ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

V.G. Soloviev

E-945

EFFECT OF PAIRING CORRELATIONS OF THE SUPERCONDUCTIVE TYPE ON THE ALPHA DECAY RATES

Дубна 1962 год

V.G. Soloviev

E-945

EFFECT OF PAIRING CORRELATIONS OF THE SUPERCONDUCTIVE TYPE ON THE ALPHA DECAY RATES

As is known [1-3] the pairing correlations of nucleons of the superconductive type strongly affect the properties of the ground- and excited states of atomic nuclei. They play therefore a great role in the beta- and gamma transitions in nuclei. Undoubtedly, the pairing correlations must influence noticeably the rates of alpha decays. The detailed theoretical investigations of the alpha transition probabilities have been carried out by many authors, e.g. [4-6]. However, the influence of the superfluidity of the ground- and excited states on the alpha decay has not been taken into account there. In this paper we formulate the theory of the alpha decay within the framework of the superfluid model of the nucleus and investigate the influence of the pairing correlations of the superconductive type both on the absolute probabilities of alpha decays and, especially on the hindrance factors F.

The matrix element of the alpha decay of the parent nucleus with $\Psi = \Psi(N)\cdot\Psi(z)$ represented as the product of the wave functions of the neutron and proton systems to the daughter one with $\Psi' = \Psi(N-2)\,\Psi(z-2)$ is given by

$$M = \Psi^*(N-2)\,\Psi(z-2)\,A\,\Psi(z)\,\Psi(N). \tag{1}$$

The operator A describing the alpha particle emission is given by

$$A = \frac{1}{4}\sum_{\substack{\nu,\nu';\omega,\omega'\\ r,r';\sigma,\sigma'}} W_{rr';\sigma\sigma'}(p,\nu\nu'\mid n,\omega\omega')\, a_{\nu r}\, a_{\nu' r'}\, b_{\omega\sigma}\, b_{\omega'\sigma'} \tag{2}$$

where $a_{\nu r}$, $b_{\omega r}$ are the proton and neutron absorption operators, $r=\pm 1$, $\sigma=\pm 1$, the states differing only in the sign of r or σ being conjugated under the time reversal. The summation ν, ν (ω, ω') is carried out over the single-particle proton or neutron levels of the average field. The function W describes both the penetration of the alpha particle through the potential barrier and the probability of its formation.

We find the matrix element of the alpha decay of an even-even nucleus between the ground states. Making use of the wave functions [2]

$$\Psi = \prod_s \left(u_s + v_s\, a^+_{s_+}\, a^+_{s_-}\right)\Psi_0 ,$$

taking into account the pairing correlations of nucleons we get

$$M = \sum_{\nu,\omega} W_{+-;+-}(p\nu,\nu)\mid n\omega,\omega)\, u_\nu(z-2)\, v_\nu(z) \prod_{s\neq\nu}\left[u_s(z-2)\,u_s(z) + \right.$$
$$\left. + \; v_s(z-2)\, v_s(z)\right]\, u_\omega(N-2)\, v_\omega(N) \prod_{s'\neq\omega}\left[u_{s'}(N-2)\,u_{s'}(N) + v_{s'}(N-2)\,v_{s'}(N)\right] \tag{3}$$

where

$$u_s(z)^2 = 1/2\left\{1 + \frac{E(s)-\lambda(z)}{\sqrt{C^2(z) + [E(s)-\lambda(z)]^2}}\right\}, \quad v_s^2(z) = 1 - u_s^2(z),$$

$C(z)$ is the correlation function, $\lambda(z)$ is the chemical potential for the ground state of the system consisting of Z protons, $E(s)$ are the single-particle energy levels. When the pairing correlations are absent (3) takes the form :

$$M = W_{+-;+-}(p,\nu = K(z),\nu = K(z) \mid n,\omega = K(N),\ \omega = K(N)) \tag{4}$$

where we denote by $K(z)$ the last filled orbital of the system consisting of Z protons when the pairing correlations are absent. $K-1$ denotes the first hole orbital, $K+1$ denotes the first particle orbital etc. From (4) it is seen that if there are no pairing correlations then the alpha particle can be formed out only of two neutrons and two protons that occupy the last filled orbitals. Since the probability of formation of the alpha particle in the nucleus is proportional to the overlap integral of the corresponding wave functions then it must change essentially in the transition from one nucleus to another due to the change in the quantum numbers of the K - level what is not observed experimentally. The effect of the pairing correlations leads to the alpha particles being formed with a noticeable probability from pairs occupying many states both higher and lower than the K - state. This means that α -decay involves an averaging of participation of many nucleon levels near the Fermi surface. This leads one, first, to the increase of the alpha decay probability and, second, to smoothing out the fluctuations in the probability of the alpha particle formation in the transition from nucleus to nucleus.

