ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

Лаборатория теоретической физики

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E – 657

PAIRING CORRELATIONS AND ONE–NUCLEON REDUCED WIDTHS OF NUCLEAR ENERGY LEVELS

Дубна 1961

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Submitted to JETP

Abstract

The account of pairing correlations between nucleons in nuclei changes the formulae for direct reactions – there appear additional factors of types (7), (8). The curves plotted in Fig. 2 show a qualitative dependence of these factors on the nuclear excitation energy. The pairing correlations impede the transitions allowed in the model of independent particles (MIP). At the same time they 'allow' the reactions which are strictly forbidden in the MIP. The qualitative agreement of these results with the experimental data is established.

The reduced widths β^2 are important spectroscopic characteristics of nuclear states. On the basis of the shell model for light nuclei, the formulae for the reduced widths were given by Lane /1/. In the collective model they were calculated by Yoshida/2/. A complete and consistent description of the problems related to the reduced widths is presented in reviews made by Lane and Macferlane, and French/3/.

Consider a certain nuclear state Ψ_s described by a set of quantum numbers s . Different decay modes (channels) of the system are characterized by a set of quantum numbers s (both the internal characteristics of the decay products and the quantum numbers of their relative motion enter k). At the boundary Σ of the nucleus (outside the range of action of nuclear forces) the Ψ_s state may be expanded in the system of functions of different channels Φ_{sk}:

$$\Psi_s(\Sigma) = \sum_k \beta_k \Phi_{sk} \tag{1}$$

The squares of the coefficients of this expansion are just the reduced widths. Otherwise (1) may be written as

$$\beta_k = \int_\Sigma \Psi_s \Phi^*_{sk} \tag{2}$$

Thus, the one-nucleon reduced width is the square of the 'projection' of the Φ_k state (the resudual nucleus + free nucleon) on the given Ψ state of the nucleus under consideration. The reduced widths are important because the cross sections for direct reactions are expressed in terms of them.

The aim of the present paper is to get the corrections to formulae for β^2 obtained according to the model of independent particles which arise by taking into account the pairing resudual (by switching off the self-consistent field) interactions between nucleons.

The wave function of the neutron or proton nuclear shells (e.g. with an even number of particles N) in the model of independent particles (MIP) may be written in a double quantized form

$$\Psi_{MIP} = \prod_{\nu=\nu_0}^{\nu_{max}} a^+_{\nu_+} a^+_{\nu_-} \Psi_0 \tag{3}$$

where Ψ_0 is the nucleon vacuum state ($a_{\nu\pm}\Psi_0 = 0$) , ν are quantum numbers characteristic of the level; $\pm$ mean that the particles at one level have opposite projections of the angular moments.

Function (3) corresponds to energy distribution shown by the dotted line in Fig. 1. The inclusion of the resudual pairing interactions alters the wave function (3). One can succeed in finding it only if the exact number of particles is not conserved (N is kept only on the average). Thus, the functions assumes the form /4/

$$\Psi = \prod_{\nu} (U_{\nu} + V_{\nu} a^{+}_{\nu+} a^{+}_{\nu-}) \Psi_{0} \tag{4}$$

where $(U_{\nu})^2$ is the probability of the absence of particles on the given level ν, whereas $(V_{\nu})^2$ is the probability of the presence of a nucleon pair on the level ν . The energy distribution shown in Fig.1 by the solid line corresponds to the new function (4).

The qualitative dependence of U^2 and V^2 on the excitation energy is shown in Fig. 2. The exact values of these coefficients for different nuclei may be obtained by solving the integral equations just as it was done for strongly deformed nuclei /5/.

A great number of one-nucleon reduced widths is determined from the cross sections for the stripping reactions (dn), (dp) /3/. When the pairing correlations are taken into account there arises an additional factor A in the expression for the stripping cross section

$$\sigma \sim | < \Psi^{*}_{f} | a^{+}_{\nu_1} | \Psi_{i} > |^2 \sim \beta^2_{\nu} A \tag{5}$$

Here Ψ_i and Ψ_f are the nuclear wave functions before and after the stripping reaction; $a^{+}_{\nu_1}$ is the creation operator of the stripped nucleon in the ν_1 state β^2 is the reduced width of the level ν_1 for the stripped nucleon in the finite nucleus.

