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REGGE-POLES AND ELASTIC SCATTERING AT HIGH ENERGIES

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Abstract

Some consequences of an asymptote of the Regge-type on high energy elastic scattering are investigated. Relations between total cross sections of various processes are obtained. It is shown that the imaginary part of the vacuum pole is a positive definite function. An approximate analytic expression is derived for the vacuum trajectory.

I. Introduction

According to Chew and Frautschi the asymptotic form of scattering amplitudes of the Regge-type can be carried over to field theory[1/].

1/ G.F. Chew, S.C. Frautschi, Phys.Rev. Lett. 6, 580 (1960); 7, 394 (1961); G.F. Chew, S.C. Frautschi,(preprint UCRL –9510 (1960).; and Phys.Rev. 123 , 1478 (1961); G.F. Chew. Preprint UCRL–10058 (1962) (with a detailed bibliography). For a futher discussion see V.N. Gribov, ЖЭТФ 41, 667(1961). V.N. Gribov, ЖЭТФ 41, 1962(1961). R. Blankenbecler and M.L. Goldberger, preprint (1961). C. Lovelace, preprint (1961). S.C. Frautschi, M.Gell-Mann, F. Zachariasen, preprint (1961). G. Domokos. Nuovo Cim. 22, 1175 (1962).

Further, it has been pointed out that this solution leads to some interesting consequences concerning cross sections of various processes and that the Regge exponent can be determined from a suitable analysis of experimental data [2/]

2/ G. Domokos. ОИЯИ, preprint D–922 (1962).

In this paper we give a derivation of the results indicated in ref. [2/] and construct an approximate analytic expression for the Regge- exponent, responsible for high energy diffraction scattering(the ' vacuum- pole'). Use is made of the results of Gribov [3/] whose formulation of the unitarity condition in terms of the generalized partial wave

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3 V.N. Gribov. ЖЭТФ, 41, 1962(1961).

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amplitudes proved to be easier to handle than that given in ref. [1/]. After this investigation has been completed, we have learned that Gell-Mann., Gribov and Pomerancuk [4/] indeoendently arrived at similar results especially

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4/ M. Gell-Mann. Phys.Rev.Lett. 8, 263 1962.

5/ V.N. Gribov, I.Ja. Pomeranchuk. Preprint ИТЭФ , N. 42 and 43 (1962).

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concerning the relations between various cross sections.

2. 'Universality' of the Vacuum- Pole

Consider two kinds of spinless particles, 'pions' and 'nucleons'. (The generalization of the following considerations to the case of particles with spin is straightforward, e.g. by using the helicity amplitudes of Jacob and Wick[6/].

6/ M. Jacob and G.C. Wick. Ann. of Physics. 7, 404 (1959). The conjecture made in ref.[/2/] that the generalization to particles with non zero spin would require additional assumptions, proved to be incorrect.

Pion-pion, pion-nucleon and nucleon-nucleon scattering in the two-pion approximation in the t -channel are shown in Fig. 1. Let $F_\ell(t)$, $G_\ell(t)$, $H_\ell(t)$ be the corresponding partial wave amplitudes (in general defined for complex ℓ) of the crossed processes, the square of the total energy in the c.m.s. being t . With the help of the generalized unitarity condition one can continue F , G , H through the elastic cut to another Riemann sheet as a function of t In fact, we have:

$$F_\ell^{II}(z) = F_\ell(z^*) = F_{\ell^*}(z)^*$$

thus unitarity can be written as:

$$F_\ell(z) - F_\ell^{II}(z) = 2i\rho\, F_\ell(z)\, F_\ell^{II}(z)$$

(analogously for G_ℓ and H_ℓ , see. Fig. 1).

Hence:

$$F_\ell^{II} = \frac{F_\ell}{1 + 2i\rho F_\ell} \qquad (2.1)$$

$$G_\ell^{II} = \frac{G_\ell}{1 + 2i\rho F_\ell} \qquad (2.2)$$

$$H_\ell^{II} = H_\ell - \frac{2i\rho\, G_\ell^2}{1 + 2i\rho F_\ell} \qquad (2.3)$$

(ρ is the two-pion phase-space factor: $\rho = \left(\frac{t-4}{t}\right)^{1/2}$ defined as an antihermitian function: $\rho^*(t^*) = -\rho(t)$ cf. Oehme [7/]).

[7/] R. Oehme. Hercegnovi lectures, 1961. (Preprint EFINS-61-55).