In order to distinguish between the effects of the pairing correlations of nucleons and those of other factors in alpha decays the following approximation is considered. The diagonal part of W is independent both of the quantum numbers of protons and neutrons, i.e.

$$W_{+-;+-}(p,\nu,\nu\mid n,\omega,\omega) = W(p\mid n), \qquad (5)$$

$$W_{+-;\sigma_1,\sigma_2}(p,\nu,\nu\mid n,\omega_1\,\omega_2) = W_{\sigma_1\sigma_2}(p\mid n\,\omega_1,\omega_2) \qquad (5')$$

$$W_{r_1 r_2;+-}(p,\nu_1,\nu_2\mid n,\omega,\omega) = W_{r_1 r_2}(p,\nu_1,\nu_2\mid n) \qquad (5'')$$

In this approximation the alpha particle formation probabilities are identical in all states whose weights are determined by the functions u_s *and* v_s. Apparently, in evaluating the effect of the pairing correlations on the alpha decay such an average treatment is correct.

The matrix element (3) is in the approximation (5)

$$M = W(p\mid n)\, R_N^{1/2}\, R_z^{1/2}, \qquad (6)$$

$$R_z^{1/2} = \sum_{\nu} u_\nu(z-2)\, v_\nu(z) \prod_{s\neq\nu} [\, u_s(z-2)\, u_s(z) + v_s(z-2)\, v_s(z)\,]\,. \qquad (7)$$

If we make use of the values of C and λ /7/ then, e.g., for the alpha decay of Cm^{244} to the ground state of P_4^{240} we get $R_N = 38$, $R_z = 45$ and $R_N \cdot R_z = 1700$. Calculations show that for nuclei in the region $230 < A < 254$ the quantities $R_N \cdot R_z$ are within the limits $1500 < R_N R_z < 3000$. In other words the pairing correlations increase the probability of alpha decay to the ground sates of even-even strongly deformed nuclei by a factor 1150-3000 Note that, perhaps, in the alpha decays of nuclei being near closed shells the decrease of the transition probabilities is partially due to vanishing of the pairing correlations of the system with a closed shell.

We find the matrix element of the alpha transition to the two-quasi-particle excited states of an even-even nucleus. Thus, for the alpha decay to a neutron state with quasi-particles occupying the orbitals f_1 and $f_2\ (f_1 \neq f_2)$ we obtain

$$M(f_1,f_2) = W_{\sigma_1,\sigma_2}(p\mid n\, f_1,\ f_2)\, R_z^{1/2}\ R_N^{1/2}(f_1,f_2) \qquad (8)$$

$$R_N(f_1 f_2) = v_{f_1}(N)^2\, v_{f_2}(N)^2 \prod_{s\neq f_1 f_2} [\, u_s(N-2,f_1 f_2)\, u_s(N) + v_s(N-2,\, f_1 f_2)\, v_s(N)\,]^2\,, \qquad (9)$$

$R_N(f_1 f_2)$ being less than unity. According to the superfluid model of the nucleus the alpha particle is in this case formed only from neutrons being in the states f_1 and f_2. The alpha decay rate is proportional to the neutron density of $v^2_{f_1}$, $v^2_{f_2}$ in these states of the parent. The hindrance factor F is then of the form

$$F = \left(\frac{W(p|n)}{W_{\sigma_1 \sigma_2}(p|n f_1 f_2)} \right)^2 \frac{R_N}{R_N(f_1 f_2)} , \qquad (10)$$

Thus, from the superfluid nuclear model it follows that the probabilities of alpha transitions to the two-quasi-particle states of even-even nuclei decrease by a factor $R_N / R_N(f_1, f_2)$ compared to the alpha decay taking place to the ground state. So, for the alpha decay of Cm^{244} to the two-quasi-particle states of Pu^{240} with energy up to 2 MeV $R_N / R_N(f_1, f_2)$ takes values in the interval 150-500. Note that in the given case the blocking effect plays an important role.

