ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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LOW-ENERGY PION-PION SCATTERING

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The results of the investigation of the equations for the low-energy pion-pion scattering, offered in paper[1], are presented in this report. In the equations the imaginary parts of partial-wave amplitudes on the unphysical cut are defined from the approximate crossing-symmetry condition, corresponding to the forward scattering.

Limiting only by s - and p -waves this condition takes a very simple form:

$$Im A_i(-z) = -\Sigma b_{ik} Im A_k(z). \tag{1}$$

Here $z = 2\nu + 1 = 2q^2/\mu_\pi o + 1$, $A_0 = A_0^0$, $A_1 = A_1^1$, $A_2 = A_0^2$ $b_{ik} = \delta_{ik} + \ell_i n_k$

$$\ell_0 = -\frac{1}{3}, \quad \ell_1 = -1/18, \quad \ell_2 = 1/6, \quad n_0 = 2, \quad n_1 = 9, \quad n_2 = -5.$$

Eq. (1) with the unitary conditions

$$Im A_i(z) = \sqrt{\frac{z-1}{z+1}} \; /A_i(z)/^2 \tag{2}$$

and p -wave threshold condition

$$A_1(1) = 0 \tag{3}$$

defines the initial system of nonlinear integral equations. Eq.(2)is valid only below the first nonelastic threshold at $z = 7$. Nevertheless, we shall use it in the whole physical interval $1 \leq z \leq \infty$. Such a model with the elastic unitarity and throwing off the highest partial waves, of course, can give a good approximation just only in the low energy region. Only in this region we shall give the physical sense.

In order to get some insights on this system properties, s -wave model for the neutral pion scattering was examined[2]. The model allows the exact solution by Castillejo-Dalita-Dyson-method (CDD). The main solution properties are following:

1. For the large z the solutions decrease not slower than $\ell n^{-1} z$ so that the equations may be written down in the non-subtracted form.

2. The scattering length can take only positive values.

3. The solutions have the arbitrariness of CDD-type, being able to have resonances only due to R -terms. The dynamical resonances do not appear.

4. There exist $0 < \lambda < \lambda_{max}$ parameter, which defines the strength of interaction, $\lambda = \lambda_{max}$ corresponding to the appearence of the bound state.

5. The phase shifts corresponding to the resonant solutions with R -terms in the limit of switching off the interaction ($\lambda \to 0$) are tending to the step functions (dotted line in Fig. 1). The solid curve corresponds to the small λ. It differs from the limiting one when $/z - z_r/ \sim \lambda$. By this :

$$Im A \simeq \pi \Gamma \delta(z - z_r) = \pi \frac{\lambda z_r}{2} \delta(z - z_r). \tag{4}$$

In the charged case we were not able to find the exact solution. However, it can be shown that the enumerated properties of the neutral model find their reflections also in the charged case.

The solutions can be determined by various methods[3,4] to have the arbitrariness of CDD-type. They allows three various asymptotic behaviour:

a) $d_1 / \ln z$ b) e_1 / z c) f_1 / z^2 (5)

Surely, the equations without the subtraction are valid for this type of solutions:

$$A_l(z) = \frac{1}{\pi} \int_1^{\infty} \frac{Im A_l(z')}{z' - z} dz' + \sum_k b_{lk} \frac{1}{\pi} \int_1^{\infty} \frac{Im A_k(z')}{z' + z} dz' \qquad (6)$$

s -waves scattering length positiveness

$$a_0 > 0 \qquad a_2 > 0 \qquad (7)$$

is an algebraic consequence of (6) and (3).

The method of the approximate constraction of the resonance [6] solutions for small λ $(\lambda = \frac{1}{3}(A_0(0) - A_2(0))$ can be obtained using the analogy with the properties of neutral model solutions[4].

