

СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА



29/viii-77

E1 - 10718

L-99

3380/2-77

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ANALYSIS OF THE PRECISION
IN EFFECTIVE MASS
AND FOUR - MOMENTUM TRANSFER
FOR FORWARD PRODUCED SYSTEMS
USING THE MAGNETIC SPECTROMETER MIS

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**ANALYSIS OF THE PRECISION
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Анализ точностей по эффективной массе и
4-импульсу на спектрометре МИС

В настоящем сообщении проведен анализ точностей по эффективной массе в интервале 0,5–1,5 ГэВ/с² и передаваемому моменту в диапазоне 0,01–0,1 для систем, рожденных вперед, в исследованиях на спектрометре МИС. Были использованы генерированные и реальные события, которые в обоих случаях прослеживались в объеме МИС. Для разных предположений о пространственных ошибках было проведено распространение ошибок на кинематические параметры частиц. В результате было найдено, что разрешение по эффективной массе составляет 22 МэВ и не зависит от диапазона масс и разрешение по передаваемому моменту составляет $2 \cdot 10^{-3}$ (ГэВ/с)².

Работа выполнена в Лаборатории ядерных проблем ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна 1977

Analysis of the Precision in Effective Mass
and Four-Momentum Transfer for Forward Produced
Systems Using the Magnetic Spectrometer MIS

A study is presented of the dependence of the resolution in effective mass in the range $0.5 \div 2.5$ GeV/c² and tetramomentum transfer t' in the range $0.01 \div 0.1$ (GeV/c)², for forward systems of 3 and 5 fast charged particles, on beam errors and instrumental and measurement errors in the MIS magnetic spectrometer.

We have used two kinds of events: Monte-Carlo events and real coherent ones, taken at 15 GeV/c and transformed to 40 GeV/c. In both cases the events are traced through the simulated MIS spectrometer.

This work was performed during the preparation of the experiment on coherent production on nuclei with incident pions of 40 GeV/c, in Serpukhov, the conclusion obtained on the basis of this analysis could be of wider interest for any charged system produced in the forward direction and analyzed in the MIS spectrometer.

As free parameters in input, at the analysis, we have taken the total r.m.s. error on point measurements in the magnetic spectrometer, ΔP , $\Delta \lambda$, $\Delta \phi$ the beam definition errors, and L/L_0 , the ratio between the length of the nuclear targets and their radiation lengths. Furthermore, we estimate the effect on the effective mass and t' due to the inclination of the film plane with respect to the x and y axis, due to the nonorthogonality of optical axis with respect to the beam line, due to magnetic field inhomogeneity and to errors in the determination of the coordinates of the fiducial mark system.

Communication of the Joint Institute for Nuclear Research. Dubna 1977

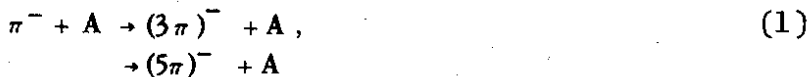
INTRODUCTION

The analysis is made of the precision in mass and tetramomentum transfer for systems produced in the forward direction with the MIS apparatus ^{/1/}.

This analysis was performed in connection with the experiment on coherent production of boson systems on the nucleus at 40 GeV/c at the Serpukhov accelerator.

The experiment on coherent production is the first experiment performed with the MIS apparatus and, at the moment, is the highest energy exclusive coherent production experiment. The detailed description of the experiment is made in the proposal (ref. ^{/2/}).

The reactions under interest are:



To better understand the experimental requirements of this kind of events we remind the main points of interest for physics:

1) Diffractive dissociation of π and K mesons enhances the forward production of the

boson system with the exchange of "natural" spin parity values. The produced system belongs to J^P series $0^-, 1^+, \dots$, consequently the diffractive hadronic production can be extensively studied with a reduced background, as compared with the free nucleon targets, diffractive reactions.

A good precision in the boson spectra and in the angular distributions will allow a partial wave analysis, for relatively small mass intervals, useful for better understanding the intrinsic nature (resonances or dynamical effects) of enhancement observed in the mass distributions of the diffracted system, both in the highest, unexplored, part of the spectrum and in the lower part, where the comparison with lower energy experimental results is interesting.

