ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ANGULAR CORRELATIONS OF LEPTONS IN THE K-MESON DECAYS

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Abstract

The angular correlation of the leptons $e\nu$ and $\mu\nu$ have been calculated in the K_{e3} and $K_{\mu 3}$ -decays. It is convenient to compare these correlations with the experimental data if the probability of detection of an electron (or a μ -meson) depends on its energy. The available experimental data agree well with a vector coupling and exclude scalar and tensor ones.

Вычислены угловые корреляции лептонов $e\nu$ и $\mu\nu$ в K_{e3} и $K_{\mu 3}$ –распадах. Эти корреляции удобно сравнивать с экспериментальными данными, если вероятность регистрации электрона /или μ –мезона/ зависит от его энергии. Существующие экспериментальные данные хорошо согласуются с векторным вариантом распадного взаимодействия и исключают скалярный и тензорный.

A large number of papers has been devoted to the theoretical analysis of the K_{e3} - and $K_{\mu 3}$ -decays. A detailed review of these works is given by Okun [1]. We note the paper by Pais and Treiman [2] in which the angular correlations of π -mesons and electrons (μ -mesons) have been calculated at a fixed energy of π -meson. The comparison of these correlations with the experimental data is independent of the assumptions about the form factors since the latter depend only on the π -meson energy in the rest system of K -mesons. However, such a comparison requires a sufficiently large statistics, and, besides, in a real experiment it is difficult to provide the identical probability of detecting electrons with different energies since only electrons possessing sufficiently low energies (in the Lab.sys.) can be identified. Thus in the recent paper [3] where a sufficiently number of events of the K_{e3} -decays has first been obtained only electrons with the energy $E_{e\,lab.} \leq$ 200 MeV were identified. This lead to a strong distortion of the spectrum of electrons so as it is impossible to analyse the experimental data without introducing corrections for the efficiency of the electron detection. Under such conditions it is found to be convenient to compare the experimental data with the angular correlations of an electron (μ -meson) and a neutrino since in this case we can analyse experimental data without introducing corrections for the abovementioned distortions. The present paper is just devoted to this problem.

If the polarization properties of particles are not taken into consideration then the probability dW for the K_{e3} decay ($K \to \pi + e + \nu$) is a function of two variables. As these variables we choose the electron energy E_e and $\cos\theta$ where θ is the angle between the directions of emission of the lepton possessing a mass (for the sake of brevity we shall speak about electron) and the neutrino. All the quantities are referred to the rest system of K -meson. With the accuracy up to the constant multipliers

$$dW(E_e, \cos\theta) = \overline{|m|^2}\, f(E_e, \cos\theta)\, dE_e\, d\cos\theta$$

where

$$f = \frac{p_e E_e E_\nu^2}{M - E_e + p_e\cos\theta}, \quad p_e = \sqrt{E_e^2 - m_e^2}, \quad E_\nu = \frac{M(W_e - E_e)}{M - E_e + p_e\cos\theta}$$

$$W_e = \frac{M^2 - m_\pi^2 + m_e^2}{2M}$$ is the maximal electron energy.

M, m_π, m_e - are the masses of K -meson, π -meson and electron respectively. An allowed region of change of the chosen variables is the rectangle $m_e \leq E_e \leq W_e$, $-1 < \cos\theta \leq 1$. $\overline{|m|^2}$ is the squared modulus of the matrix-element which is averaged over the spin states. If the K -meson spin is zero, then the matrix element can be represented in the form

$$m = \frac{1}{\sqrt{2}}\Big[g_s \bar{u}_e (1+\gamma_5) v_\nu - \frac{i g_{V1}}{M} K_\alpha \bar{u}_e \gamma_\alpha (1+\gamma_5) v_\nu +$$

$$+ \frac{i g_{V2}}{M} (p_e + p_\nu)_\alpha \bar{u}_e \gamma_\alpha (1+\gamma_5) v_\nu + \frac{i g_T}{M^2} K_\alpha (p_e + p_\nu)_\beta \bar{u}_e \sigma_{\alpha\beta} (1+\gamma_5) v_\nu \Big]$$

$g_{s,V,T}$ are the functions of $E_\pi = M - E_e - E_\nu$. From the conservation of the combined parity it follows that they are real.

