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О глубоконеупругом e-p рассеянии в области $\omega \simeq 1$

Детально исследуется глубоконеупругое e-p рассеяние в области $\omega \simeq 1$. Выполненный анализ в этой области показывает, что необходимо введение нарушения скейлинга. Рассмотрены параметризации q^2-зависимости структурных функций, основанные на асимптотически свободной теории поля, партонной модели с неточечными партонами и масштабно-инвариантной партонной модели. Показано, что имеющиеся в этой области данные могут быть удовлетворительно описаны во всех трех подходах. С целью проверки соотношения Дрелла-Яна проведен совместный анализ данных по глубоконеупругому и упругому e-p -рассеянию.

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On Deep Inelastic e-p Scattering in the Vicinity of $\omega \simeq 1$

In order to distinguish between the different theoretical approaches the deep inelastic e-p scattering data in the regions of $\omega \simeq 1$ have been analysed. The q^2-dependence of the structure functions arising in asymptotically free field theories, parton models with pointless partons and scale invariant parton model have been used. The analysis showed that the considered data cannot favour any of the discussed theoretical schemes. A combined analysis of the deep inelastic and elastic e-p scattering data has been carried out. The validity of the Drell-Yan relation has been checked.

The investigation has been performed at the Laboratory of Nuclear Problems, JINR.

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I. INTRODUCTION

In recent experiments on deep inelastic $e(\mu)-N$ scattering performed at Stanford[1] and Batavia[2] rather large deviations from scaling of the structure functions νW_2 and $2MW_1$ have been observed. It has been reported[3] that the q^2-dependence of the structure functions is different for the different regions of the scaling variable ω. For small $\omega (\omega<3)$, νW_2 decreases with q^2 and for large values of $\omega (\omega>9)$ νW_2 is a raising function of q^2. A similar behaviour of $2MW_1$ has been found too.

A detailed analysis of the q^2-dependence of the structure functions is necessary for comprehension of the physical nature of Bjorken scaling. As is well known, different theoretical approaches predict different forms of scale breaking.

In this paper we analyse the data in order to answer the question: can the available experimental data favour any of the existing theoretical approaches? In our anaylsis we considered only the threshold region $(\omega \simeq 1)$*. Restricting to this region only we have the following reasons:

*The analysis of the data for $\omega \simeq 1$ we had began in ref.[4]. In the present paper the recent data on the cross sections measured at 50 and 60 degrees are included. Also, instead of making any predictions about R, as in ref.[4], here we shall use parametrizations on the two independent structure functions νW_2 and $2MW_1$.

i) at $\omega \simeq 1$, we can use a rather simple form of ω-dependence for the structure functions:

$$\nu W_2 \sim \left(1-\frac{1}{\omega}\right)^p \; ; \quad 2MW_1 \sim \left(1-\frac{1}{\omega}\right)^q .$$

ii) the threshold behaviour of νW_2 is connected with the asymptotic behaviour of the nucleon form factor G_M at $q^2 \to \infty$ (the Drell-Yan relation)[5].

iii) in the vicinity of $\omega \simeq 1$ in many theoretical approaches explicit expressions for the q^2-dependence of νW_2 can be obtained.

The cross sections for deep inelastic e-p scattering measured [1] at SLAC at laboratory scattering angles of θ = 26, 34, 50 and 60 degrees ($10 \leq q^2 \leq 30$ $(\mathrm{GeV/c})^2$, $5.3 \leq \nu \leq 18.3$ (GeV)) have been used in our analysis. These are the data, which kinematically cover the region $\omega \simeq 1$ we are interested in. In our analysis we restricted ourselves to $\omega \lesssim 1{,}4$ *.

As soon as the threshold region of deep inelastic scattering is connected with the asymptotic behaviour of the nucleon form factors (i.e.) the elastic e-p cross sections measured at $6 \leq q^2 \leq$ ≤ 25 $(\mathrm{GeV/c})^2$ [7] have been included in our analysis, too. The validity of the Drell-Yan relations has been checked and information about the scale breaking parameters arising in the different theoretical models is obtained.