We consider a favoured alpha decay of odd nuclei in which the quasi-particle occupies one and the same orbital in the parent and daughter. When the odd neutron is on the orbital f the matrix element is given by

$$M(f) = W(p|n) R_z^{1/2} \cdot R_{N+1}^{1/2}(f), \qquad (11)$$

$$R_{N+1}^{1/2}(f) = \sum_{\omega \neq f} u_\omega(N-1, f) v_\omega(N+1, f) \prod_{s \neq \omega, f} [u_s(N-1, f) u_s(N+1, f) + v_s(N-1, f) v_s(N+1, f)]. \qquad (12)$$

The hindrance factor takes the form

$$F = \frac{M(N)^2 + M(N+2)^2}{2M(N+1, f)^2} = \frac{R_N + R_{N+2}}{2R_{N+1}(f)} \qquad (13)$$

The hindrance factor for the favoured alpha decays associated with the blocking effect changes in the limits in the transuranic region

$$1{,}2 < \frac{R_N + R_{N+2}}{2R_{N+1}(f)} < 3. \qquad (14)$$

As a rule, $F > 3$ can not be explained in such a manner. The comparison of the calculated values of F with the experimental data are given in Table I. The values of F placed in brackets have been calculated on the basis of data in /11/ taking into account the fact that besides the alpha particles emitted with $\ell = 0$ a fraction of them is emitted with $\ell = 2$ and $\ell = 4$. From Table I it is seen that the calculations are in a satisfactory agreement with the experimental data.

We consider unfavoured alpha decays in which the quasi-particle passes from one state to another. When the neutron passes from the state f_2 to the state f_1 ($f_1 \neq f_2$) the matrix element is

$$M(f_1, f_2) = W_{\sigma_1, -\sigma_2}(p|n f_1 f_2) R_z^{1/2} R_{N+1}(f_1 f_2)^{1/2}, \qquad (15)$$

where

$$R_{N+1}(f_1, f_2) = u_{f_2}(N-1, f_1)^2 v_{f_1}(N+1, f_2)^2 \times$$

$$\times \prod_{s \neq f_1 f_2} [u_s(N-1, f_1) u_s(N+1, f_2) + v_s(N-1, f_1) v_s(N+1, f_2)]^2. \qquad (16)$$

When the alpha decay of an odd N-nucleus is unfavoured the alpha particle is readily formed out of proton pairs that occupy the orbitals near the Fermi surface, and of neutrons occupying the states f_1 and f_2. Therefore the unfavoured alpha decays are strongly hindered compared to the favoured ones. The hindrance factor is of the form

$$F = \left[\frac{W(p \mid n)}{W_{\sigma_1, -\sigma_2}(p \mid n f_1, f_2)} \right]^2 \frac{R_N + R_{N+2}}{2 R_{N+1}(f_1, f_2)} . \qquad (17)$$

The values of $\frac{R_N + R_{N+2}}{2 R_{N+1}(f_1, f_2)}$ for alpha transitions to the ground- and hole states of strongly deformed nuclei lie between 50 and 130. This same ratio is 200-800 for alpha decays to the particle $K+2$-state and it will exceed 10^3 when alpha decays take place to the $K+3$-and higher states. The unfavoured alpha decays to the particle excited states are more hindered compared to the transitions taking place to the hole states.

The role of pairing correlations in (17) is demonstrated in Table 2. It is seen that the pairing correlations are responsable only for a partial decrease of the given transition probabilities. The decrease of the probability of the alpha particle formation from nucleons occupying different orbitals and also some other phenomena are described by the ratio $\left(\frac{W(p \mid n)}{W_{\sigma_1 - \sigma_2}(p \mid n f_1 f_2)} \right)^2$.

Since the pairing correlations contribute considerably to F then instead of the systematization of F's depending on the quantum numbers of the states f_1 and f_2 /12/ we should made the systematization of $\left(\frac{W(p \mid n)}{W_{\sigma_1, -\sigma_2}(p \mid n f_1, f_2)} \right)^2$.

The unfavoured alpha decays yield information on non-diagonal parts of W which can be used to calculate the hindrance factor for alpha decays both of odd-odd nuclei and taking place to the two-quasi-particle states of even-even nuclei.

Thus, the pairing correlations of nucleons of the superconductive type affect strongly the absolute probabilities of alpha decays to the ground states of even-even nuclei and those of the favoured decays in odd nuclei as well as, especially, the values of the hindrance factors in the unfavoured alpha decays and alpha transitions to the two-quasi-particle levels of even-even nuclei. The account of the pairing correlations leads to an improved agreement between theory and corresponding experimental data.

The obtained results are explaned directly by those properties of the ground- and excited states which follow from the superfluid nuclear model. The numerical calculations carried out use the properties of the ground- and excited states of /7/ and are completely unambigous, since they are no new parameter.

Note that the method suggested can easily be generalized to the case of the alpha decays taking place to the collected levels $0+, 2+, 0-$.

In conclusion I expresses my gratitude to N.N. Bogolubov and T.V. Voros for the interesting discussion and to N.A. Busdavina for working out the routine and making numerical calculations.

