ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ANALYTIC PROPERTIES OF THE ELASTIC $\pi\pi$ SCATTERING AMPLITUDE IN THE ℓ PLANE

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1. Recently it has been shown that if the partial wave amplitudes of elastic $\pi\pi$ scattering were meromorphic functions of ℓ, then the results of Regge[1] obtained in the framework of quantum mechanics can be carried over to field theory, without any appreciable difficulty[2]. In the present note we investigate the aforementioned analytic properties.

2. Let us write down for the amplitude integral equations of the Chew-Mandelstam type[3]. Put: $A^I_\ell(\nu) = D^I_\ell(\nu)^{-1} N_\ell(\nu)$, then D^I_ℓ obeys the following equation:

$$D^I_\ell(\omega) = 1 + 1/\pi \int_1^\infty d\omega' K(\omega,\omega') f^I_\ell(\omega') D_\ell(\omega') \tag{1}$$

and

$$N^I_\ell(\nu) = -1/\pi \int_1^\infty \frac{d\omega'}{\omega'+\nu} f^I_\ell(\omega') D^I_\ell(\omega') \tag{2}$$

where $\omega = -\nu$, $K(\omega,\omega') = \frac{2}{\omega'-\omega}\left[\left(\frac{\omega'}{\omega'-1}\right)^{1/2} Q_0\left(\left(\frac{\omega'}{\omega'-1}\right)^{-1/2}\right) - \right.$

$$\left. -\left(\frac{\omega}{\omega-1}\right)^{1/2} Q_0\left(\left(\frac{\omega}{\omega-1}\right)^{1/2}\right)\right]$$

$$f^I_\ell(\nu) = 1/\nu \sum_{I'} a_{II'} \int_0^{-\nu-1} A^{I'}_s\left(\nu', 1+2\frac{\nu+1}{\nu'}\right) P_\ell\left(1+2\frac{\nu'+1}{\nu}\right) d\nu' \tag{3}$$

(we follow the notation of ref.[3]).

The system of equations (1)-(3) can be solved by iteration.

Let us choose some zeroth approximation for A^I_s such that $\lim_{\nu\to\infty} f^I_\ell(\nu) = 0$. (This is necessary for the self-consistency of the system[4]). Then it is easy to see that eq. (1) is of the Fredholm type. Obviously, f^I_ℓ is an entire function of ℓ (because P_ℓ is entire in ℓ), and therefore, D^I_ℓ and N^I_ℓ, consequently, A^I_ℓ, meromorphic in ℓ. (This can be seen by writing down the Fredholm solution of (1)).

3. In order to calculate f^I_ℓ in the second approximation let us

represent the first approximation of $Im\ A_\ell^I$ by a Watson-Sommerfeld integral. (This is possible in view of the properties established in 2)).

$$f_\ell^I(\nu) = \frac{1}{4\pi\nu i}\sum_{I'} a_{II'}\int_0^{-\nu-1} d\nu'\, P_\ell\left(1+2\frac{\nu'+1}{\nu}\right)\left\{\int_{-i\infty}^{i\infty}\frac{\lambda' d\lambda'}{\cos\lambda'\pi}\, Im\, A_{\lambda'}^{I'}(\nu')\times\right.$$

$$\left.\times P_{\lambda'-1/2}\left(-1-2\frac{\nu+1}{\nu'}\right)\left(1+(-1)^{I'+1}\sin\pi\lambda'\right)+\Sigma\right\}. \qquad (3)$$

Here $\lambda' = \ell' + 1/2$ and Σ stands for the contribution of poles in the λ' - plane. The integral in λ' exists at least as a principal value one. The integral equation for the second approximation of D_ℓ^I is of the Fredholm type, therefore D_ℓ^I in the second approximation is meromorphic in ℓ.

4. By means of the procedure sketched above, one can show that in every step of the iteration procedure, A_ℓ^I is meromorphic in ℓ, and that the poles in the ℓ plane are determined by the roots of D_ℓ^I. From this result it does not follow immediately that - provided the iteration converges - the amplitude itself is meromorphic.

One can show, however, that if $|A_\ell^I(\eta\nu)|$ for $\nu > 0$ converges uniformly in ℓ, then the amplitude is meromorphic in ℓ.

Knowing the analytic properties of A_ℓ^I, one can choose the 0^{th} approximation in the most convenient way, e.g. choosing a Regge-pole. This choice corresponds to the suggestion of Wong[5].

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