It turns out that one can obtain the resonance solutions different by their structure. We where not able to obtain the solution with resonance only in one partial wave. In any resonant solution the resonance in A_0 is necessary. Therefore, physically interesting two-resonant solution contains resonances in A_0 and A_1 (sol.1) . The triresonant solutions in A_0 , A_1 and A_2 (sol. 2), as well as such with a great number of resonances are possible. We shall limit ourselves solutions by I and II. Solution 1 depends on three arbitrary parameter: λ and on two resonance positions $z_0 = 2\nu_0 + 1$ and $z_1 = 2\nu_1 + 1$. Solution II contains five arbitrary parameters: λ , z_0 , z_1 and also the resonance and zero positions z_2 and $x_0 > z_2$ in A_2 -wave. Solution II turns into solution I, when $x_0 \to z_2$. For small λ these solutions are described by simple equations[4-5].

The various iterarive procedures[3,4] can be used for to obtain the solutions at non-small λ . They have been obtained on the base of modified N/D method with the help of numerical calculations by electronic computer of the Institute for Mathematics of Academy of Sciences, Siberian Branch, USSR

The numerical calculations, made for the fixed value of ρ -resonance energy $z_1 = 13,5$ $(M_\rho = 730_{Mev})$ resulted the next important facts:

1 . There exist the upper limiting value $\lambda = \lambda_{Max} \approx 0,4$ (depending on z_0) corresponding to the bound state appearing in A_0 . When $\lambda \to \lambda_{Max}$, $a_0 \to \infty$.

2. The deviations from δ -approximation formulas of papers[4,5] for A_1 and A_2 can be described with the help of λ -renormalization. The A_0 -wave changes more essentialy; its scattering length can be approximated by equation

$$a_0 \simeq \frac{5\lambda}{1 - \lambda/\lambda_{Max}} . \qquad (8)$$

3. The resonance in A_1 remains narrow up to $\lambda = \lambda_{Max}$. The full ρ -meson width 2γ can not exceed 40 MeV. 2γ -dependence on a_0 for different z_0 for solution I is plotted in Fig. 2. The corresponding curves for the solution II lie below the curves of Fig. 2. It is seen from this figure that for $a_0 < 2,5$; $2\gamma < 30$ Mev.

In the next Fig. the calculated cross-sections $\sigma_{\pi\pi} = \frac{1}{3}(2\sigma_{\pi^+\pi^-} + \sigma_{\pi^\pm\pi^0})$ (Fig. 3a), $\sigma_{\pi^\pm\pi^0}$ (Fig. 3b), and $\sigma_{\pi^\pm\pi^\pm}$ (Fig. 3c) are given for some parameters values. The solid curves correspond to sol. 1 for $\lambda = 0,2$; $z_1 = 13,5$; $z_0 = 13,5$; the dotted ones - to sol. II for $\lambda = 0,2$; $z_0 = 100$; $z_2 = 11,8$; $x_0 = 25$; $z_1 = 13,5$.

Experimental data from papers[7,8] and paper[9], are given in Fig. 3a and 3b. It is seen from these Figs., that, generally speaking, the calculated curves do not contradict to the experimental data.
However due to experimental errors it is untimelinessly to speak about the agreement degree.

Now we shall explain the parameters choice. As it was stated ρ -resonance position in all the solutions is fixed at the point $z_1 = 13,5$ ($M_\rho = 730\,Mev$). λ -parameter is chosen equal to 0,2 for getting not too narrow ρ -resonance and a reasonable a_0 scattering length. At $\lambda = 0,2$ $a_0 = 2,0$ for solution I and $a_0 = 1,8$ for sol. II. In Fig. 3a the full resonance width equals 30 MeV for sol. I and 20 MeV for sol. II. In sol. I A_0 -resonance position consides with that of ρ -resonance. It makes the resonance wider only by 10%. There are no any data on the presence of low-energy A_0 -resonance. Taking this resonance away (putting $z_0 > 25$)our curves change a little. They may change more if shifting this resonance into $z_0 < 12$ -region. In sol. II we introduced A_2 -resonance at $z_2 = 11,8$($M = 690\,Mev$). This was done according to the following reasons. The analysis of pion angle distributions in reactions $\pi^{\pm} + p \to p + \pi^{\pm} + \pi^0$in the neighbourhood of resonance shows/9,10,11/, that the intenference term of A_1 and A_2 -waves, has the negative sign below the resonance, and the positive one above it. As in our solutions always $a_2 > 0$, we can explain this sign only by putting A_2 resonance, located below ρ -resonance. The vague indications to the resonance existence in A_2 wave, at chosen energy, were obtained in paper/12/. As it is seen in Fig. 3a, this resonance introduction makes the agreement with experiment in 600-700 MeV region better .