2) A good resolution in the 4-momentum transfer t' , allows:

a) to separate coherent production from incoherent one (as it is well known this is impossible to make event by event, due to the lack in nuclear recoil energy and direction measurements, but it is performed on the statistical basis with the cut in the t' distributions);

b) to check in detail the validity, at this energy, of the Glauber theory, modified for coherent production on the nucleus ^{/3/};

c) to extract the A dependence of the differential cross section and consequently the elementary unstable boson system-nucleon total cross section and the dependence effective mass of the boson system. As is well known the results of the analysis, performed at lower energies ^{/4/}, show very peculiar characteristics and they have originated

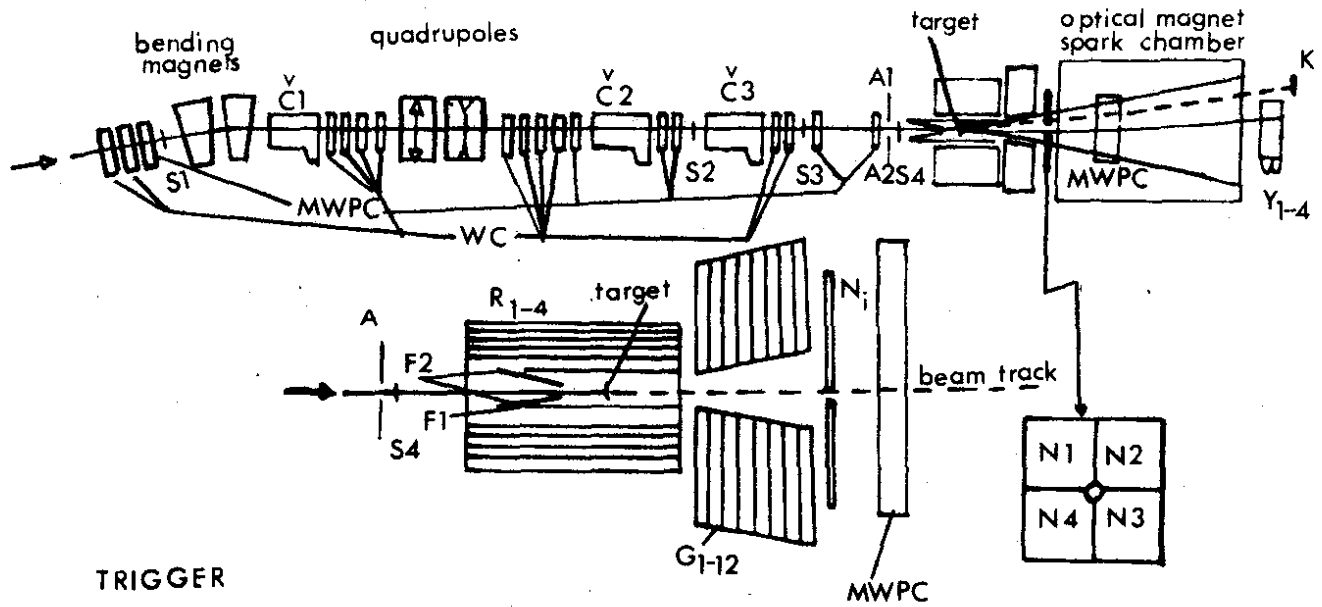
many theoretical speculations on the basis of space-time propagation of the boson system through nuclear matter (ref.^{15/}).

So the study of the multimeson production at high energy in the forward direction would impose the definite requirements to the experimental accuracy. The first minimum in the diffractive differential cross-section, e.g., for lead is of about $0.01(\text{GeV}/c)^2$ therefore, for reliable extraction of processes (1.,2) one would need a resolution in t' of about $10^{-3}(\text{GeV}/c)^2$ and for effective masses $\sim 50 \text{ MeV}/c^2$.

EXPERIMENTAL SET-UP

A general view of the set up is given in the figure. It has 50 optical spark chambers $1\text{m} \times 1\text{m}$ with a 2 mm spacing. The useful volume is $1.5 \times 1.3 \times 5 \text{ m}^3$. The radiation length is 100 m. Each plate of the spark chamber has a thickness 16μ of AL and 56μ of maylar. In front of each pair of spark chambers there is a copper disk (8 mm thick) with a central window 60 cm in diameter to materialize γ rays.