Poles in (2.1) to (2.3) arise, when

$$S_\ell = 1 + 2i\rho F_\ell = 0\,. \qquad (2.4)$$

The solution of (2.4), written as $L = L(t)$, gives the corresponding Regge- trajectory.

It is easily seen that (2.4) is equivalent to condition (4') of ref. [1/], on writting $F_\ell = [\rho\,(ctg\,\delta_\ell - i)]^{-1}$ and remembering that $ctg\,\delta_\ell$ is a Hermitian function of ℓ . One sees that through unitarity, the same pole appears in all the three amplitudes. In particular, taking the $I = 0$ state in the t -channel, one must find - among others - the vacuum- trajectory, thus concluding that the asymptote of all the three processes is determined by the same pole [1/]. The meromorphic part of the amplitudes can be found by differentiating (2.4) in the neighbourhood of its root:

$$1 + 2i\rho F_\ell = 2i\rho \left[\frac{\partial F_\ell}{\partial \ell}\right]_{\ell = L} (\ell - L) + \ldots \qquad (2.5)$$

Thus:

$$F_\ell^{II} = \frac{F_L}{2i\rho \left[\frac{\partial F_\ell}{\partial \ell}\right]_{\ell = L}} \cdot \frac{1}{\ell - L} + \ldots \qquad (2.6)$$

$$G_\ell^{II} = \frac{G_L}{2i\rho \left[\frac{\partial F_\ell}{\partial \ell}\right]_{\ell = L}} \cdot \frac{1}{\ell - L} + \ldots \qquad (2.7)$$

$$H_\ell^{II} = \frac{-G_L^2}{\left[\frac{\partial F_\ell}{\partial \ell}\right]_{\ell = L}} \cdot \frac{1}{\ell - L} + \ldots \qquad (2.8)$$

Denote the residues in (2.6) to (2.8) by f , g , h , respectively, then one obtains the following relation between them:

$$\frac{f}{g} = -2i\rho F_L \frac{g}{h}$$

or, in view of (2. 4):

$$g^2 = fh. \qquad (2.9)$$

Now, on the basis of the results of refs. [1,3] and formulas (2.6) to (2.8) it is clear that (2.9) can be continued out from the region $4 < t < \infty$. In particular, let us go back to the first sheet, insert (2.6) to (2.8) into a Watson-Sommerfeld integral and let $s \to \infty$, $t \to 0$. If $L(0) = 1$,then by the optical theorem $f(0)$, $g(0)$, $h(0)$ are proportional to the asymptotic values of the $\pi\pi$, πN , NN total cross sections, respectively.

Thus, it follows from (2.9) that for $s \to \infty$

$$\sigma_{\pi N}^2 = \sigma_{\pi\pi}\sigma_{NN}. \qquad (2.10)$$

If one includes the scattering of other particles (e.g. K , Y) into the scheme, the above considerations can be repeated without any essential change. Instead of reproducing the argument once more, we simply list some relations, analogous to (2.10)

$$\sigma_{KN}\sigma_{KY} = \sigma_{K\pi}\sigma_{\pi Y}$$

$$\sigma_{YN}^2 = \sigma_{YY}\sigma_{NN} \qquad (2.11)$$

$$\sigma_{\pi N}\sigma_{YY} = \sigma_{\pi Y}\sigma_{YN}.$$

It is evident from the foregoing considerations that the same formulae hold for the imaginary parts of the amplitudes for $t \neq 0$ as well [2]. It follows from eq. (2.4) (or the equivalent eq. (4') of ref [1]) that $Im L \neq 0$ for $t > 4$. In other words, the Regge- trajectory cannot come out to the first sheet in the physical region of the t -channel.

Now, it is easy to see that one must have $Im L > 0$. In fact, $L(t)$ possesses a spectral representation [1,5] with at most one substraction, e.g.

$$L(t) = L(0) + \frac{t}{\pi}\int_4^\infty \frac{dt' \, Im L(t')}{t'(t'-t)}. \qquad (2.12)$$

Therefore, for $t < 4$

$$sgn \frac{d^n L}{dt^n} = sgn\, Im L \qquad (n = 1, 2, \ldots). \qquad (2.13)$$

If $sgn\, Im L = -1$ and $L(0)=1$ (constant cross section at infinity) then for $t < 0$ the amplitude violates the Froissart - condition [8]; hence $Im L > 0$, in complete analogy with potential scattering [9].

