ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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Abstract

Some experiments with the three and four pion annihilations of polarized antiprotons on hydrogen are suggested to define space and charge parities of the proton-antiproton system.

1

In the paper by Okonov and the author [1] the task has been raised to test whether the space and charge parities of the system are namely such as those of the Dirac particle and antiparticle. For this purpose experiments with two-pion annihilations $\tilde{p} + p \to \pi + \pi$ have been suggested. At present the weight of this channel is known to be very small[2] (one two-pion annihilation per four hundred $p\tilde{p}$ -annihilations apparently)[1)]. In this paper it is shawn how to define a variant of the space and charge parities of the $p\tilde{p}$ system from the experiments with three- and four pion annihilations by making use of polarized antiprotans [2].

Let us note that the variant of parities (non-Dirac one) that forbids absolutely two-pion annihilations is also discussed. The reason is that experimental difficulties do not allow to consider the presence of such annihilations to be well established. Indeed, it is necessary to exclude the annihilation $\tilde{p} + p \to \pi + \pi$ plus π° — meson or γ - quantum of a low energy (the abovementioned variant does not forbid such annihilations).

On the other hand the absence of the two-pion annihilations would not a definite evidence in favour of this variant (since other explanations are possible).

The experiments suggested will allow us to test the hypothesis by Okonov [6] which explains the suppression of the two-pion channel by assuming that $\tilde{p}p$ annihilates mainly in the singlet state.

2

The variants as in [1] are denoted by $\{\pi_{\tilde{p}p}, C_{\tilde{p}p}\}$ where $\pi_{\tilde{p}p}$ is the product of intrinsic space parities $\tilde{p}$ and p and $C_{\tilde{p}p}$ is the multiplier $+1$ or $+1$ in the expression $1(-1)^{l+s}$ for the charge parity of the $\tilde{p}p$ system.

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1) This fact does not contradict with the simplest statistical theories of multiple production that give a correct mean multiplicity if volume of the interaction is taken to be equal to ten Fermi volumes, i.e. about $10 \cdot 4\pi/3\,(h/m_{\pi}c)^3$. [3,4] Apparently it agrees worse with statistical theories which assume the ordinary volume of the interaction $4\pi/3\,(h/m_{\pi}c)^3$. Taking into account the $\pi\pi$ interaction and other arguments these theories give a correct mean multiplicity but at the same time the weight of the two-pion channel turns out to be of about several per cent, see for example [5] .

In the Appendix it is shown that the angular distribution $\sigma(\theta,\phi,\theta)$ of the reaction $\tilde{p}+p\to\pi^++\pi^-+\pi^0$ in the variant $\{-1,-1\}$ (which forbids the two-pion annihilation) must vanich at the points $\theta=\theta'=90^\circ$, $\phi=0^\circ$ or 180°, while in other variants it is not obligatory that $\sigma(90^\circ,0^\circ,90^\circ)$ equals zero. θ' is the angle between the direction $\vec{p}_a$ of the incident beam and total momentum $\vec{p}'$ of the π^+ and π^- mesons (in the c.m.s. of the reaction); θ is the angle between $\vec{p}'$ and the mamentum $\vec{p}$ of the π^+ meson relative to the Lorentz system where the total momentum of π^+ and π^- equals zero; ϕ is the angle between the vectors $[\vec{p}_a . \vec{p}']$ and $[\vec{p}' . \vec{p}]$, see Appendix. $\sigma(\theta,\phi,\theta')$ means the angular distribution integrated over the azimutal angle ϕ' of the vector $\vec{p}'$ (see Appendix). In this case it is of no importance whether $\tilde{p}$ or p are polarized or not.

If $\sigma(90^\circ,0^\circ,90^\circ)=0$ then the integral of $\sigma(\theta,\phi,\theta')$ over a certain neighbourhood of the paint $(90^\circ,0^\circ,90^\circ)$ can not exceed a certain fraction of the total cross section σ of the three-pion channel .

One can show that in the same variant the angular distribution $\sigma(\theta_-,\phi_-;\theta_+,\phi_+,\theta')$ of the reaction $\tilde{p}+p\to\pi^-+\pi^-+\pi^++\pi^+$ (integrated over ϕ' must vanish when ϕ_-,ϕ_+ equal 0° or 180° and $\theta_-=\theta_+$. Since this is true for any values of the angles θ and $\theta_-(=\theta_+)$ then the integral

$$\int_0^\pi d\cos\theta' \int_0^\pi d\cos\theta_- \sum_{a,\beta=0,\pi} \sigma(\theta_-,a,\ \theta_-,\beta;\theta') \tag{1}$$

must also vanish.