The method of analysis we have used is considered in detail in ref. [8]. According to it the free parameters in the structure functions and the form factors are determined through minimization of the functional form χ^2, into which directly measured in experiment cross sections enter.

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* Using the results of ref. [6], one can show that at values of $\omega \lesssim 1{,}4$ the main contribution to the expansion of νW_2 in powers of $(1-\frac{1}{\omega})$ comes from the first term.

II. DIFFERENT SCHEMES OF SCALE BREAKING

Different theoretical models predict different forms for the q^2-dependence of the structure functions. In this paper we consider deviations from scaling in the range of $\omega \simeq 1$ arising in the following theoretical approaches:

1) asymptotically free gauge field theories [9],
2) parton model with logarithmic form of scale-breaking [10],
3) parton model with pointless partons [11],
4) scale invariant parton model [12].

Correspondingly the following expressions for the structure functions νW_2 and $2MW_1$ at $\omega \simeq 1$ and for the proton form factor G_M at $q^2 \to \infty$ arise [10-13]:

1)
$$\nu W_2 = a(1-x)^p \left(\ln\frac{q^2}{\mu^2}\right)^{G(0.69+4\ln(1-x))} \qquad (1a)$$

$$2MW_1 = b(1-x)^q \left(\ln\frac{q^2}{\mu^2}\right)^{G(0.69+4\ln(1-x))}$$

$$G_M = c\left(\frac{1}{q^2}\right)^r \left(\ln\frac{q^2}{\mu^2}\right)^{\frac{G}{2}\left(0.69-4\ln\frac{q^2}{W_t^2}\right)} \qquad (1b)$$

2)
$$\nu W_2 = a(1-x)^p \left(\ln\frac{q^2}{\mathrm{GeV}^2}\right)^{-G_1} \qquad (2a)$$

$$2MW_1 = b(1-x)^q \left(\ln\frac{q^2}{\mathrm{GeV}^2}\right)^{-G_1}$$

$$G_M = c\left(\frac{1}{q^2}\right)^r \left(\ln\frac{q^2}{\mathrm{GeV}^2}\right)^{-\frac{G_1}{2}} \qquad (2b)$$

3)
$$\nu W_2 = a(1-x)^p \left(1+\frac{q^2}{M_G^2}\right)^{-2}$$

$$2MW_1 = b(1-x)^q \left(1+\frac{q^2}{M_G^2}\right)^{-2} \qquad (3a)$$

$$G_M = c\left(\frac{1}{q^2}\right)^r \left(1+\frac{q^2}{M_G^2}\right)^{-1} \qquad (3b)$$

4) $$\nu W_2 = a(1-x)^p \left(\frac{GeV^2}{q^2}\right)^A$$

$$2MW_1 = b(1-x)^q \left(\frac{GeV^2}{q^2}\right)^A \qquad (4a)$$

$$G_M = c\left(\frac{1}{q^2}\right)^{r+A} \qquad (4b)$$

Here

$$x = \frac{1}{\omega} = \frac{q^2}{2M\nu} .$$

As has been pointed out in refs.[10-13] in all the models considered above the generalized Drell-Yan relation holds:

$$r = \frac{p+1}{2} . \qquad (5)$$

The check of this relation is an important test of the constituent models of nucleon. We fit to the data when both p and r are considered as free parameters as well as when p and r are related through eq. (5).

III. Results of Analysis

In this section we shall present the results of our fit of the data. As has been indicated above we considered the kinematical range $\omega \le 1.4$ in inelastic scattering (50 experimental points) and $q^2 \ge 6\ (GeV/c)^2$ in elastic e-p scattering (10 experimental points). We assumed that the scaling relation $G_M = \mu_p G_E$ between the charged and magnetic

form factors holds. The experimental data at high q^2 values are consistent with it[14].

Fits to the data were made for the structure functions and the form factors given by eqs. (1), (2), (3) (a,b,c,G, etc., were free parameters). In searching for the right scaling variable, three approaches to the study of the deviations from scaling were used:

1) when ω is considered a scaling variable
2) when ω is replaced by ω' :

$$\omega' = \frac{2M\nu + M^2}{q^2} , \qquad (6)$$

M is the nucleon mass, and

3) when ω is replaced by ω'' :

$$\omega'' = \frac{2M\nu + a}{q^2} , \qquad (7)$$

where a is a free parameter.