γ_α are Dirac matrices, $\sigma_{\alpha\beta} = \dfrac{\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha}{2i}$, $\gamma_\alpha^+ = \gamma_\alpha$

$k,\ p_\nu$ are the four-momenta of the K-meson and the neutrino.

$$\overline{|m|^2} = g_{vi}^2 (1 + \beta_e \cos\theta) + \tilde{g}_s^2 (1 - \beta_e \cos\theta)$$

$$+ \frac{g_T^2}{M^2} \{ (1 - \beta_e \cos\theta) [(E_\nu - E_e)^2 - m_e^2] + \frac{2m_e^2 E_\nu^2}{E_e} \} +$$

$$+ 2 \tilde{g}_s g_{vi} \frac{m_e}{E_e} + \frac{2 \tilde{g}_s g_T}{M} [(1 - \beta_e \cos\theta)(E_\nu - E_e) + \frac{m_e^2}{E_e}]$$

$$+ 2 g_{vi} g_T \frac{m_e}{M} [\frac{E_e}{E_\nu} + \beta_e \cos\theta].$$

Here $\beta_e = \dfrac{p_e}{E_e}$, $\tilde{g}_s = g_s - \dfrac{m_e}{M} g_{v2}$.

We shall compare with the experimental data not the quantity dW but $\dfrac{dW}{f\, dF_e} = \overline{|m|^2}\, d\cos\theta$ since it is this quantity that depends on the type of interaction.

Now we consider the K_{e3} -decay. Then

$$\overline{|m|^2} = g_{vi}^2 (1 + \cos\theta) + g_s^2 (1 - \cos\theta) + \frac{g_T^2}{M^2} (1 - \cos\theta)(E_\nu - E_e)^2 +$$

$$+ \frac{2 g_s g_T}{M} (1 - \cos\theta)(E_\nu - E_e).$$

We have neglected the terms $\dfrac{m_e}{M}$, $\dfrac{m_e}{E}$ and put $\beta_e = 1$. As it is seen from the given expression $\overline{|m|^2}$ depends quite simply on $\cos\theta$ and is independent of the electron energy for the scalar and vector iuteractions, if g_i is assumed to be constant. The correspouding curves are given in Fig. 1 . A characteristic feature of the curves of the tensor type of interaction is their vanishing for $\cos\theta = 1$.

From the fact that $\overline{|m|^2}$ is independent of E_e for the scalar and vector interactions, it follows that in order to make comparison with theoretical predictions we may sum up the experimental data referred to different electron energies, i.e. make use of an one-dimensional diagram instead of a two-dimensional one. It is also clear that the dependence of $\dfrac{dW}{f\, dE_e}$ on $\cos\theta$ remains unchanged if this expression is multiplied by a certain function $\phi(E_e)$ which characterizes the efficiency of the electron detection depeading on E_e. In fact the probability of the detection depends on $E_{e\,Lab.}$, but it is easily seen that practically this dependence can be formulated as a dependence on E_e. Indeed, $E_{e\,Lab.}$ is expressed as a function of E_e, the angle α (α is the angle between the direction of the electron emission and direction of flight of the K -meson in the K -meson rest system) and of the K -meson velocity v . But dW is independent neither of α nor of v . Therefore $\frac{dW}{fdE_e}\phi(E_e, \alpha, v)$ has the same dependence on $\cos\theta$ as $\frac{dW}{f\,dE_e}$ has. The account of the probability of detection of electron leads only to that the events with different electron energy appear with different statistical weights. This leads only to a change in the dispersion (see Appendix). The aforesaid is strictly applicable only to the scalar and vector interactions where g_s and g_v are constant. However, a weighted summation of the distributions with different E_e can not alter the qualitative behaviour of the curves for the tensor interaction too, namely the vanishing for $\cos\theta = +1$.

Fig. 2a gives a hystogram for $\overline{\frac{dW}{f\,dE_e}}$ *, plotted according to 142 events of the K_{e3} - decays, given in [3]. The hystogram is normalized to the same area as the curves of Fig. 1 . It is evident that the experimental data agree only with the vector interaction.