Recently Grashin and Shalamov/16/studying the process $\pi^- + n \to \pi^- + \pi^- + p$ have obtained rather reliable evidance for A_2 -resonance existence at 500-600 MeV.

It follows from the Figures, the main difference between our curves and the experimental data takes place in the close neighbourhood of the resonance. The experimental resonance peak, as a rule, lies not only below the theoretical one but also has the greater area. It must be said that the experimental resonances have the width of the order of experimental resolution. For example, the angular distributions correspond to the more narrow resonances. Besides, the cross-sections obtained by direct use of Chew-Low formula, in some cases differ by 50-100% from the ones,obtained by more accurate extrapolation method (see Fig. 3b). Therefore we hope that the further increase of experiment accuracy will make the agriment with the suggested theory better. The solution discussed, formaly have the power asymptotes of (5b)-type. However it can be proved/4/ that for each of the examined solutions we have the solution with (5a) logarithm asymptotic, very close to it in the low-energy region. This close solution can be obtained by CDD- zero shifting from the infinite to a finite but high-energy point.

In order the solving to be more easy we have chosen the power asymptotics. However it has no direct physical sence, as we deal with low energy model. It follows that, if to wish, we may consider our solutions to the logarithmic asymptotic, what is in quality correspondance with the diffraction peak in the high-energy region.

The remarkable property of the obtained solutions is ρ -resonance narrowness (2γ < 40 MeV).

This solution property is the result of the bound state absence in A_0-wave, and also the result of threshold condition for ρ -wave (3), written down for the non-subtracted equations (6) :

$$\frac{1}{\pi}\int_1^\infty \frac{Im A_1(z')}{z'-1}\,dz' = \frac{1}{\pi}\int_1^\infty \frac{dz'}{z'+1}\left\{ \frac{1}{9} Im A_0 - \tfrac{1}{2} Im A_1 - \frac{5}{18} Im A_2 \right\}. \tag{9}$$

Here the eq. (9) is the 'correlation condition' between various waves in our theory. It can be seen directly from (9), that the large A_0 -wave is necessary for ρ -resonance existence. In the end, the correlation condition (9) is the reflection of the dissapearence of $Re A_1$ as $z \to \infty$. This property can be based on the correspondance with the diffraction scattering picture. Therefore, any modification of the low-energy equations, satisfying the mentioned correspondance criterion, will give the correlation condition different from (9) only by some high-energy contributions. In the case the contributions being small, ρ -meson width upper limit will change a little.

For instance, if to accurate eq. (6) by taking the crossing-symmetry conditions for $Im A_l$ not only imaginary parts of forward (backward) scattering amplitude, but also their first derivatives with respect to angle, we shall obtain the eq. (2.10) of paper /13/. It is not difficult to see, that the correlation condition for these equations leads to the changing of 2γ not more than 6%.

Now we shall discuss a question on the accuracy of our equations (6). In these equations deriving, they were spoken to have been obtained from the usual dispersion relations for the forward (backward) scattering, and one of the essential approximations to be in approximation of real part of the scattering amplitude by some number of partial waves. This approximation was criticized as the most narrow point of our scheme.

It can be shown, our equations can be obtained by some other way which do not contain the direct approximation of $Re A_l$. Putting aside the elastic unitary approximation for the partial waves (2), note, that the only step necessary for turning the dispersion relation for partial wave A_l into equ ation, consists in the determination of its imaginary part on the unphysical cut. This imaginary part in our method is defined by using the crossing-symmetry for the forward (backward) scattering. So, in this method the approximations in the real part of amplitude are caused only by approximations in the imaginary part on the unphysical cut.