1. The Scaling Variable ω.

In the first approach we used ω as a scaling variable. We tried to fit the data assuming exact scaling and using the following parametrizations for the structure functions in the threshold region:

$$\nu W_2 = a(1-x)^p$$
$$2MW_1 = b(1-x)^q . \qquad (8)$$

Our analysis showed that these functions provide very poor fit to the data ($\chi^2/\bar{\chi}^2 = 278/46$). Hence deviations from scaling are needed. The fit parameters for the structure functions and the form factor for each of the theoretical approaches (1), (2), (3), (4) are listed in Table 1. When the parametrizations based on asymptotically free field theories have

Table I

The results of analysis of the data [1,7] (variable ω)

	Asymptotically free gauge theories		The logarithmic scale breaking		The parton model with pointless partons		The scale invariant parton model	
1	2	3	4	5	6	7	8	9
a	0.27±0.05	1.77±1.14	3.38±0.41	12.17±8.04	1.78±0.22	3.48±2.22	1.26±0.16	11.81±7.75
p	0.33±0.19	1.55±0.43	2.01±0.10	2.84±0.43	2.51±0.09	2.95±0.42	1.47±0.13	2.87±0.43
b	0.92±0.10	0.62±0.09	20.40±3.55	15.87±3.35	3.94±0.49	3.47±0.60	20.98±3.67	14.01±2.86
q	0.97±0.16	0.69±0.17	3.02±0.07	2.85±0.10	2.89+0.07	2.80±0.10	3.15±0.07	2.83±0.10
c	0.07±0.01	0.07±0.01	0.34±0.03	0.33±0.03	0.31±0.03	0.31±0.03	0.44±0.04	0.43±0.03
r	—	0.65±0.10	—	1.49±0.05	—	1.75±0.04	—	1.18±0.06
b_1			2.39±0.15	2.40±0.15				
M_t	—	—	—	—	5.01±0.28	5.00±0.28		
A	—	—	—	—			0.82±0.05	0.85±0.05
G	0.45±0.03	0.44±0.03						
$\chi^2/\bar{\chi}^2$	73/54	72/53	73/54	70/53	60/54	59/53	75/54	65/53

been used, μ^2 has been fixed as follows μ^2 = 0.25, 1 [9], and 5 GeV^2. The value of the parameter G strongly depends on the choice of μ^2. For μ^2 = = 0.25 GeV^2 [15], G = 0.66±0.04, which is four times bigger than G=4/27 [16], which is expected in a model with three colour quartet, for μ^2 = 5 GeV^2, G=0.18± ± 0.01, which is close to the value expected by the theory. Parameter A which characterizes the anomalous dimensions equals A = 0.82±0.05. This value is two times bigger than the one obtained in refs. [4,17]. The data at $W \geq 1.8$ GeV^2, μ = 1 GeV^2 have been used (W is the invariant mass of the final hadrons). The parametrizations (1a, 2a, 3a, 4a) have been fitted to the data at $W \geq 2$ GeV and $W \geq 2.3$ GeV too. The values of the parameters obtained in these cases are consistent within the error with those given in Table 1. Note that even in the range $W > 2.3$ GeV the scaling parametrization (8) is a poor fit to the data. As is seen from Table 1, imposing relation (5) on the parameters the data are fitted well. If we consider p and r as free parameters, the data are also fitted well. Nevertheless, relation (5), i.e., the Drell-Yan relation, is fulfilled (within the error) for all cases, but (4).

2. The Scaling Variable ω'

We shall present here the results of fitting to the data when ω' is chosen as a scaling variable. An attempt to fit to the data using eq. (8) with ω replaced by ω' gave a poor fit.