We have the following definitions for the angles: θ' and ϕ' denote spherical angles of the total momentum $\vec{p}'$ of the two π^- mesons (in the c.m.s. of the reaction) with respect to the following three orts z_a, y_a, x_a : the ort z_a is parallel to the incident beem, the ort y_a is directed, for example, along the antiproton polarization vector. θ_- and ϕ_- are spherical angles of the momentum $\vec{p}_-$ of one of the π^- mesons (in the Lorentz system where the total momentum of the two π^- mesons equals zero) reckoned from the orts $z'\ y'\ x'$; the ort $z'\parallel\vec{p}'$; the ort $y'\parallel[\vec{z}_a\cdot\vec{p}']$. θ_+ and ϕ_+ are reckoned from the same orts $z'\,y'\,x'$.

The formula (A.2) of the Appendix provides the expression of $\vec{p}_-$ in terms of the momenta $\vec{p}_1^{\,-}$ and $\vec{p}_2^{\,-}$ of the two π^- mesons relative to the c.m.s. of the reaction.

3.

Let us note that the study of the angular distribution of the annihilation pions allows one to test whether the space, charge and combined parities are conserved in annihilation process.

Namely, it follows from the invariance under space reflection I and from the fact that $\tilde{p}, p$ and π mesons have definite parities that

$$\sigma(\theta, \phi, \theta') = \sigma(\theta, -\phi, \theta')$$

for the three-pion annihilation and

$$\sigma(\theta_-, \phi_-; \theta_+, \phi_+, \theta') = \sigma(\theta_-, -\phi_-; \theta_+, -\phi_+; \theta') \qquad (2)$$

for the four-pion one, see [7].

From the invariance under charge conjugation C it follows

$$\begin{aligned} \sigma(\theta, \phi, \theta') &= \sigma(\pi - \theta, \phi; \pi - \theta') \\ \sigma(\theta_-, \phi_-; \theta_+, \phi_+; \theta') &= \sigma(\theta_+, -\phi_+; \theta_-, -\phi_-; \theta') \end{aligned} \qquad (3)$$

Finally from the invariance under IC it follows that

$$\begin{aligned} \sigma(\theta, \phi, \theta') &= \sigma(\pi - \theta, -\phi; \pi - \theta') \\ \sigma(\theta_-, \phi_-; \theta_+, \phi_+; \theta') &= \sigma(\theta_+, \phi_+; \theta_-, \phi_-; \theta') \end{aligned} \qquad (4)$$

The angular distribution possess these properties in any variant of parities.

4

The presence of polarized antiprotons [2] allows us to suggest the following simple experiment to determine the $(\tilde{p}p)$ space parity. It is necessary to compare the signs of the quantities η_1, and (see Appendix), defined for the reaction $\tilde{p} + p \to \pi^+ + \pi^- + \pi^0$

as follows

$$\Delta_1 = \int^{\pi/2} d\cos\theta' \int_0^{\pi} d\cos\theta \left[\int_{-\pi/2}^{\pi/2} d\phi' \, H(\theta,\theta',\phi') - \int_{\pi/2}^{3\pi/2} d\phi' \, H(\theta,\theta',\phi') \right]$$

$$\Delta_2 = \int_{\pi/2}^{\pi} d\cos\theta' \int_0^{\pi} d\cos\theta \left[\int_{-\pi/2}^{\pi/2} d\phi' \, H(\theta,\theta',\phi') - \int_{\pi/2}^{3\pi/2} d\phi' \, H(\theta,\theta',\phi') \right] \tag{5}$$

Here $H(\theta,\theta',\phi') = F(\theta,0;\theta',\phi') + F(\theta,180^\circ;\theta',\phi')$ where $F(\theta,\phi;\theta',\phi')$ - is the angular distribution of the reaction in the case when $\tilde{p}$ (or p) is polarized. Thus, it is necessary to choose cases when $\vec{p}$ is coplanar with the vectors $\vec{p}_a$ and $\vec{p}\,'$ (i.e. $\phi = 0^\circ$ or 180°); to devide this statistic into four parts and compare the signs of the right-left asymmetry for the cases of the backwards and forward emission of the π^0 mesons in the c.m.s. The magnitude of the allowed dispersion of angles ϕ near 0° and 180° depends on the absolute magnitude of Δ_1 and Δ_2 and on the antiproton energy. From the Table 1 it is seen that the relative sign of Δ_1 and Δ_2 defines the variant of the space parity.