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[8] M. Froissart. Phys.Rev. 123, 1053 (1961).

[9] T. Regge. Nuovo Cim. 18, 947 (1960).

We conclude this section by remarking that arguments, like those, leading to our formulae (2.6) to (2.8) are familiar in the theory of multichannel reactions (cf, e.g. ref[7]). Actually it was this analogy which led us in deriving the factorization property of the residues. The poles, we find, correspond exactly to the well-known resonance-poles, with the only difference that now we are working on the section of the four- dimensional complex (ℓ , t) manifold: (ℓ -complex, t -real), while the usual resonance theory operates with another one (ℓ real- and integer - , t -complex).

3. Approximate Determination of the Vacuum Trajectory[10]

The properties of the vacuum trajectory, exposed in the previous section, together with some other simple properties

[10] The ideas, which the present section is based upon, have been briefly described in a previous paper (G. Domokos, preprint ЖЭТФ E-961 (1962) and ДАН СССР (to be published). However, the concrete solution, we obtained there there, violates Froissar t's condition[8].

allow one to obtain an approximate analytic expression for $L(t)$. First of all, we remark that for $t > 4$ we must have $Re\,L < 2$. In fact, it follows from the results of ref.[8] that one can make at most two substractions in S in the double spectral integral of the pion-pion amplitude. If the amplitude is to have an asymptote of the form $f(t)\,S^{L(t)}$ then we have in the asymptotic region:

$$\frac{S^N t^M}{\pi^2} \int_{s_0}^{\infty}\int_{4}^{\infty} \frac{ds'\,dt'\,\rho(s',t')}{s'^N t'^M (s'-s)(t'-t)} \approx$$

$$\approx \frac{S\;\;t}{\pi^2} \int_{s_0}^{\infty}\int_{4}^{\infty} \frac{ds'\,dt'}{s'^N t'^M (s'-s)(t'-t)}\; Im\,(f(t')\,s'^{L(t')})$$

$$(s_0 >> 4, \quad 0 \le M \le 2, \quad 0 \le N \le 2).$$

The integral over s' converges if $\max Re\,L < N$ which proves our statement. It follows further, with the help of (2.10) that $Re\,L < 2$, everywhere. Second, it follows from unitarity (cf. e.g. ref.[5]) that for $t \to 4 + 0$

$$Im\,L = 0((t-4)\ \ )$$

with $\Lambda = L(4) + ½$. Third, Suranyi has shown[11] that at least in the two-particle approximation in the t -channel,

[11] P. Suranyi, private communication.

$$\lim_{t \to \infty} L(t) = -1. \qquad (3.3)$$

Instead of reproducing Suranyi's arguments here, we remark that (3.3) has a simple physical interpretation. It has been shown in ref./5/ that for scattering on a potential, having a behaviour $\approx r^{x}$ for small r, $L(t)$ tends to $-3/2 + x/2$ as t tends to infinity. Now, (3.3) corresponds to $x = 1$ indicating that small distances don't play an important role - a result, not unexpected in the two- meson approximation.

Actually, it follows from (3.3) that $Im L \to 0$ at $t \to \infty$, for if one had $Im L \to$ const, ReL would go (logarithmically) to minus infinity. (A behaviour $Im L \to \infty$ is not allowed in view of eq. (2.12)).

We now made the Ansatz:

$$Im L(t) = (t-4)^{\Lambda} \sum_{n=N_0}^{N} c_n t^{-n} \qquad (t > 4) \tag{3.4}$$

According to the previous discussion, $N_0 > \Lambda$, further, obviously, $\Lambda > 3/2$ (as, according to the results of Sec. 2, $L(t)$ monotonously increases in $0 < t < 4$ and $L(0) = 1$). We choose therefore $N_0 = 2$ and we shall see that this gives a self- consistent result. We take one more coefficient ($N = 3$). Inserting (3.4) into (2.12), the integration can be carried out e.g. by applying Feynman's trick. We now arrive at an expression with three unknown parameters (c_2, c_3, Λ); c_2 and c_3 can be fixed by imposing the condition