Let us assume that the structure functions are given by eqs. (1a), (2a), (3a) and (4a), in which ω is replaced by ω'. The results of the analysis of the data on deep inelastic e-p scattering restricted to $\omega' < 1.4$; $W \geq 1.8$ GeV (44 points) and the elastic e-p scattering at $q^2 \geq 6$ $(GeV/c)^2$ are presented in Table II. As is seen from Table II, in each of the four forms of scale breaking using the variable ω' we can fit the data well both when p and r are

Table II

The results of analysis of the data[1,7] (variable ω')

	Asymptotically free gauge theories		The logarithmic scale breaking		The parton model with pointless partons		The scale invariant parton model	
1	2	3	4	5	6	7	8	9
a	0.40±0.08	2.09±1.67	40.26±23.71	6.00±4.74	1.06±0.12	4.20±3.27	0.99±0.11	6.77±5.35
p	1.55±0.24	2.73±0.60	5.06± 0.33	3.49±0.57	2.79±0.09	3.79±0.57	2.24±0.15	3.58±0.57
b	2.11±0.33	1.48±0.29	4.86± 0.85	8.01±2.03	4.12±0.59	3.17±0.59	10.79±2.39	7.44±1.80
q	2.57±0.22	2.29±0.24	3.33±0.10	3.51±0.14	3.61±0.10	3.41±0.13	3.79±0.10	3.47±0.13
c	0.15±0.03	0.14±0.03	0.41±0.03	0.37±0.03	0.36±0.03	0.35±0.03	0.42±0.03	0.41±0.03
r	—	1.24±0.12	—	1.75±0.05	—	1.89±0.04	—	1.58±0.07
σ	0.24±0.04	0.25±0.04	—	—	—	—	—	—
σ_1	—	—	1.02±0.16	1.22±0.18	—	—	—	—
M_G	—	—	—	—	8.13±0.76	8.00±0.74	—	—
A	—	—	—	—	—	—	0.41±0.06	0.44±0.06
$\chi^2/\bar{\chi}^2$	62/46	58/45	76/46	61/45	56/46	53/45	63/46	57/45

independent free parameters and when they fulfill eq. (5).

3. Scaling Variable ω''

Recently in many analyses $\omega'' = \frac{2M\nu + a}{q^2}$ (a - a free parameter) has been used as a scaling variable. Our analysis showed that the data in the threshold region can be fitted well, when the following parametrizations for the structure functions and the form factors

$$\nu W_2 = a(1-x'')^p$$

$$2MW_1 = b(1-x'')^q \qquad (9)$$

$$G_M = c\left(\frac{1}{q^2}\right)^r$$

have been used. Here $x'' = \frac{1}{\omega''}$. The data for $\omega'' \leq 1.4$ (25 points) were used in these fits. The parameters are reported in Table III. The second column corresponds to fit to the data when relation (5) is imposed on p and r , and the third column corresponds to independent p and r . In both cases we obtain a good fit. When p and r are considered as free parameters they satisfy within the error eq. (5), as expected by the Drell-Yan relation. The value of q , which characterized the threshold behaviour of $2MW_1$ equals $q = 4.39 \pm 0.20$. This value is consistent with the results reported in ref.[18,19].

Note that using functions (9) we obtained a good fit to the data in the region $\omega \leq 1.4$ too. ($\chi^2/\bar{\chi}^2 = 73/52$). The values of the fit parameters did not change ($q = 4.40 \pm 0.11$; $p = 3.06 \pm 0.07$; $r = 2.02 \pm 0.04$).

4. Remarks

a) It is well known that the structure functions $2MW_1$ and νW_2 in the parton model with the only

Table III
The results of analysis of the data [1,7] (variable ω'')