In order to make sure that in fact g depends weakly on E_π we put the distribution $\frac{dW}{fdE_e}$ separately for the cases E_e < 100 MeV and E_e > 100 MeV (62 and 80 events respectively). For a strong and monotonous dependence of g on E_π these distributions would be different since the change in the E_π -meson energy ΔE_π depends on E_e when the $\cos\theta$ changes from -1 up to 1. For $E_e <$ 100 MeV $\Delta E_\pi <$ 87 MeV, and for 100 MeV $< E_e <$ 215 MeV 87 MeV $< \Delta E_\pi <$ 130 MeV. From Fig. 2b and 2c it is seen that these distributions are practically not distinguished. Fig. 3 gives the values of the form factor g^2_{vi} depending on $\cos\theta$ averaged over E_e >100 MeV. For the sake of comparison the curves of the dependence of g^2_{vi} on $\cos\theta$ are plotted for E_e = 100 MeV, which are obtained from the theory with an intermediate boson [4], where $g_{vi} \sim \frac{1}{M_B^2 - M^2 - m_\pi^2 + 2ME_\pi}$, M_B is the boson mass. It is seen that the values of $M_B <$ 600 MeV are unlikely.

Up to this point of our presentation we considered the case when only one of the possible types of interaction takes place. If all the three types are present then

$$\frac{dW}{f\,dE_e} = g_v^2\,(1+\cos\theta) + [\,g_s + \frac{E_\nu - E_e}{M}\,g_T\,]^2\,(1-\cos\theta).$$

If g_i 's are constant then from the comparison with the experimental curve it follows that $\overline{g_{vi}} \neq 0$,

$$\overline{[\,g_s + \frac{E_\nu - E_e}{M}\,g_T\,]^2} \approx 0 \;.$$

. And, consequently $\overline{|\,g_s + \frac{E_\nu - E_e}{M}\,g_T\,|} \approx 0$

i.e. $g_s \approx 0$, $g_T \approx 0$.

In conclusion we note that the suggested method of treatment of experimental data may be applied to the μ - decay too, although $|m|^2$ depends in this case on E_e since $\beta_e \neq 1$:

$$\frac{dW}{f\,dE_e} = g_{vi}^2\,(1+\beta_e\cos\theta) + \frac{m_e}{M}\,g_{v2}^2\,(1-\beta_e\cos\theta) - 2\,g_{vi}\,g_{v2}\frac{m_e}{E_e} \;.$$

The distribution summed over E_e will be of the same form but with the average weighted quantities $\overline{\beta_e}$, $\overline{(\frac{m_e}{E_e})}$ if g_{vi}, g_{v2} are constant. After having plotted $\frac{dW}{f\,dE}(\cos\theta)$ we may choose one of the two possible solutions for the ratio $\frac{g_{v2}}{g_{vi}}$ which are got if we take into consideration that $dW_{e3} \approx dW_{\mu 3}$ (see [1]). Indeed, if $\frac{g_{v2}}{g_{v1}} \approx 15$ and E_e < 200 MeV, then $|m|^2$ does not depend practically on $\cos\theta$.

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* The line above denotes the averaging.

If $\frac{f_{\nu 2}}{f_{\nu 1}} = -0{,}5$ then the dependence of $\overline{|m|^2}$ on $\cos\theta$ is represented by the inclined line with the tangent of the angle formed by this line and the axis $\cos\theta$ being

$$\frac{0{,}9\,\bar{\beta}_e}{1{,}1 + \left(\frac{m_e}{E_e}\right)} \quad (> 0{,}4\,\bar{\beta}_e).$$

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Appendix

The Calculation of Statistical Errors

For each part of the hystogram we plot the quantity $\Sigma \frac{\Delta n_i}{f_i}$ corresponding to $\int \frac{dW}{f\,dE_e} dE_e\, d\cos\theta$, where the sum and the integral are extended to the chosen interval in which $\cos\theta$ and E_e change. Here Δn_i is the experimentally measured number of cases appearing in the given small region $\Delta E_e \Delta \cos\theta$. Since this region can be chosen sufficiently small, then