The light is collected on the film camera by five pairs of mirrors which simulate the virtual camera positions, The length of the optical axis of the stereoscope cameras is 280 cm. The optical path length from the centre of the chamber to the virtual camera position is 920 cm. The stereoscopic factor is 3,3. The target is situated 80 cm upstream the first spark chamber and its thickness is chosen so as to reduce the multiscattering effect on momentum and angular errors to a value small in comparison



TRIGGER

BEAM $T1 = S1S2S3S4AF2(\overset{V}{C}1 + \overset{V}{C}2)\overset{V}{C}3$

INTERACTION $T2 = 11K$

COHERENT $T = T2F1R, G1, N1, MWPC, 2Y1, Y1$

Figure

with the measurement error. The target is surrounded by a veto-counter system to avoid protons from the break-down of the nucleus and π^0 -production. A system of lead scintillator sandwiches defines an acceptance cone of $\pm 14^\circ$ for the MIS.

The direction and $\frac{\Delta p}{p}$ of the incident meson, which creates the trigger, is determined by ten MWPC and twelve wire spark chambers placed along the beam transport system, a part of them is upstream and downstream of two bending magnets.

RESOLUTION

When analysing the accuracy we take into account different sources of errors, namely, the inaccuracy of the beam and the spatial resolution in the chamber.

The main contributions to the spatial resolution, that is the precision we can obtain in reconstructing a spark in the chamber, are the following:

- a) the spark formation mechanism,
- b) the distortion due to the optical apparatus, the quality of the film, the development process and the precision of the apparatus for the automatic reading of the photographs. We call all these errors the "measurements errors".

We suppose to have corrected the systematic errors due to the well known effects of drift and staggering of the single spark.

For the total y and z error, we have:

$$\sigma_y^2 = \sigma_{y\text{sp}}^2 + \sigma_{y\text{meas}}^2 \cdot 1/2$$

$$\sigma_z^2 = \sigma_{z\text{sp}}^2 + \sigma_{z\text{meas}}^2$$

where $\sigma_{y\text{sp}}$ and $\sigma_{z\text{sp}}$ are the random spark fluctuation in space assumed to be equal, $\sigma_y, \sigma_{z\text{meas}}$ are the measurement errors in y and z . The factor $1/2$ takes into account the two independent measurements of the same spark (see ref.^{/6/}).

The z coordinate is calculated by means of the differences $y_2 - y_1$ of the y coordinate in two view:

$$z = \lambda (y_2 - y_1),$$

where λ is the stereoscopic factor of the optical system.

For the "CERN Little Omega" chamber^{/6/} it has been found that $\lambda=4,75$, whereas for the MIS spectrometer we have estimated λ to be equal to 3.3.

We must notice that σ_y, σ_z depend on the azimuthal angle ϕ which the track forms with the x -axis (normal to the plates).

Remembering that ϕ is very small in our events, we can concentrate our attention on σ_{oy} and σ_{oz} , the first terms of σ_y and σ_z for $\phi=0,1$.

On the basis of the result of the little Omega apparatus at CERN (ref.^{/6/}) and taking into account the different demagnification factor in that apparatus and in the MIS we have assumed for the ratio $\sigma_{y\text{meas}} / \sigma_{oy} = 0,75$; so we have:

$$\sigma_{oz} \approx \sqrt{2} \lambda \sigma_{y\text{meas}} = 3,5 \sigma_{oy}$$

The range of variability which we have chosen for σ_{oy} , is between 200 and 600 μm .

As other free parameters of the present analysis are taken the errors of the p, λ, ϕ , momentum and the angles of the incident particle, and the ratio L/L_0 between the length L of the different nucleus targets and their radiation lengths L_0 .

The field of variability which we have used for these parameters will be specified below.

Errors of the particle momenta due to the spatial resolution

The errors on the co-ordinates $(1/\rho, \lambda, \phi)$ of a particle (which characterize its momentum), simply propagate from the errors σ_y, σ_z .