1	2	3
a	0.53 ± 0.12	0.15 ± 0.28
p	3.04 ± 0.08	2.73 ± 1.31
b	4.43 ± 1.10	5.60 ± 2.52
q	4.39 ± 0.20	4.57 ± 0.33
c	0.41 ± 0.03	0.41 ± 0.03
r	$\frac{p+1}{2}$	2.02 ± 0.04
d	1.93 ± 0.22	2.00 ± 0.21
$\chi^2/\overline{\chi^2}$	35/29	35/28

spin 1/2 charged constituents obey the Callan-Gross relation:

$$2MW_1 = \omega \nu W_2 . \qquad (10)$$

In fitting to the data with eqs. (1), (2), (3) and (4) we have assumed that $2MW_1$ and νW_2 have the same q^2 -dependence. As a, b, p and q are free parameters, eq. (10) has not been assumed. An important test of the parton model is the check of the Callan-Gross relation. Table IV presents the values of the ratio $R' = \frac{\omega \nu W_2}{2MW_1}$ evaluated for the four parametrizations discussed (p and r are considered independent). As is seen from the Table, the values of R' in the range $\omega \le 1.4$ within the error do not contradict eq. (10), but (1a).

b) Our analysis showed that in order to obtain a good fit to the data in the threshold region it is necessary to introduce scale breaking terms in the expressions in ω and ω' as well. The present data do not allow us to distinguish between the four considered functional forms. Naturally, the question arises in what kinematical region shall we be able to distinguish between them. The answer to this question is of a special interest in connection with the planned experiments on deep inelastic scattering of high energy muons on protons [20]. In Table V we have listed the cross sections, calculated with eqs. (1a), (2a), (3a) and (4a) for some values of E, ω and q^2, in the intervals $20 \le E \le 300$ GeV, $30 \le q^2 \le 480$ $(GeV/c)^2$ and $1.12 \le \omega \le 1.17$. The values of the parameters for eqs. (1a)-(4a) have been fixed in Table I. Even at $E \simeq 100$ GeV the cross sections, obtained by eqs. (1a)-(4a) differ strongly.

IV. CONCLUSIONS

A detailed test of scaling based on the threshold region of deep inelastic e-p scattering is performed. The ω-dependence of the structure functions in this region is rather simple and one may expect to obtain information about their q^2-dependence. The

Table IV

Value of $R' = \frac{\omega r W_2}{2MW_1}$ near threshold

ω	Asymptotically free gauge theories	The logarithmic scale breaking	The parton model with pointless partons	The scale invariant parton model
1.10	0.40±0.17	0.86±0.39	0.77±0.33	0.84±0.37
1.20	0.73±0.10	0.94±0.13	0.92±0.13	0.99±0.07
1.30	1.05±0.07	1.01±0.08	1.05±0.08	1.03±0.08
1.40	1.36±0.20	1.09±0.18	1.16±0.18	1.12±0.18

Table V

The differential cross-sections for inelastic e-p scattering (in 10^{-40} cm^2/GeV-sr) calculated by assuming (1a), (2a), (3a) and (4a)

E (GeV)	q^2 (GeV)^2	ω	Asymptotically free gauge theories	The logarithmic scale breaking	The parton model with pointless partons	The scale invariant parton model
20	32	1.12	428.8	448.0	414.0	428.0
40	64	1.15	89.3	109.1	63.5	89.2
60	96	1.16	33.7	43.6	17.2	31.6
80	128	1.17	16.8	22.4	6.4	14.8
100	160	1.17	9.6	13.3	2.9	8.1
150	240	1.17	3.5	5.2	0.6	2.7
300	480	1.17	0.6	1.0	0.05	0.4

behaviour of νW_2 at $\omega \simeq 1$ can be related to the asymptotic behaviour of the proton form factor at high q^2. The data on deep inelastic e-p scattering for $\omega \le 1.4$ and elastic e-p scattering for $q^2 \ge 6$ (GeV/c) have been fitted too. There have been used parametrizations of q^2-dependence of the structure functions νW_2 and $2MW_1$ based on:

1) asymptotically free gauge theories /9/
2) parton model with logarithmic scale breaking /10/
3) parton model with pointless partons /11/
4) scale-invariant parton model /12/.

The analysis showed that deviations from scaling in ω are necessary. However, the avaliable data can be fitted equally well with either of the considered parametrizations. The Drell-Yan relation does not contradict the data too. The scaling variables ω' and $\omega'' = \frac{2M\nu + a}{q^2}$ (a - a fit parameter) have been used too. Deviations from scaling in ω' are necessary, as well as in ω. The use of ω'' allows one to fit to the data without introducing any additional q^2-dependence.

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