$\Delta n_i = 0$ or 1. Therefore $\Sigma \frac{\Delta n_i}{f_i} = \Sigma_{\text{over exp. points}} \frac{1}{f_j}$

To estimate the errors it is necessary to know the dispersion of the quantity $\Sigma \frac{\Delta n}{f}$, $D\left(\Sigma \frac{\Delta n}{f}\right)$. It can be obtained in the following manner.

$$\frac{\Delta n}{f} = \phi(\cos\theta) \sim \overline{|m|^2}$$

Let for the sake of simplicity $\phi(\cos\theta)$ be constant in the given interval of $\cos\theta$. Then the average number of particles $\overline{\Delta n}$ appearing in the small region $\Delta E_e \Delta \cos\theta$ is $C f \bar{N} \cdot \Delta E_e \Delta \cos\theta$, where $\bar{N}$ is the mean number of particles entering into the whole interval of $\cos\theta$ under consideration. $C = (\Sigma f \Delta E_e \Delta \cos\theta)^{-1}$ since $\overline{\Sigma \Delta n} = \bar{N}$. The quantity Δn is subjected to the Poissan's law, i.e. the dispersion $D(\Delta n) = \overline{\Delta n}$. Then $D\left(\Sigma \frac{\Delta n}{f}\right) = \Sigma \frac{\overline{\Delta n}}{f^2} =$

$$= \bar{N} \frac{\Sigma \frac{\Delta E_e \Delta \cos\theta}{f}}{\Sigma f \Delta E_e \Delta \cos\theta}$$

i.e.

$$D\left(\Sigma \frac{\Delta n}{f}\right) = \bar{N} \frac{\int \frac{1}{f}\, dE_e\, d\cos\theta}{\int f\, dE_e\, d\cos\theta}.$$

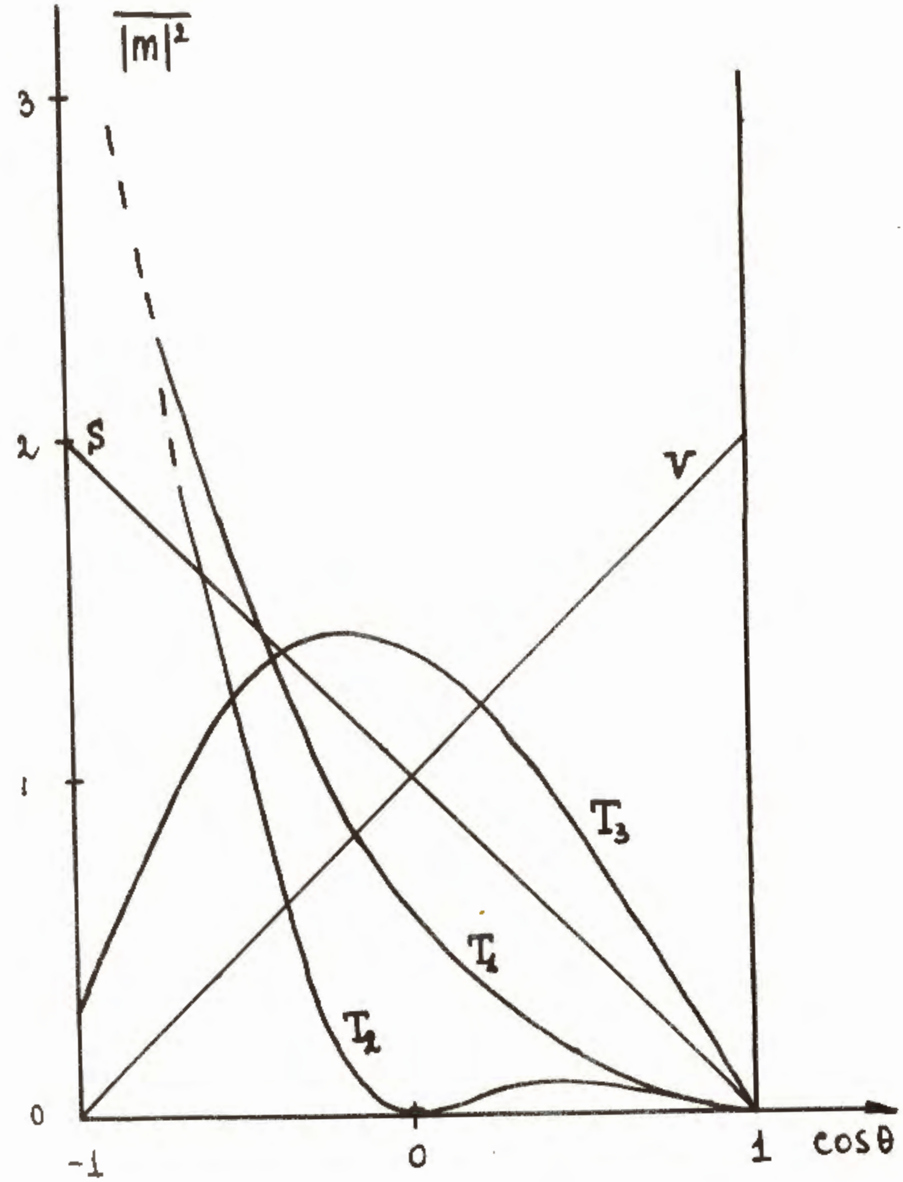
This formula has just been used to evaluate the errors plotted in the hystograms. In our case the quantity f has been normalized by the condition