$L(\infty) = -1$, $L(4) = \Lambda - ½$, (3.5) giving then finally:

$$L(t) = 1 + \left\{ \frac{\Lambda^2 + \frac{1}{2}\Lambda - 6}{C_2^{\Lambda}} \left[\frac{1-(-\nu)^{\Lambda}}{(\nu+1)^2} - C_1^{\Lambda}\frac{1}{\nu+1} + C_2^{\Lambda} \right] + \frac{\Lambda^2 + \frac{1}{2}\Lambda - 4}{C_3^{\Lambda}} \left[\frac{1-(-\nu)^{\Lambda}}{(\nu+1)^3} - C_1^{\Lambda}\frac{1}{(\nu+1)^2} + C_2^{\Lambda}\frac{1}{\nu+1} - C_3^{\Lambda} \right] \right\} \tag{3.6}$$

Here $\nu = ¼(t-4)$ and the C_K^{Λ} are binomial coefficients: $C_k^{\Lambda} = \frac{\Gamma(\Lambda+1)}{k!\,\Gamma(\Lambda - k + 10)}$.

We want to determine Λ by requiring that the trajectory should go as high as possible, without violating condition (3.1). (This corresponds to the 'principle of maximal strength of the interaction' - often discussed by Chew). If one now begins to vary Λ, one finds that at $\Lambda = \Lambda_{crit} \approx 1{,}65$ the derivative of $Re L$ at $t = 4$ changes sign, going over to negative values. Physically this phenomenon can be understood as follows. Increasing Λ means increasing the centrifugal barrier; now the higher the barrier the smaller is the 'effective well depth' arising from the attractive force plus centrifugal repulsion). At a certain height of the barrier the well becomes so 'shallow' that no bound state with zero energy can be formed. This critical height corresponds to our value of Λ_{crit} the 'attractive potential' being fixed by the spectral representation and by (3.4),(3.5). (Actually, the situation is a bit more complicated, as the attractive force itself depends on the parameter Λ).

The 'optimal' value of Λ (in the sense qualified above) is $\Lambda_0 \approx 1.58$. The corresponding curves $L_1 \equiv Re L(t)$ and $L_2 \equiv Im L(t)$ are plotted in Fig. 2 a,b.

4. Discussion and Conclusions

Summarizing our results, we see that some general requirements (constancy of total cross sections, behaviour of the vacuum trajectory at infinity etc) together with properties established on the basis of unitarity and Mandelstam representation (like the analytic properties of L) allow one to predict some features of high energy elastic scattering.

However, as soon as one is going to extract numerical results from the theory, these principles prove to be insufficient in each concrete case and must be supplemented by some additional information, like the 'principle of maximal strength' or similar. Whether this remains the case even when one manages to deal with many- particle amplitudes as well and if so, how can one- if at all - formulate this principle in sufficient generality, is an open question as yet.

Instead of dwelling on this subject further, we want to make some comments on the results obtained in the previous two sections.

One of the remarkable features of an asymptote of the Regge-type is that the real part of the amplitude does not vanish except forward scattering. In fact, an elementary calculation shows that asymptotically the real part is $ctg\frac{\pi L}{2}$ times the imaginary one. For forward scattering $(L(0) = 1)$ this factor vanishes, showing that the real part is of lower order in S than the imaginary one[12/], but in any other direction it gives a non-vanishing contribution.

12/ Cf. e.g. G. Domokos. Acta Phys. Hung.13, 89 (1961).

A second comment concerns the solution for the vacuum - trajectory obtained in Sec. 3.

It is obvious that for $t < 0$ the curve must somewhere intersect the t axis (in our case this occours at $t < -100$).

If the residue of the pole not vanished there, this would correspond to an S -wave bound state with imaginary mass and to a violation of unitarity. One would be inclined to say that this phenomenon is due to the two-pion approximation only and including many-particle-states it would disappear. However, experimental data seem to indicate that such an intersection really occours[13/].

13/ S.C. Frautschi, M. Gell-Mann, F. Zachariasen. Preprint 196? .

If this fact proves to be true, then the only way out is the vanishing of the residue of the pole. The residue must have at least a second order root (in view of the factor $ctg\frac{L\pi}{2}$), therefore in the neighbourhood of the root of L , the amplitude must be pure real.

Finally we want to remark that the procedure by which we obtained the vaccum trajectory, obviously can be extended to obtain others as well. If one e.g. assumes that some - or all - elementary particles are but manifestations of Regge-trajectories then an approximate expression for the corresponding trajectory can be constructed along the same lines as described in Sec. 3.

In conclusion the author would like to express his sincere thanks to Academician N.N. Bogolubov, Prof. A.A.Logunov and Mr. P. Suranyi for several worthful discussions and critical remarks.