Here we only give the final results in the form of the error matrix on $(1/\rho, \lambda, \phi)$ assuming the maximum correlation in errors $1/\rho$ and ϕ

$$\begin{pmatrix} \left(\frac{8\sigma_s}{L^2 \cos^2 \lambda}\right)^2 & 0 & -32 \frac{\sigma_s^2}{L^3 \cos^3 \lambda} \\ 0 & \left(\frac{4\sigma_z}{L \cos \lambda}\right)^2 & 0 \\ -32 \frac{\sigma_s^2}{L^3 \cos^3 \lambda} & 0 & \left(\frac{4\sigma_s}{L \cos \lambda}\right)^2 \end{pmatrix} \quad (3)$$

where σ_s is the error on the sagitta of the track projected in the xy -plane and it is given by:

$$\sigma_s = 3,5 \frac{\overline{\sigma_y(\phi^2)}}{\sqrt{N}}$$

Errors on the particle momenta
due to the multiple scattering

These errors, independent of the previous ones, are added quadratically to the measured ones to give the total errors on the momentum.

The matrix of errors on $(1/\rho, \lambda, \phi)$ is ^{/7/}

$$\begin{pmatrix} \frac{4}{3} \frac{K}{L \cos^4 \lambda} & 0 & -\frac{K}{6 \cos^3 \lambda} \\ 0 & \frac{KL}{3} & 0 \\ -\frac{K}{6 \cos^3 \lambda} & 0 & \frac{KL}{6 \cos^2 \lambda} \end{pmatrix} \quad (4)$$

where $K = \frac{1}{2} \left(\frac{C}{p\beta} \right)^2$ with $C = 0.021 / \sqrt{L_c}$, L_c - radiation length of the chamber (in cm), p - particle momentum (in GeV/c).

A similar procedure is made also in order to take into account the multiple scattering in the target. Due to the fact that we use different nuclear targets, we have introduced as a further free parameter the ratio L/L_0 between the length of the target and its radiation length.

SIMULATION PROGRAM

The program used to evaluate the experimental errors on $M_{3\pi}$ and 4-momentum transfer to the nucleus (in reactions (1), (2)), simulates the experimental apparatus.

As far as input events are concerned, two kinds have been used:

- i) events entirely simulated by the Monte-Carlo method.
- ii) events transformed at the required energy (40 GeV) from events of a previous experiment at lower energy (15 GeV). These events have the same mass - distribution of 15 GeV ones, but they are more forward peaked, because in the previous experiment the events were already cut with an acceptance cone of 22° ; this corresponds, at 40 GeV, to an acceptance cone of about 8° .

The events are traced in the MIS spectrometer, their errors are calculated from matrices (3) and (4) and propagated by using the GRIND method to obtain errors in the effective mass and 4-momentum transfer.

In this program the pattern recognition problem is neglected but the only condition in order that the track is reconstructed is that it has seven sparks at least.

RESULTS

In order to obtain the errors:

ΔM^* , error on the final 3(5) pions the invariant mass M^* ,

Δt , error on the total quadromomentum transfer,

$\Delta t'$, error on $t' = t - t_{\min} = q_+^2$ *

* $q_+ \sim p \theta$, where p and θ are, in the lab. system, the momentum and the angle with respect to the incident beam; of the outgoing pions center of mass system $\Delta q_+ = \theta \Delta p + p \Delta \theta$,

and $\Delta t' = 2q_+ \Delta q_+$. From here it is easy to see that the error in t' has not a Gaussian distribution.

we have made the analysis with different groups of experimental parameters. The four parameters in input were: σ_y , the error in space in y direction, $\Delta p/p$, the relative error of the incident beam momentum, $\Delta\lambda = \Delta\phi$, the error on the angular direction of the incident beam particle, L/L_0 , the ratio between the target length and its radiation length.

We took this basic set of values:

$$\sigma_y = 0.03 \text{ cm}; \quad \Delta\lambda = \Delta\phi = 0.15 \text{ mrad}$$

$$\Delta p/p = 0.3\%; \quad \frac{L}{L_0} = \frac{1}{10}$$

The field of variability for σ_y , in the analysis is between 0 and 0.06 cm, with steps of 0.02 cm. For $\Delta p/p$, there were chosen three values: zero, to see the effect of the measurement errors; 2%, the spread of the beam in the MIS channel, and 0.3%, which is the region of precision which can be reached with the present set up of wire chambers and MWPC's on the beam.

