss process differs in its nature from the electromagnetic one, being caused by the nuclear interactions. Such process should be postulated for other hadrons too in their traversing the atomic nuclei. It is remarkable that only these nucleons are involved for explanation of the proton emission process which are situated in the neighbourhood of close to the path inside an atomic nucleus.

As has been pointed out in some of our previous work, the fast protons are of nearly constant energy, $E_p \approx 80$ MeV, independent on the proton and on the pion multiplicities in pion-nuclei collision events /6,7/. Their transversal momenta and their angular distributions do not depend on the proton and pion multiplicities too /7/. Thus, it is very probable that the protons can be ejected from the nucleus as a result of absorption of the low energy pions knocked out monotonously, by the high-energy pion in its passing through an atomic nucleus, in some process which may be named the pionization process in nuclear matter, in analogy with the ionization process one.

REFERENCES


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