In the case of the four-meson annihilation the all afore-mentioned is true for Δ_1 and Δ_2 defined as

$$\Delta_1 = \int_0^{\pi} d\cos\theta_- \int_0^{\theta_-} d\cos\theta_+ \int_0^{\pi} d\cos\theta' \left[\int_{-\pi/2}^{\pi/2} d\phi' \, H - \int_{\pi/2}^{3\pi/2} d\phi' \, H \right]$$

$$\Delta_2 = \int_0^{\pi} d\cos\theta_- \int_0^{\theta_-} d\cos\theta_+ \int_0^{\pi} d\cos\theta' \left[\int_{-\pi/2}^{\pi/2} d\phi' \, H - \int_{\pi/2}^{3\pi/2} d\phi' \, H \right] \tag{6}$$

Here H means the sum $\sum_{\alpha,\beta=0,\pi} F(\theta_-,\alpha;\theta_+,\beta;\theta',\phi')$ (comp. formula (1)): it is implied that $\tilde{p}$ (or p) are polarized.

Note that the discovery of the reliable cases of the two-pion annihilation and the coincidence of the signs of Δ_1 and Δ_2 would mean that the variant of parities is a Dirac one.

Table 1

1	2	3	4
variant	Relative sign of Δ_1 and Δ_2	$(R+L)-(U+D)$	$I(\theta,\theta')$
$\{-1,+1\}$ Dirac variant	+		
$\{-1,-1\}$	+	vanished at points	$I(90^\circ, 90^\circ) = 0$
$\{+1,+1\}$	−	$\theta = \theta' = 90^\circ$ $\phi = 0^\circ,\ 180^\circ$	
$\{+1,-1\}$	−		$I(90^\circ, 90^\circ) = 0$

5

More difficult experiments (when both antiprotons and protons of the target are polarized) would enable us to define the variant of the charge parity. We can inquire whether the value

$$(R+L)-(U+D) = \int_{-\pi/4}^{\pi/4} d\phi' \, G(\theta,\phi,\theta',\phi') + \int_{3\pi/4}^{-3\pi/4} d\phi' \, G - \int_{\pi/4}^{3\pi/4} d\phi' G - \int_{-3\pi/4}^{-\pi/4} d\phi' G \tag{7}$$

vanished (see Table 1) at points $\theta = \theta' = 90^\circ$, $\phi = 0^\circ$ or 180° where $G(\theta,\phi;\theta',\phi')$ is the angular distribution of $\tilde{p} + p \to \pi^+ + \pi^- + \pi^0$ in the case when both antiprotons and protons are polarized along the same direction perpendicular to the beam.

The distinctive indication of the variant $C_{\tilde{p}\,p} = -1$ is the vanishing at the point $\theta = \theta' = 90^\circ$ of the function

$$I(\theta,\theta') = \int_0^{2\pi} d\phi \, [\int_0^{2\pi} d\phi' \, G(\theta,\phi;\theta',\phi') - \int_0^{2\pi} d\phi' \, F(\theta,\phi;\theta',\phi')] \tag{8}$$

See Table 1. Instead of F one can use the angular distribution of the unpolarized $\tilde{p}$ and p.

6

From the space and charge-parity selection rules in the Dirac variant it follows that the two-meson annihilation may go only through the triplet states of the $\tilde{p}p$ system, see, for example, the Table 1 in [1]. Okonov [6] suggested the following hypothesis to explain the supression on the two-pion annihilations: the annihilation of $\tilde{p}\,p$ is possible only in the singlet state of this system. Then the angular distribution of the annihilation π mesons must not depend on the azimuth ϕ' even if antiproton are polarized. In particulary we must have $\Delta_1 = \Delta_2 = 0$ and $I(\theta,\theta') = 0$. Quite generally any dependence of the angular distribution of the annihilation products on the polarization of $\tilde{p}$ or p must be absent according to this hypothesis.

In conclusion I express my gratitude to Prof. M.A. Markov, V.I. Ogievetzky and E.O. Okonov for the discussion.