Analogous consideration allows us to choose for $\Delta\lambda = \Delta\phi$ the values of 0.; 0.15; 0.25; 0.50 mrad. For L/L_0 the values are 0, 1/50, 1/5, 1/2; the determination of the length L of the target should be done in those range of values, as a compromise, for each different nuclear target and incident particles, high interaction rate, low empty target rate and multiple scattering effects. For example for 40 GeV/c L/L_0 ratio is around 1/7 to get a compatible contribution to momentum resolution as compare with measurement ones.

Table 1a

 M^* , t , t' errors - 3 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi = \Delta\lambda$ (mrad)	L/L ₀	ΔM^*				Δt	and $\Delta t'$					
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$			$M^* < 1.4 \text{ GeV}$			$M^* > 1.4 \text{ GeV}$		
					$t' < .1$	$t' > .1$	$t' < .1$	$t' > .1$		$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$
1	.03	3	.15	0	21.2± .4	19.8± 1.1	24.2± .7	23.3± 1.3	Δt 0.0055± 0001	.0078± .0002	.0100± .0004	.0110± .0004	.0130± .0005	.0180± .0015	
									$\Delta t'$ 0.0024± 0001	.0058± .0002	.0087± .0005	.0028± .0002	.0063± .0004	.013 ± .001	
2	.03	3	.15	$\frac{1}{50}$	21.5± .4	20.3± 1.1	24.6± .7	23.7± 1.2	Δt 0.0055	.0079	.0100	.110	.0130	.0180	
									$\Delta t'$ 0.0024	.0059	.0089	.0028	.0064	.013	
3	.03	3	.15	$\frac{1}{10}$	22.5± .3	21.5± 1.1	25.7± .7	24.8± 1.2	Δt 0.0056	.0080	.011	.011	.013	.018	
									$\Delta t'$ 0.0025	.0061	.0093	.0029	.0066	.013	
4	.03	3	.15	$\frac{1}{5}$	23.7± .3	22.9± 1.1	27.0± .7	26.1± 1.1	Δt 0.0056	.0082	.011	.011	.013	.18	
									$\Delta t'$ 0.0026	.0063	.0099	.0029	.0068	.013	
5	.03	3	.15	$\frac{1}{2}$	26.9± .3	26.4± 1.1	30.4± .7	29.4± 1.1	Δt 0.0057	.0086	.012	.011	.013	.019	
									$\Delta t'$ 0.0029	.0069	.011	.0032	.0074	.015	

 ΔM^* in MeV; t , t' , Δt , $\Delta t'$ in $(\text{GeV}/c)^2$

n.b.

the statistical errors on Δt and $\Delta t'$ are quoted only for the first 'run'. in the other cases the errors are of the same order as the quoted ones

Table 1b

 M^* , t , t' errors - 3 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi - \Delta\lambda$ (mrad)	L/L_0	ΔM^*				Δt and $\Delta t'$						
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$		$M^* < 1.4 \text{ GeV}$			$M^* > 1.4 \text{ GeV}$			
					$t' < .1$	$t' > .1$	$t' < .1$	$t' > .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	
6	.03	0	.15	$\frac{1}{10}$	$22.5 \pm$	$21.6 \pm$	$25.7 \pm$	$24.8 \pm$	Δt	.0039	.0068	.0097	.0075	.0095	.015
					.4	1.1	.7	1.2	$\Delta t'$.0025	.0061	.0093	.0029	.0065	.013
7	.03	20	.15	$\frac{1}{10}$	$22.5 \pm$	$21.6 \pm$	$25.7 \pm$	$24.8 \pm$	Δt	.026	.028	.030	.056	.057	.062
					.3	1.8	.7	1.2	$\Delta t'$.0025	.0061	.0100	.0029	.0066	.013
8	.03	3	.25	$\frac{1}{10}$	$22.6 \pm$	$21.6 \pm$	$25.7 \pm$	$24.8 \pm$	Δt	.0056	.0083	.012	.011	.013	.018
					.4	1.1	.7	1.2	$\Delta t'$.0027	.0065	.010	.003	.007	.014
9	.03	3	.5	$\frac{1}{10}$	$22.5 \pm$	$21.6 \pm$	$25.7 \pm$	$24.8 \pm$	Δt	.0061	.0097	.0149	.012	.014	.020
					.3	1.0	.7	1.2	$\Delta t'$.0034	.0082	.014	.0038	.0088	.0167
10	.03	3	.0	$\frac{1}{10}$	$22.5 \pm$	$21.5 \pm$	$25.7 \pm$	$24.8 \pm$	Δt	.0055	.0078	.010	.011	.013	.017
					.3	1.1	.7	1.2	$\Delta t'$.0024	.0058	.0087	.0028	.0063	.013