$$\int \frac{dE_e\, d\cos\theta}{f} = 2$$

It is clear that the quantity $\bar{N}$ can be evaluated only approximately by replacing $\bar{N}$ the experimentally measured number of cases for the giving interval of $\cos\theta$

It is easy to obtain more exact formula taking into account the variation of :

$$D\left(\Sigma \frac{\Delta n}{f}\right) = \frac{\bar{N} \int \frac{\phi}{f}\, dE_e\, d\cos\theta}{\int \phi f\, dE_e\, d\cos\theta}.$$

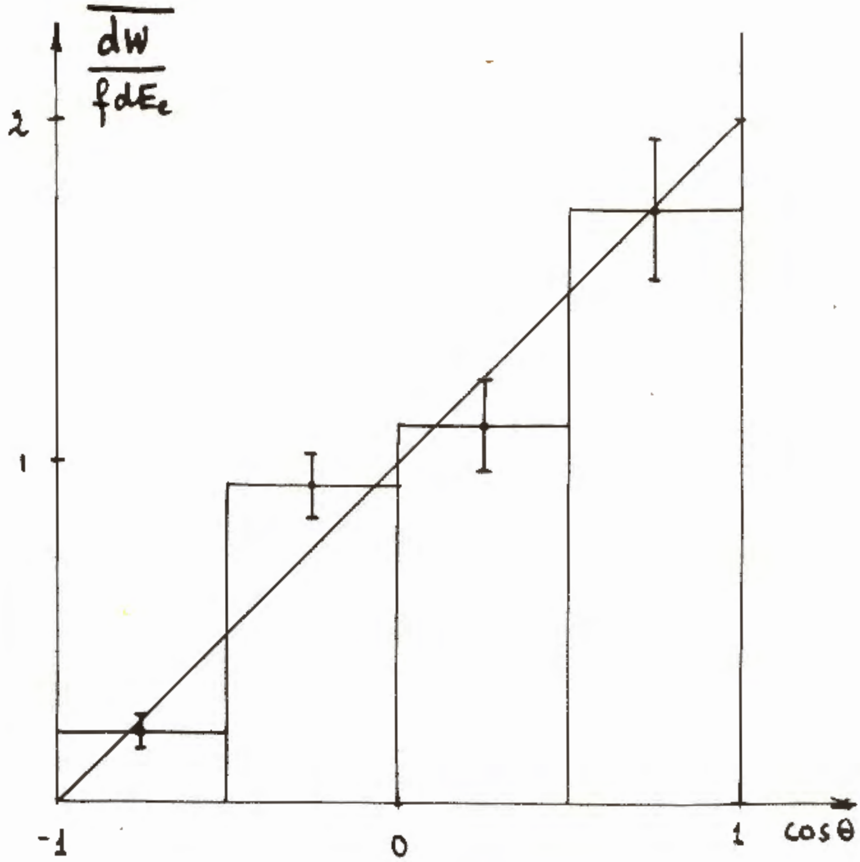


F i g. 1.