Appendix

Three-Pion Annihilation

1. For the reaction $\tilde{p} + p \to \pi^+ + \pi^- + \pi^\circ$ the space and charge parity selection rules are of the form

$$\pi_{\tilde{p}p}(-1)^{l_a + l + l'} = +1 \qquad C_{\tilde{p}p}(-1)^{l_a + s + l} = +1 \tag{A.1}$$

l_a is the orbital angular momentum of the $\tilde{p}p$ system;
l is the orbital angular momentum of the $(\pi^+ \pi^-)$ system;
l' is the orbital angular momentum of the $(\pi^+\pi^-) - (\pi^\circ)$ system;
s is the total spin of $\tilde{p}$ and p. There are no any simple consequences of (A.1) which had took place in the case of the two-pion annihilation (such as the prohibition of the s-state annihilation) and which allowed us to distinguish the variants $\{\pi_{\tilde{p}p}, C_{\tilde{p}p}\}$ by the energy behavior of the cross section. The determination of the variant, generally speaking, needs experiments with polarized antiprotons and protons.

2. We will obtain from (A.1) some relations between the amplitudes $\langle \vec{p}\vec{p}' \mid R \mid \vec{p}_a m_a m_b \rangle$ of transition from the initial state characterized by the presence of $\tilde{p}$ and p (their relative momentum being $\vec{p}_a$ and their spin projections on the $\vec{p}_a$ direction being m_a and m_b respectively) into the final three-pion state which may be caracterized by momenta $\vec{p}$ and $\vec{p}'$. $\vec{p}$ denotes π^+ momentum in the Lorentz system, where the total momentum of π^+ and π^- is equal to zero (the momentum of π^- in this system is $-\vec{p}$). In the c.m.s. of the reaction this total momentum is $\vec{p}'$ [7,8]. If $\vec{p}_1, \vec{p}_2$ and $\vec{p}_3$ are the momenta of π^+, π^- and π° in the c.m.s. of the reaction than

$$\vec{p} = \vec{p}_1 + (\vec{p}_1 + \vec{p}_2)\left[\frac{(\vec{p}_1 + \vec{p}_2)\cdot\vec{p}_1)}{(\vec{p}_1 + \vec{p}_2)^2}\left(\frac{E_{12}}{\kappa_{12}} - 1\right) - \frac{\sqrt{p_1^2 + \kappa^2}}{\kappa_{12}}\right] \tag{A.2}$$

$$\vec{p}' = \vec{p}_1 + \vec{p}_2 = -\vec{p}_3$$

where $E_{12} = \sqrt{p_1^2 + \kappa^2} + \sqrt{p_2^2 + \kappa^2}$ and $\kappa_{12} = \sqrt{E_{12}^2 - (\vec{p}_1 + \vec{p}_2)^2}$, see [7].

(κ is the mass of the π meson, the velocity of light is the unit of velocity). In [7] we have obtained the following expression for $< \vec{p}\vec{p}'|R|\vec{p}_a\, m_a\, m_b >$:

$$<\vec{p}\vec{p}'|R|\vec{p}_a\, m_a\, m_b> = R_{ma,mb}(\theta,\phi;\theta',\phi') = \sum_{l,j,m} (2j+1)\sqrt{2l+1}/4\pi\sqrt{4\pi} \tag{A.3}$$

$$D^{l}_{o,m}(-\pi,\theta,\pi-\phi)\, D^{J}_{m,m_a+m_b}(-\pi,\theta',\pi-\phi')<lm|R^{J}|m_a m_b>$$

where

$$<lm|R^{J}|m_a m_b> = \sum_{l',l_a s} \sqrt{\frac{2l'+1}{2J+1}}\, C^{Jm}_{lm\, l'0} <ll'|R^{J}|s\, l_a> \cdot \tag{A.4}$$

$$\sqrt{\frac{2l_a+1}{2J+1}}\; C^{s\, m_a+m_b}_{\frac{1}{2}m_a\, \frac{1}{2}m_b}\; C^{J\, m_a+m_b}_{s\, m_a+m_b\, l_a 0}$$

θ' and ϕ' are spherical angles of $\vec{p}$ with respect to the axes z_a, y_a, x_a, the axis z_a being parallel to $\vec{p}_a$; θ and ϕ are $\vec{p}$ spherical angles with respect to the axes $z' y' x'$: the axis z' is parallel to $\vec{p}'$; the axis y' is parallel to $[\vec{p}_a \cdot \vec{p}']$ see [7]. By definition

$$D^{l}_{m,n}(\Phi_2,\Theta,\Phi_1) = e^{-im\Phi_2}\, \mathcal{P}^{l}_{mn}(\cos\Theta)\, e^{-in\Phi_1}$$

where the function $\mathcal{P}^{l}_{mn}$ differs from the function P^{l}_{mn} defined in [9] by the multiplier i^{n-m} and coincides with the function $d^{l}_{m,n}$ defined in [10].