 ΔM^* in MeV; t , t' , Δt , $\Delta t'$ in $(\text{GeV}/c)^2$

Table 1c

 M^* , t , t' errors - 3 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi = \Delta\lambda$ (mrad)	L/L_0	ΔM^*				Δt and $\Delta t'$						
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$		$M^* < 1.4 \text{ GeV}$			$M^* > 1.4 \text{ GeV}$			
					$t' < .1$	$t' > .1$	$t' < .1$	$t' > .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	
11	0.	3	.15	$\frac{1}{10}$	$9.3 \pm .2$	$9.2 \pm .6$	$11.0 \pm .4$	$10.6 \pm .5$	Δt	.0043	.0051	.0065	.0091	.0094	.011
									$\Delta t'$.0011	.0026	.0047	.0013	.0028	.0055
12	.02	3	.15	$\frac{1}{10}$	$17.5 \pm .3$	$16.8 \pm .9$	$20.2 \pm .7$	$19.5 \pm .9$	Δt	.0049	.0067	.0088	.010	.011	.015
									$\Delta t'$.0019	.0046	.0074	.0023	.0050	.010
13	.04	3	.15	$\frac{1}{10}$	$27.8 \pm .4$	26.5 ± 1.3	$31.4 \pm .8$	30.3 ± 1.5	Δt	.0063	.0095	.013	.013	.015	.02
									$\Delta t'$.0031	.0076	.011	.0036	.0082	.016
14	.06	3	.15	$\frac{1}{10}$	$38.4 \pm .5$	36.6 ± 1.9	43.2 ± 1.1	41.6 ± 2.1	Δt	.0080	.013	.017	.015	.0189	.028
									$\Delta t'$.0044	.010	.016	.0049	.011	.022
15	.06	3	.15	$\frac{1}{10}$	$31.5 \pm .4$	31.7 ± 1.6	$38.2 \pm .8$	37.4 ± 1.6	Δt	.0089	.013	.017	.020	.022	.032
									$\Delta t'$.0042	.010	.0147	.0051	.0115	.0239

 $^*H = 12 \text{ Kg}$. ΔM^* in MeV; $t, t', \Delta t, \Delta t'$ in $(\text{GeV}/c)^2$

Table 2a

 M^* , t , t' errors - 5 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi = \Delta\lambda$ (mrad)	L/L_0	ΔM^*				Δt and $\Delta t'$						
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$		$M^* < 1.4 \text{ GeV}$			$M^* > 1.4 \text{ GeV}$			
					$t' < .1$	$t' > .1$	$t' < .1$	$t' > .1$		$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$
1	.03	3	.15	0	17.9 \pm	14.9 \pm	25.5 \pm	26.4 \pm	Δt	.0069 \pm	.0074 \pm	.0150 \pm	.0130 \pm	.0100 \pm	.0200 \pm
					1.1	.1	.5	.9	.0004	.0005	.0005	.0004	.0004	.0007	
2	.03	3	.15	$\frac{1}{50}$	18.3 \pm	15.4 \pm	25.9 \pm	26.7 \pm	Δt	.0069	.0075	.015	.013	.016	.020
					1.6	.1	.5	.9	.0019	.0045	.014	.0028	.0067	.012	
3	.03	3	.15	$\frac{1}{10}$	19.3 \pm	17.3 \pm	27.1 \pm	27.8 \pm	Δt	.0070	.0078	.016	.013	.017	.02
					1.6	.3	.4	.9	.0021	.0050	.015	.0029	.0071	.013	
4	.03	3	.15	$\frac{1}{5}$	20.6 \pm	19.3 \pm	28.6 \pm	29.1 \pm	Δt	.0070	.0082	.017	.013	.017	.021
					1.6	.5	.4	.9	.0022	.0056	.016	.0031	.0075	.014	
5	.03	3	.15	$\frac{1}{2}$	24.0 \pm	24.5 \pm	32.4 \pm	32.4 \pm	Δt	.0072	.0091	.019	.014	.017	.022
					1.8	.9	.4	1.0	.0026	.0069	.018	.0034	.0085	.016	

 ΔM^* in MeV; $t, t', \Delta t, \Delta t'$ in $(\text{GeV}/c)^2$

nb.