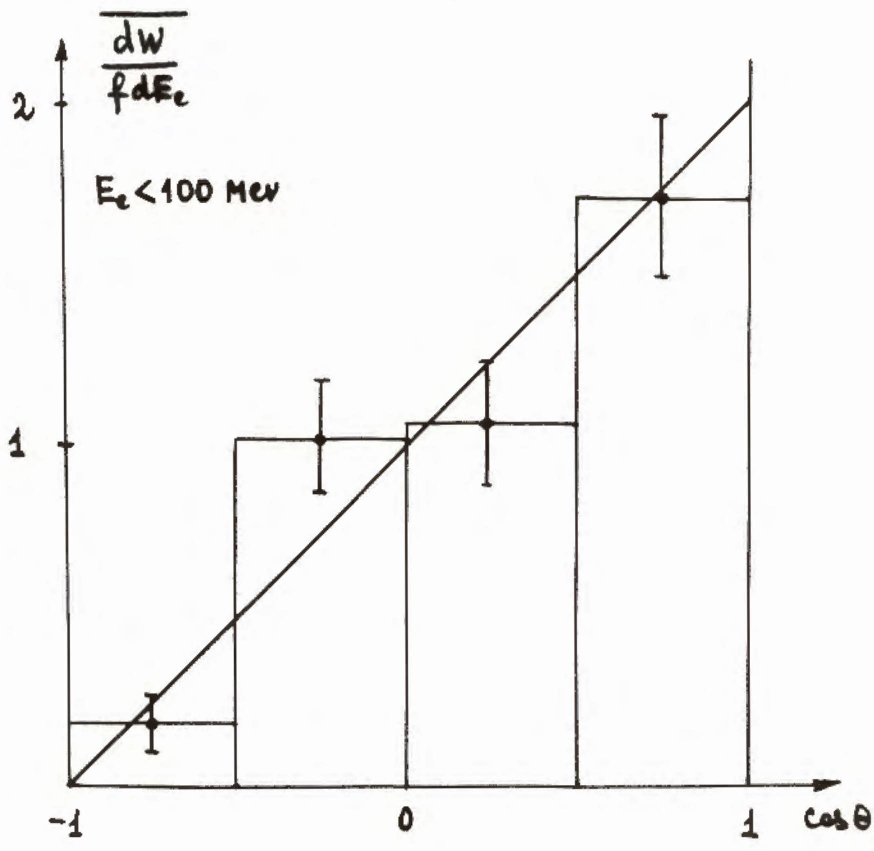
The dependence of $\overline{|m|^2}$ *on* $\cos\theta$

S, V correspond to vector and scalar interactions.

T_1, T_2, T_3 *correspond to the tensor type of interaction for* E_{e1} = 100 MeV, E_{e2} = 130 MeV, E_{e3} = 200 MeV.



F i g. 2a.



F i g. 2b.