We shall obtain the factor A for the case when during the stripping process the neutron is added to the nucleus having an even number of neutrons. Then

$$\begin{aligned} \Psi_i &= \prod_{\nu} (U^{i}_{\nu} + V^{i}_{\nu} a^{+}_{\nu_-} a^{+}_{\nu_+}) \Psi_0 \\ \Psi_f &= \prod_{\nu \neq \nu_1} (U^{f}_{\nu} + V^{f}_{\nu} a^{+}_{\nu_-} a^{+}_{\nu_+}) a^{+}_{\nu_1} \Psi_0 \end{aligned} \tag{6}$$

With the aid of these functions we get

$$A = (U^{i}_{\nu_1})^2 \prod_{\nu \neq \nu_1} (U^{i}_{\nu} U^{f}_{\nu} + V^{i}_{\nu} V^{f}_{\nu})^2 \tag{7}$$

The factor $(U^{i}_{\nu_1})^2$ shows that the cross section of stripping reaction in this case is proportional to the probability that the energy level ν_1 will be left empty in the residual nucleus. The expression under the product sign is due to the change on other levels of a neutron shell. The contribution from the change in the proton shell is small, and we neglect it here.

If the neutron is added to the nucleus having an odd number of neutrons and forms a pair, then one can obtain for A

$$A = (V^f_{\nu_1})^2 \prod_{\nu \neq \nu_1} (U^i_\nu U^t_\nu + V^i_\nu V^f_\nu)^2 \qquad (8)$$

If the neutron fails to form a pair the corresponding wave function is:

$$\Psi_f = \prod_{\nu \neq \nu_1} (U^f_\nu + V^f_\nu a^+_{\nu_+} a^+_{\nu_-}) (U^f_{\nu_1} a^+_{\nu_{1-}} a^+_{\nu_{1+}} - V^f_{\nu_1}) \Psi_{0_f} \qquad (9)$$

then the expression for A is obtained from (9) by substitution $(V^f_{\nu_1})^2$ for $(U^f_{\nu_1})^2$. Just in analogous manner it is possible to consider other cases.

In the model of independent particles direct reactions are strictly forbidden, when a nucleon settled on a level below the Fermi surface since in MIP all levels with the energy < E_F are occupied totally. (Such processes are interpreted usually as direct reactions with the excitation of the target nucleus). The account of residual interactions 'allow' such processes since the levels below the Fermi surface are occupied only with the probability $V_\nu^2 \leq 1$ (see Fig. 1).

The cross section of such a process is proportional to the quantity

$$\sigma \sim \mathcal{B} \beta^2_{\nu\, ex} \qquad (10)$$

where $\beta^2_{\nu\, ex}$ is the one-nucleon reduced width for the system: excited nucleus of the target plus nucleon on the level

The factor $\mathcal{B}$ is of the form similar to (7) (when residual interactions are taken into account the processes during which the nucleon settles on the level with energy $> E_F$ and $< E_F$ do not differ essentially. However in this case to the factor $(u^i_{\nu})^2$ in Fig. 2 there corresponds a part of the curve which is on the left of E_F.

Since the reduced widths have been calculated from the experimental data by the formulae which do not take into account pairing correlations, the factors responsible for these correlations,, must enter the numerical values for the reduced widths. (See Table 1 of Macferlane and French paper /3/).

Let us consider the behaviour of the reduced widths for a nucleon in a given nucleus depending on the excitation energy of the levels. We shall consider the ratios β^2/β^2_{ground}. Let us be concerned with the simplest case when an odd nucleon during the stripping reaction is added to the free shell. In this case, in the model of independent particles $\beta^2/\beta^2_{ground} = 1$ for all the levels. According to (7) (see also the behaviour of the function u^2 in Fig. 2. The factor under the product sign does not essentially affect the qualitative dependence of A on the excitation energy of the level) the pairing correlations lead to an increase of $\beta^2_\nu/\beta^2_{ground}$ with energy. This is confirmed by the experimental data. In the F^{17} nucleus there is one more proton in addition to the twofold closed magic nucleus. The widths are known for the levels - ground state and excited state 0.5 MeV $(S_{1/2})$: $\dfrac{\beta^2(0,5\ MeV)}{\beta^2_{ground}} \sim 4$.

In the O^{17} nucleus (one neutron + O^{16}) $\dfrac{\beta^2(0.87\ MeV\ S_{1/2})}{\beta^2_{ground}(d_{5/2})} \sim 2$ A small value of the relative width for the level 4.56 MeV $(p^{-1}_{1/2}\ d^2_{5/2})$ is due to a change in the configuration of the core during the stripping reaction. This is in agreement with a small value of the correlation factor (10) . In Ca^{41} nucleus (one neutron besides the twofold magic frame Ca^{40})

$\dfrac{\beta^2(\ 1.95\ MeV\ 2\ p_{3/2})}{\beta^2_{ground}(f_{7/2})} \sim 1.5.$ The relative widths for the levels 2.677 MeV and 3.405 MeV (the configuration in both cases is $2\ S_{1/2}\ f^2_{7/2}$) are small in view of the change of the core $(E_\nu < E_F\ (10))$. A gradual increase of the relative width with excitation can be observed for eight levels of a nucleus K^{40} (the configuration of the first four levels is $d^7_{3/2}\ f_{5/2}$ and of the rest $d^7_{3/2}\ f_{5/2}$).