The statistical errors on Δt and $\Delta t'$ are quoted only for the first 'run'. In the other cases the errors are of the same order as the quoted ones

Table 2b

 M^* , t , t' errors - 5 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi - \Delta\lambda$ (mrad)	L/L_0	ΔM^*				Δt and $\Delta t'$						
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$		$M^* < 1.4 \text{ GeV}$				$M^* > 1.4 \text{ GeV}$		
					$t' < .1$	$t' > .1$	$t' < .1$	$t' > .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	$t' < .01$	$.01 < t' < .05$	$.05 < t' < .1$	
6	.03	0.	.15	$\frac{1}{10}$	19.4 ± 1.6	$17.3 \pm .3$	$27.1 \pm .5$	$27.8 \pm .9$	Δt	.004	.006	.015	.008	.011	.015
									$\Delta t'$.0021	.0050	.015	.0029	.007	.013
7	.03	20	.15	$\frac{1}{10}$	19.4 ± 1.6	$17.3 \pm .3$	$27.1 \pm .4$	$27.8 \pm .9$	Δt	.037	.035	.040	.072	.081	.091
									$\Delta t'$.0021	.0051	.015	.0030	.0071	.013
8	.03	3	.25	$\frac{1}{10}$	19.4 ± 1.6	$17.3 \pm .3$	$27.1 \pm .4$	$27.8 \pm .9$	Δt	.0070	.0082	.017	.014	.017	.021
									$\Delta t'$.002	.006	.015	.0031	.0075	.014
9	.03	3	.5	$\frac{1}{10}$	19.4 ± 1.6	$17.3 \pm .3$	$27.1 \pm .4$	$27.8 \pm .9$	Δt	.0073	.0099	.016	.013	.017	.023
									$\Delta t'$.0030	.0079	.018	.0038	.0093	.017
10	.03	3	0	$\frac{1}{10}$	19.4 ± 1.6	$17.3 \pm .3$	$27.1 \pm .4$	$27.8 \pm .9$	Δt	.0069	.0076	.016	.013	.016	.020
									$\Delta t'$.0020	.0047	.014	.0028	.0068	.012

 ΔM^* in MeV; t , t' , Δt , $\Delta t'$ in $(\text{GeV}/c)^2$

Table 2c

 M^* , t , f , errors - 5 prongs events

RUN	σ_y (cm)	$\Delta p/p$ (%)	$\Delta\phi - \Delta\lambda$ (mrad)	L/L_0	ΔM^*				Δt and Δf						
					$M^* < 1.4 \text{ GeV}$		$M^* > 1.4 \text{ GeV}$		$M^* < 1.4 \text{ GeV}$			$M^* > 1.4 \text{ GeV}$			
					$f' < .1$	$f' > .1$	$f' < .1$	$f' > .1$		$f' < .01$	$.01 < f' < .05$	$.05 < f' < .1$	$f' < .01$	$.01 < f' < .05$	$.05 < f' < .1$
11	.0	3	.15	$\frac{1}{10}$	$8.7 \pm .8$	$9.3 \pm .4$	$12.5 \pm .3$	$11.7 \pm .4$	Δt	.0059	.0063	.0088	.011	.013	.016
									Δf	.0011	.0031	.0066	.0014	.0035	.0065
12	.02	3	.15	$\frac{1}{10}$	15.3 ± 1.2	$13.8 \pm .3$	$21.6 \pm .4$	$21.8 \pm .7$	Δt	.0065	.0071	.013	.013	.015	.018
									Δf	.0017	.0042	.011	.0023	.0056	.010
13	.04	3	.15	$\frac{1}{10}$	23.6 ± 2.0	$20.9 \pm .3$	$32.8 \pm .5$	33.9 ± 1.1	Δt	.0076	.0087	.019	.014	.018	.023
									Δf	.0025	.0059	.018	.0036	.0056	.016
14	.06	3	.15	$\frac{1}{10}$	32.3 ± 2.8	$28.7 \pm .4$	$44.3 \pm .7$	46.4 ± 1.5	Δt	.0090	.011	.026	.017	.022	.029
									Δf	.0034	.0078	.026	.0049	.012	.021
15	.06	3	.5	$\frac{1}{10}$	25.6 ± 2.2	$22.8 \pm .2$	$35.3 \pm .4$	39.2 ± 1.6	Δt	.0088	.0095	.023	.017	.022	.028
									Δf	.0029	.0065	.022	.0043	.010	.019