We multiply the summand in the right hand part of (A.4) by $C_{\vec{p}p}(-1)^{l_a+s+l}$ which is equal to $+1$ owing to (A.1) and use the following properties of the Clebsch-Gordan coefficients:

$$C^{J\, m_a+m_b}_{s m_a+m_b\, l_a 0} = (-1)^{J-s-l_a}\, C^{J-m_a-m_b}_{s-m_a-m_b\, l_a 0} \tag{A.5}$$

$$C^{s\, m_a+m_b}_{\frac{1}{2}m_a\, \frac{1}{2}m_b} = C^{s-m_a-m_b}_{\frac{1}{2}-m_b\, \frac{1}{2}-m_a}$$

We find that the right hand side of (A.4) is then the matrix element $< lm\,|R^J|\,-m_b\;\;-m_a>$ multiplied by $C_{\tilde{p}p}(-1)^{l+J}$ i.e.

$$< lm\,|R^J|\,m_a\;m_b\;> = C_{\tilde{p}p}(-1)^{l+J} < lm\,|R^J|\,-m_b - m_a> \qquad \text{(A.6)}$$

Let us now substitute the right hand side of (A.6) for $< lm\,|R^J|\,m_a\;m_b\;>$ in the right hand side of (A.3) and use the equations (for the integral j)

$$D^j_{m,n}(-\pi, \Theta, \pi-\Phi) = (-1)^{j-m} D^j_{m,-n}(-\pi, \pi-\Theta, \pi+\Phi) = (-1)^{j-n} D^j_{-m,n}(-\pi, \pi-\Theta, \pi-\Phi) \qquad \text{(A.7)}$$

We obtain a following consequence of the invariance under charge conjugation C :

$$R_{m_a m_b}(\theta, \phi; \theta' \phi') = C_{\tilde{p}p} R_{-m_a, -m_b}(+\pi-\theta, \phi, \pi-\theta', -\phi') \qquad \text{(A.8)}$$

In the same manner we can obtain (see [7] and [11]) the consequence of the invariance under space reflection:

$$R_{m_a, m_b}(\theta, \phi; \theta', \phi') = \pi_{\tilde{p}p}(-1)^{m_a+m_b} R_{-m_a\;-m_b}(\theta, -\phi; \theta', -\phi') \qquad \text{(A.9)}$$

The ϕ' dependence of $R_{m_a\,m_b}(\theta, \phi, \theta', \phi')$ is known, see (A.3). Therefore, the equation (A.8) and (A.9) can be divided by the same multiplier $\exp.i(m_a + m_b)\phi'$. In the following we will not write down the argument ϕ' .

The relations (A.8) and (A.9) and some of their combinations are written in the Table II for different variants. The following notations are taken: $R_{-1/2, -1/2}(\theta, \phi, \theta') = a$ and analogously $R_{-1/2, +1/2} = b$, $R_{+1/2, -1/2} = c$, and $R_{+1/2, +1/2} = d$. Then $R_{-1/2, -1/2}(\theta, -\phi, \theta') = a'$ and analogously for b, c and d; $R_{-1/2, -1/2}(\pi-\theta, \phi, \pi-\theta') = \tilde{a}$; $R_{-1/2, -1/2}(\pi-\theta, -\phi, \pi-\theta') = \tilde{a}'$ and analogously for b, c and d.