In the most simple approximation (6), this fact is masked by $Re A_l$ having the same crossing-symmetry as $Im A_l$. But in the next approximation, where $Im A_l$ on the left-hand cut, is defined by taking the first derivatives (in the crossing-symmetry conditions), the picture is changing. Here $Im A_l$ have a local crossing-symmetry of the type (1) (see formula (2.9) of the paper /13/). But at the same time $Re A_l$ has no simple crossing-symmetry.

So, the applicability range of our equations is not limited by any reasons connected with the small Lehmann ellipsis. This fact can be established also in the differential method frame. The expressions for partial waves in terms of the amplitudes and their derivatives values at $\cos \theta = \pm 1$ can be obtained /13/ in this method by integration by parts of the usual definition of partial wave. The restriction by the first few terms is caused by a smallness of the residual term (following from the smallness of the higher partial waves), but not due to the reasons of convergency of the series in a whole.

Note in conclusion that the method used does not allow to obtain the higher partial waves; the spectral functions to be introduced to calculate them consistently. Therefore, the next step to accurate out scheme is to take into account the spectral functions. It can allow to estimate the degree of accuracy of our equations as well.

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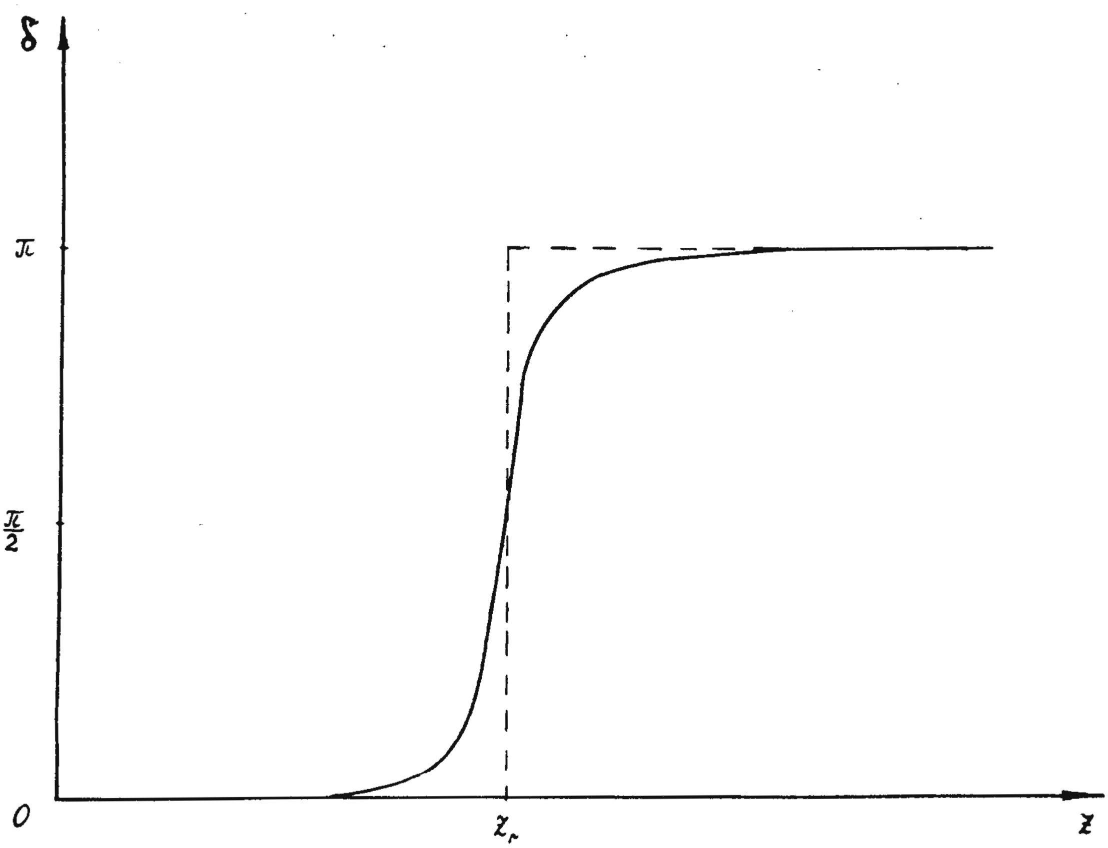