Concerning the results, exposed in Sec. 3, the author gratefully acknowledges the benefit of a pleasant discussion with Dr. V.N. Gribov and Prof. I.Ja. Pomeranchuk.

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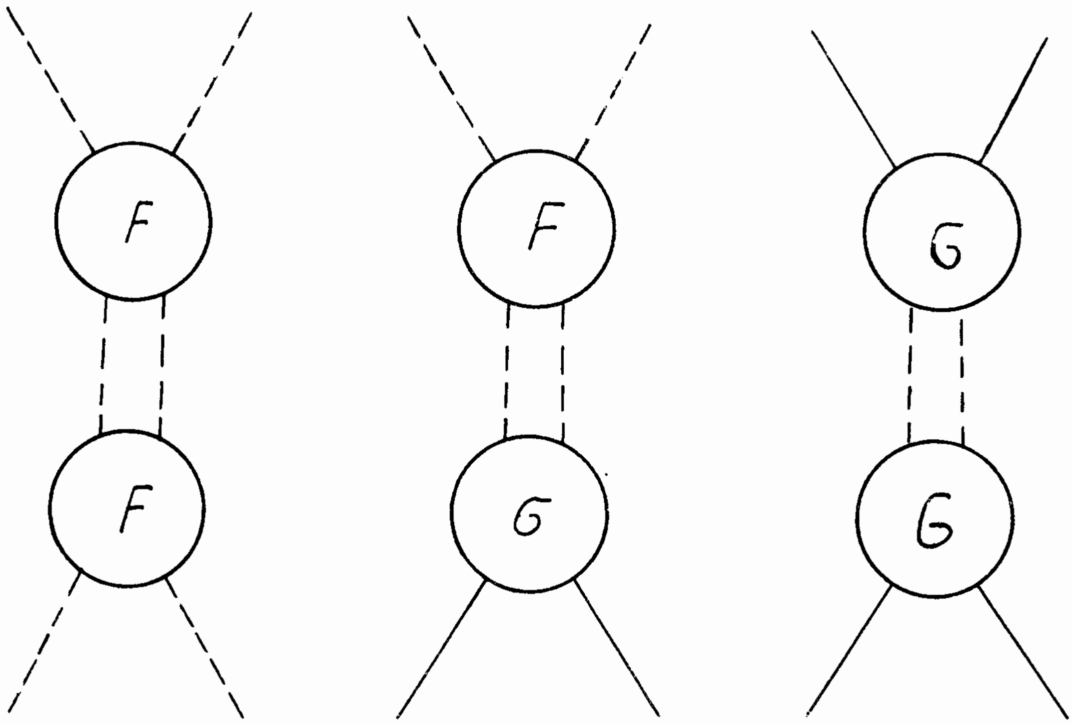