One can interpret in an analogous manner the widths in the nuclei B^{12}, C^{13}, Zn^{69}, S^{33}. . When there are more than one particle on the last orbit, it is more difficult to interpret the experimental data. However, the effect of the pairing correlations may be felt in these cases either. For instance, for the two levels in the G^{54} nucleus, the magic by neutrons core + two paired neutrons on the $2\ p^2_{3/2}$ shell:

$\dfrac{\beta^2(0.36\ MeV\ )}{\beta^2_{ground}} \sim 0.44$, what corresponds to the correlations factor (8) (see also the behaviour of the function V^2 in Fig. 2).

In conclusion the authors take the opportunity of thanking V.G. Neudachin, V.G. Soloviev, N.I. Piatov and V.I. Furman for the discussion of the results.

A Caption to Fig. 2

The coefficients V_ν^2 and U_ν^2 are plotted against the excitation energy (i.e., against ν). The factors under the product sign in (7), (8) and do not essentially change the qualitative dependence of the correlation corrections on ν. For odd nuclei the energy of the ground state lies in the vicinity of the Fermi surface E_F , for even nuclei (all the nucleons are paired) the ground state is below the surface E_F.

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Recieved by Publishing Department on January 19, 1961.

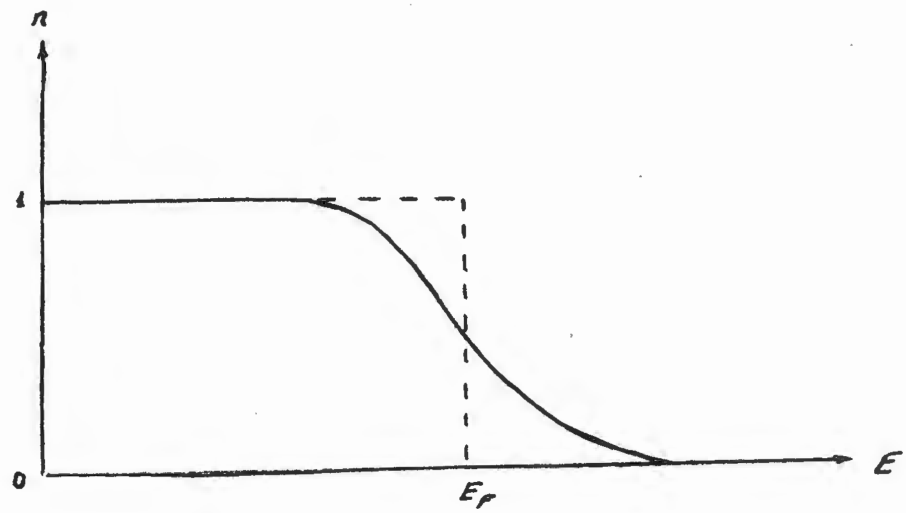

Fig. 1.

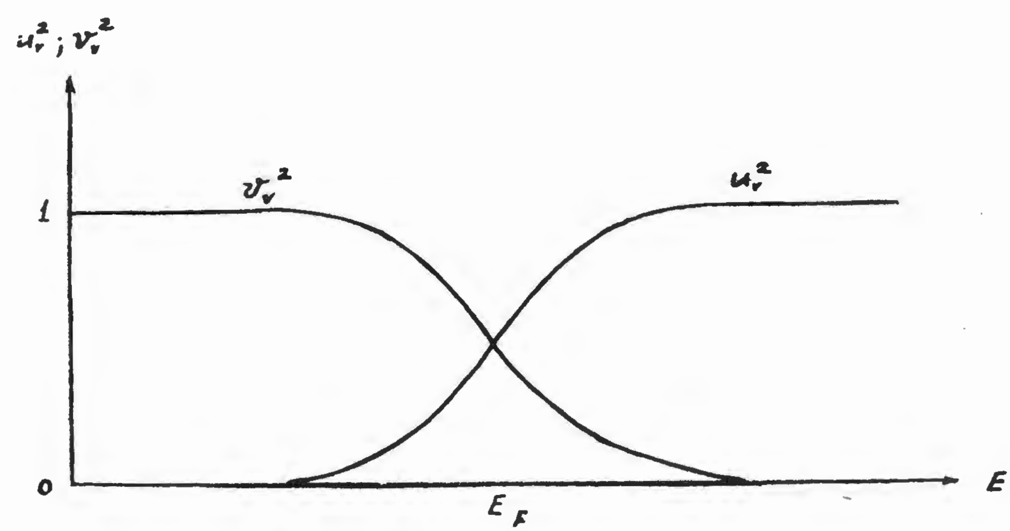

Fig. 2.