 $^2\text{H} = 12 \text{ Kg}$ ΔM^* in MeV; $t, f, \Delta t, \Delta f$ in $(\text{GeV}/c)^2$

Every set of values is a run from 1 to 15 for the 3 prongs (tables 1a,b,c) and from 1 to 15 for the 5 prongs (tables 2a,b,c).

We show in tables for run 1-7 the results of the analysis performed on real events, transformed at 40 GeV/c. We determine that there is no practical difference between the use of real events or simulated Monte-Carlo events.

For the basis set of values (run 3), we have:

- a) $\Delta M_{3\pi}^*$ of the order of 22 MeV/c² for 3π invariant mass less than 1.4 GeV/c² and 25 MeV/c² for $M_{3\pi}^*$ more than 1.4 GeV/c². As it is possible to see in the tables the effect of the separation between events with t' less or more than 0.1 (GeV/c)² is small.
- b) the relative error of $\Delta t'/t'$ is of 25% for $t' \sim 0.01$ (GeV/c)², 20% - for $0.01 < t' < 0.05$ and 12% - for $0.05 < t' < 0.1$ and for $M^* \leq 1.4$ GeV/c². At higher masses there is a systematic increase of $\Delta t'/t'$ of the order of 2-3%.

The results for the 5 pions, as it is possible to see from table 2, run 3, follow closely those of the 3 pions system. Looking at different set of parameters, we can infer that: σ_y is clearly the more important parameter, compare run N.3 with run N.14 ($\sigma_y = .06$), $\frac{\Delta p}{p}$ has practically no influences on t' and on M^* ; it influences only t (compare run 6 and 7). L/L_0 increase all the errors of about 20-30% going from $\frac{L}{L_0} = \frac{1}{10}$ to $\frac{L}{L_0} = \frac{1}{2}$. $\Delta\lambda = \Delta\phi$ is important for if bigger than 0.25 mrad (compare run N.8, 9,10 with run N.3).

We traced the events in a constant magnetic field $H = 17$ KG.

To see the influences of this assumption run N.15 is equal to run N.14, but with $H=12$ KG:, there is a small improvement in ΔM^* , (due to longer tracks in useful volume) but practically no change Δt and $\Delta t'$.

ANALYSIS OF THE SOURCE OF SHIFTING IN MASS DETERMINATION

Sources of shifting in mass have been studied by using space reconstruction of K^0 events.

The inclination of the film plane to the x and y axis, which are perpendicular to the optical axis, was taken equal to 0.5 mrad. For the homogeneous magnetic field and $\sigma_y = 0$ the mass of the K^0 was determined with the systematic shift of 0.2 MeV and for $t' \sim 3 \cdot 10^{-5}$ (GeV/c)². The inclination of optical axis to the beam line gives the systematic shift in K^0 mass equal to 0.15 MeV and no influence on t' .

Inhomogeneous magnetic field (8 %) gives errors in K^0 mass equal to 8.5 MeV.

We have considered also the influence on the mass of the errors in the determination of x and y coordinate of fiducial system. It has been found that space errors σ_x and σ_y equal to $100 \mu\text{m}$ give no shift in K^0 mass while $\sigma_x = \sigma_y = 1 \text{ mm}$ give a shift of 0.17 MeV.

So we can conclude that the parallelity of the optical axis is rather good and that we need accurate space measurements of the fiducial marks.

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Received by Publishing Department
on June 6, 1977.