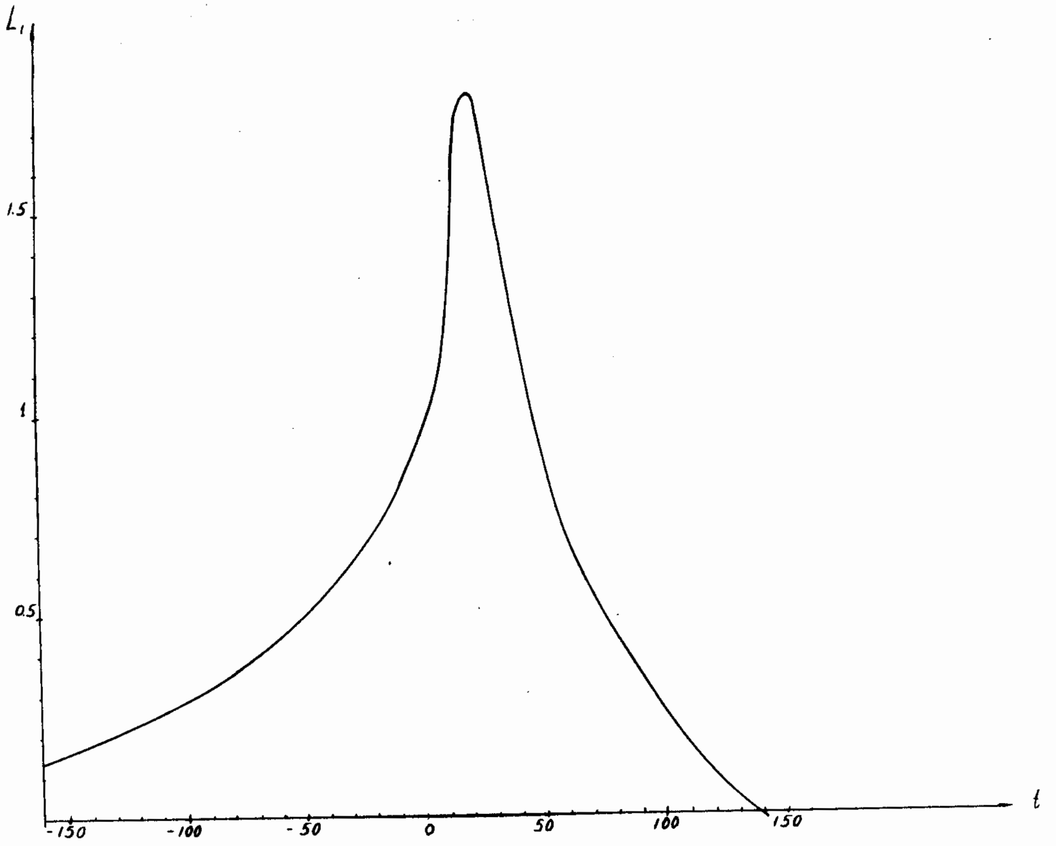
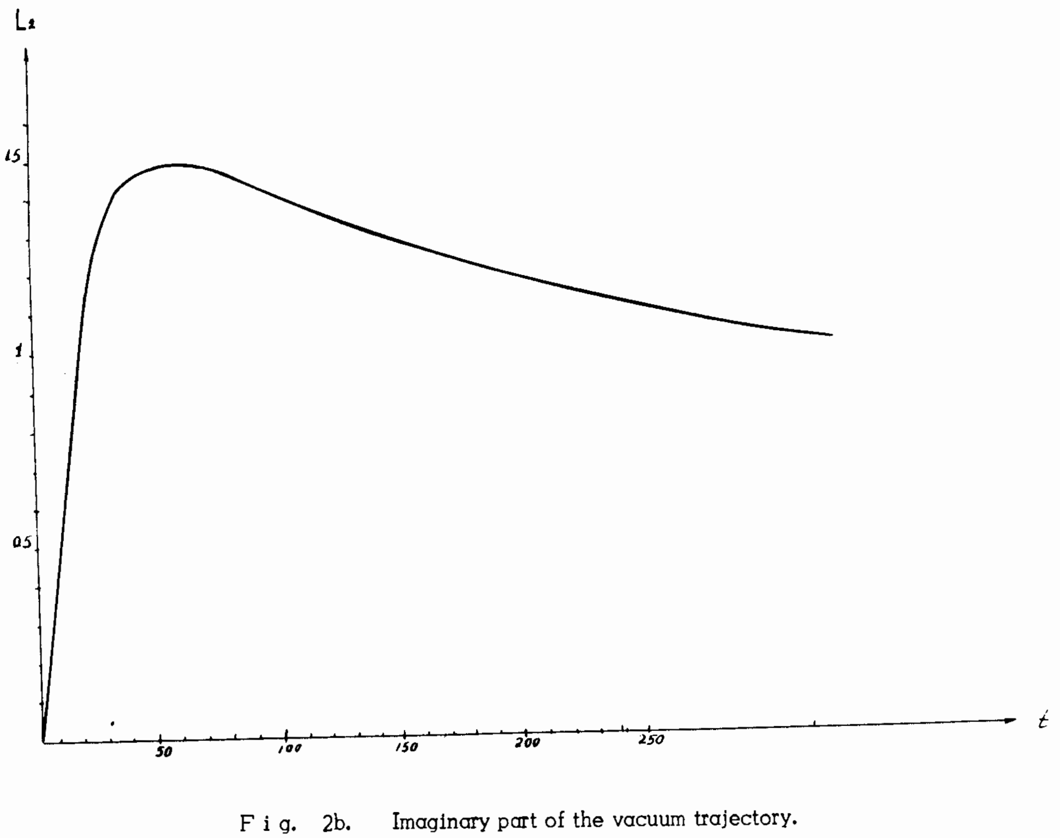


F i g. 1. Generalized diagrams for unitarity condition.



F i g. 2a. Real part of the vacuum trajectory.



F i g. 2b. Imaginary part of the vacuum trajectory.