Fig. 1

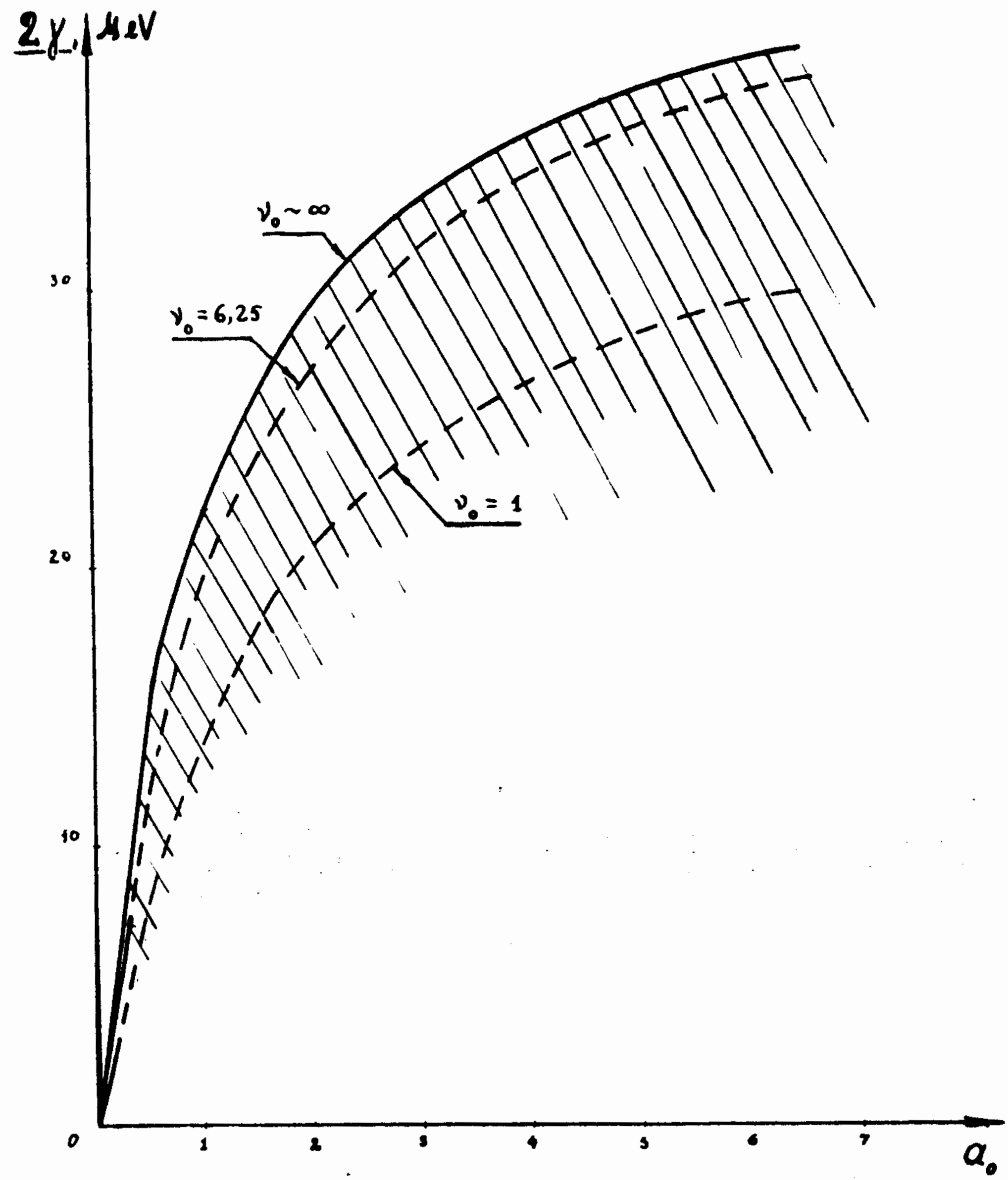