Table II

Variant	I	C	IC
$\{-1,+1\}$ Dirac variant	$a=d'$ $b=-c'$	$a=\tilde{d}$ $b=\tilde{b}$ $c=\tilde{c}$	$a=\tilde{a}'$ $d=\tilde{d}'$
$\{-1,-1\}$	$a=d'$ $b=-c'$	$a=-\tilde{d}$ $b=-\tilde{b}$ $c=-\tilde{c}$	$a=-\tilde{a}$ $d=-\tilde{d}'$
$\{+1,+1\}$	$a=-d'$ $b=c'$	$a=\tilde{d}$ $b=\tilde{b}$ $c=\tilde{c}$	$a=-\tilde{a}$ $d=-\tilde{d}'$
$\{+1,-1\}$	$a=-d'$ $b=c'$	$a=-\tilde{d}$ $b=-\tilde{b}$ $c=-\tilde{c}$	$a=\tilde{a}'$ $d=\tilde{d}'$

3. If the beam and target are polarized in some direction $\vec{y}_a$ (perpendicular to $\vec{p}_a$) then the angular distribution of π^+, π^-, π^0 is of the form (comp. for ex. [11]):

$$G(\theta,\phi;\theta',\phi') \propto W(\theta,\phi,\theta') + \sqrt{2}\,\tilde{P}_y[-Jm\,\tilde{W}_-\cos\phi' + Re\,\tilde{W}_{y-}\sin\phi'] +$$

$$+\sqrt{2}\,P_y[-Jm\,W_-\cos\phi' + Re\,W_-\sin\phi'] - \tag{A.10}$$

$$-\tilde{P}_y P_y[Re\,W_{-,-}\cos 2\phi' + J_m\,W_{-,-}\sin 2\phi' + Re\,W_{-,+}]$$

The magnitudes $\tilde{P}_y$ and P_y of the antiproton and target proton polarization are defined usually as the mean value of the y -component of the spin operator $\vec{\sigma}/2$. The coefficients W in (A.10) are expressed in terms of the transition amplitudes a, b, c, d as follows:

$$W(\theta,\phi,\theta') = 1/4\,[\,|a|^2 + |b|^2 + |c|^2 + |d|^2\,]$$

$$\tilde{W}_-(\theta,\phi,\theta') = \frac{1}{2\sqrt{2}}\,(ac^* + bd^*) \tag{A.11}$$

$$W_- = \frac{1}{2\sqrt{2}}\,(ab^* + cd^*)$$

$$W_{-,-} = ad^*/2\ , \qquad W_{-,+} = -bc^*/2$$

From the Table II one can see that $b = +\tilde{b}$ or $b = -\tilde{b}$ depending on the variant, i.e. b does not change its sing (or change) when substituting $\pi - \theta$ and $\pi - \theta'$ for θ and θ' respectively. We will call the function b even (or odd) with respect to the point $\theta = \theta' = 90^\circ$ (for any value of ϕ). From the Table II and (A.11) it follows that when $\pi_{\tilde{p}p} = -1$ and $\phi = 0^\circ$ and 180° the coefficients $\tilde{W}_-$ and W_- are even functions with respect to the point $\theta = \theta' = 90^\circ$ and when $\pi_{\tilde{p}p} = +1$ they are odd ones. The functions $W_{-,-}$ and $W_{-,+}$ being in all variants even ones in some variants vanish at the points $\theta = \theta' = 90^\circ$, $\phi = 0^\circ$ and 180° or on the line $\theta = \theta' = 90^\circ$, ϕ is arbitrary. Further we can see that in the variant $\{-1, -1\}$ all functions a, b, c, d and consequently all W-coefficients vanish at the points $\theta = \theta' = 90^\circ$, $\phi = 0^\circ$ or 180°. The quantity

$\int_0^{2\pi} d\phi' \, G(\theta, \phi, \theta', \phi') = \sigma(\theta, \phi, \theta')$ vanishes in this point too, see Section 2 of the basis text.

Now we must get from the experimental data the multipliers of $\cos\phi'$ and $\cos 2\phi'$ in Eq. (A.10) and also the quantity $\tilde{p}_y\, p_y\, \mathrm{Re}\, W_{-,+}$. In some cases it is not necessary to know the behaviour of these coefficients as functions of θ, ϕ, θ' but it is sufficient to deal with some integrals of them over these angles. The necessary manipulations are just the means for distinguishing between variants. They are exposed in the Sections 2, 4, 5.

Note that to determine $\pi_{\tilde{p}p}$ it would be sufficient to use only the $\dot{I}$ invariance. However, in this case it turns out to be necessary to know signs of the $\tilde{p}$ and p polarizations (i.e. whether $\tilde{p}_y$ and p_y are positive or negative quantities) comp. [12] .

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