Table 1

Favoured alpha decays

State	α − decays	F exper.	F calcul.	State	α − decays	F exper.	F calcul.
9/2 −[734]	$^{151}Cf^{249} \rightarrow ^{149}Cm^{245}$	1,8 [8]	1,8	3/2 −[521]	$_{97}Bk^{245} \rightarrow _{95}Am^{241}$	1,7 [8]	1,7
7/2 +[624]	$^{149}Cm^{245} \rightarrow ^{147}Pu^{241}$	2,2 [8]	2,0	5/2 −[523]	$_{95}Am^{243} \rightarrow _{93}Np^{239}$	1,1 [8]	1,7
5/2 +[622]	$^{147}Cm^{243} \rightarrow ^{145}Pu^{239}$	1,5 [8]	2,2				(1,4)
		2 [9]	(1,8)	5/2 −[523]	$_{95}Am^{241} \rightarrow _{93}Np^{237}$	1,3 [8]	1,7
						2 [9]	(1,4)
1/2 +[631]	$^{145}Cm^{241} \rightarrow ^{143}Pu^{237}$	2,7 [8]	2,1	5/2 −[523]	$_{95}Am^{239} \rightarrow _{93}Np^{235}$	2,3 [8]	1,7
1/2 +[631]	$^{145}Pu^{239} \rightarrow ^{143}U^{235}$	2,5 [8]	2,1	5/2 +[642]	$_{93}Np^{237} \rightarrow _{91}Pa^{233}$	3,8 [8]	1,4
		1,7 [10]				5, [13]	

Table 2

Unfavoured alpha decays

f_2	α − decays	f_1	F exper.	$\frac{R_N + R_{N+2}}{2R_{N+1}(f_1,f_2)}$
9/2 − [734] K	$Cf^{249} \rightarrow Cm^{245}$	7/2 + [624] K	833 [8]	80
		5/2 + [622] K−1	251 [8]	80
5/2 + [622] K	$Cm^{243} \rightarrow Pu^{239}$	1/2 + [631] K	3220 [8]	50
		7/2 − [743] K−1	143 [8]	60
7/2 − [743] K	$Pu^{237} \rightarrow U^{233}$	5/2 + [633] K	1059 [8]	70
3/2 − [521] K	$Bk^{245} \rightarrow Am^{241}$	5/2 − [523] K	342 [8]	60
		5/2 + [642] K−1	29,4 [8]	57
3/2 − [521] K	$Bk^{243} \rightarrow Am^{239}$	5/2 − [523] K	682 [8]	60
		5/2 + [642] K−1	68 [8]	57
5/2 − [523] K	$Am^{243} \rightarrow Np^{239}$	5/2 + [642] K	1570 [8]	120
		3/2 − [521] K+2 }	1526 [8]	760
		1/2 − [530] K −1 }		120
5/2 + [642] K	$Np^{237} \rightarrow Pa^{233}$	1/2 − [530] K	3400 [13]	115

References

1. V.G. Soloviev. Dokl. Acad. Nauk 133, 325 (1960); JETP , 40, 654 (1961).
2. V.G. Soloviev. Isv. Acad. Nauk, ser. phys. 25, 1198 (1961).
3. V.G. Soloviev. Mat.Fys. Skr. Dan. Vid. Selsk., 1. N. 11 (1961). Preprint JINR, J-801 (1961).
4. A. Bohr, P.O. Froman, B.R. Mottelson. Dan.Mat.Fys.Medd. 29, 10 (1959).
5. I. Perlman, J. Rasmussen, Alpha radioactivity. IL (1959).
6. V.G. Nosov. JETP, 39, 141 (1960).
7. T. Voros, V.G. Soloviev, T. Siklos, Isv. Akad. Nauk, ser. phys. 26, (1962).
8. J.O. Rasmussen. Phys.Rev., 115, 1675 (1959).
9. B. Mottelson, S.G. Nilsson. Mat.Fys.Skr. Dan. Vid. Selsk. I, N. 8, (1959).
10. B.S. Dselepov, R.B. Ivanov, V.G. Nedovesov. JETP, 41, 1725 (1961).
11. A. Sandulescu, M. Iosifescu. Nucl.Phys., 26, 209 (1961).
12. O. Prior. Aktiv for Fysik, 16, 15 (1959).
13. S.A. Baranov, V.M. Kulakov, P.S. Samoilov, A.G . Selenkov, J.F. Rodionov. JETP, 41, 1733 (1961).

Received by Publishing Department
on March 19, 1962 .

Note Added in Proof

Prof. Rasmussen informed me that Mang and himself made the calculation of the alpha-decay rates in even-even nuclei taking into account the pairing correlations.