Fig. 2

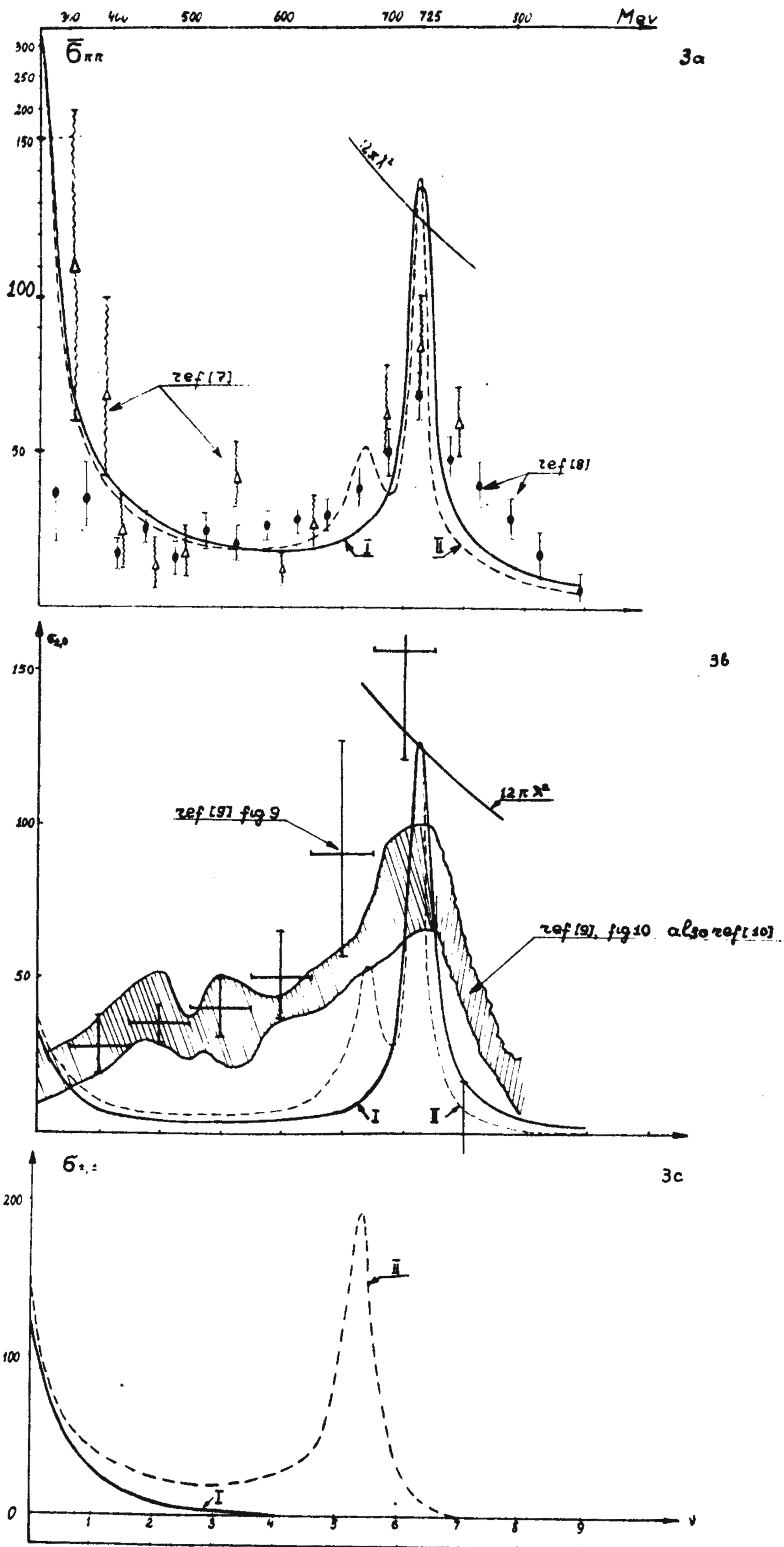

Fig. 3