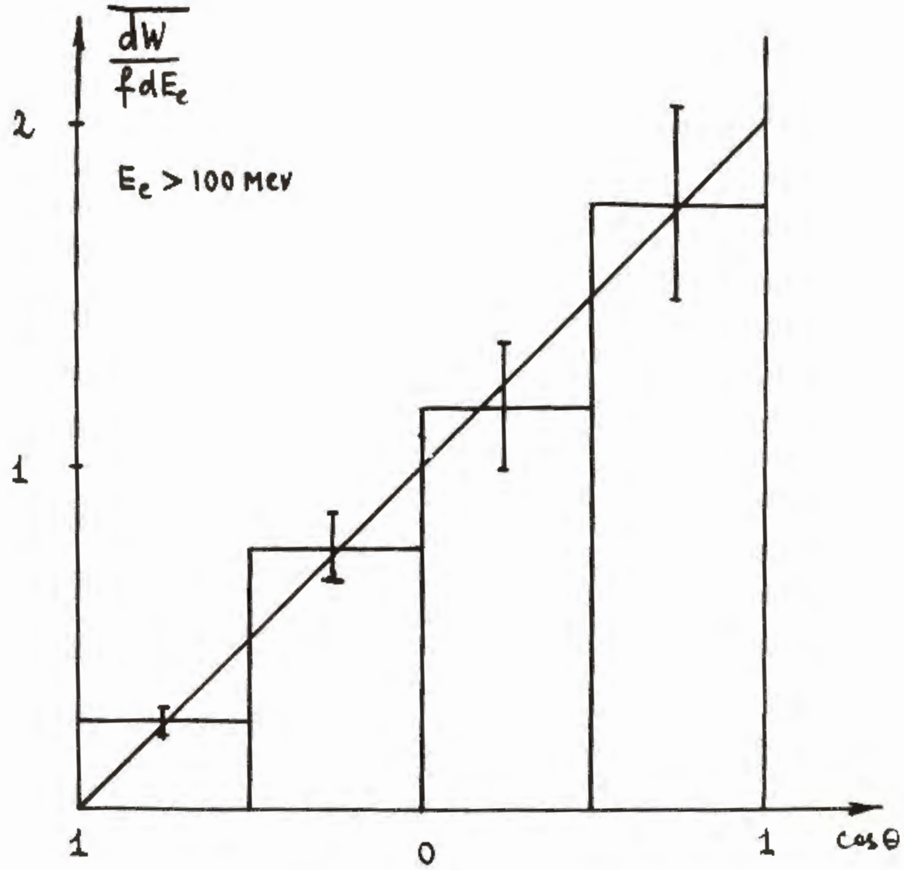

dW / f dE_e
2
E_e > 100 MeV
1
1
0
1
cosθ

Fig. 2c.

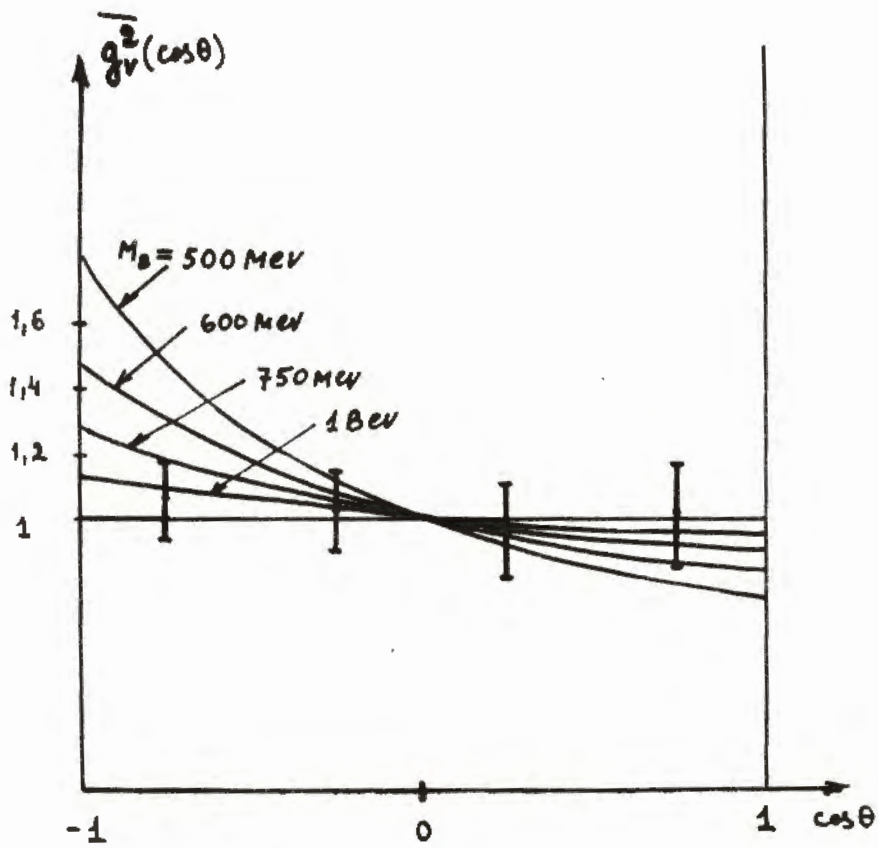

g_v^2(cosθ)
M_0 = 500 MeV
600 MeV
750 MeV
1 BeV
1,6
1,4
1,2
1
-1
0
1
cosθ

Fig. 3.

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