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Лаборатория теоретической физики

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ON PROPERTIES OF A NUMBER OF STRONGLY-DEFORMED NUCLEI

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ON PROPERTIES OF A NUMBER OF STRONGLY–DEFORMED NUCLEI

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БИБЛИОТЕКА

Abstract

By using the improved scheme of single-particle levels of the self-consistent field, certain properties of strongly deformed nuclei in the region $150 < A < 190$ have been investigated on the basis of the superfluid nucleus model. By comparing calculated pairing energies with experimental data the pairing interaction constants having average values equal to $G_N = 0.024\hbar\omega_0 = 0.18$ MeV and $G_Z = 0.026\hbar\omega_0 = 0.20$ MeV have been found. The density of the single-particle low-energy levels of odd mass nuclei has been calculated which agrees with experimental data and is about twice as large as those presented by Nilsson's schemes. The regularities in the behaviour of the low-excited states of even-even nuclei have been noted. It has been shown the calculation error which is due to the conservation of number of particles on the average does not exceed 6 percent.

The superfluid model of a nucleus/1/ basing on the unified or shell models takes into account residual short-range interactions of nucleons in the nucleus by means of the variational principle/3/. To describe such interactions we use the Hamiltonian of the type

$$H = \sum_{s\sigma} (E_s - \lambda)\, a^+_{s\sigma} a_{s\sigma} - G \sum_{s,s'} a^+_{s+} a^+_{s-} a_{s'-} a_{s'+} \qquad (1)$$

where E_s are single-particle levels of the self-consistent field; λ is a parameter playing the role of the chemical potential; (s,σ) are quantum characteristics of the level; G is the short-range pairing interaction constant. Note the number of particles is conserved on the average. The investigation of properties of transuranic elements has been carried out in/2/ where the basic equations of the problem are given.

In the present paper we investigate the properties of the strongly-deformed nuclei in the region $150 < A < 190$ on the basic of the superfluid nucleus model.

By comparing the calculated pairing energies with experimental data let us define the constants of pairing interactions G, calculate spectra of the single-particle levels for odd-mass nuclei as well as for some even-even nuclei and compare them with experimental values.

Modification of the Nilsson Scheme

As single-particle levels E_s we use in calculating the levels of the Nilsson scheme/4/. The analysis of the scheme based on the well-known spectroscopic data/5,10,12/ showed that the proton shell in general is described satisfactoryly by the scheme given in/6/. We make essential modifications in the scheme of neutron levels for $82 < N < 126$, namely:

a) all the eigenvalues of neutrons with $N=6$ are increased by $0.25\hbar\omega_0$ (that corresponds to the parameter $\mu = 0.33$), except i 13/2, which are increased by $0.06\hbar\omega_0$ ($\mu = 0.42$);

b) the subshell h 11/2 is decreased by $0.3\hbar\omega_0$ ($\mu = 0.65$);

c) the eigenvalues f 5/2 are increased by $0.06\hbar\mathring{\omega}_0$ (μ = 0.42);

d) the eigenvalues p 3/2 are decreased by $0.01\hbar\mathring{\omega}_0$ with the agreement with the experimental data on nuclei with 109 neutrons;

e) the level ½ - [521] as in paper/6/ is increased by $0.04\hbar\mathring{\omega}_0$. The modified Nilsson scheme plotted in Fig. 1 gives correct ground states for all nuclei with $93 \leq N \leq 109$ and for some nuclei the necessary sequence of the first excited levels.

The numerical solutions of basic equations given in/2/ have been made on the computer 'Strela'. The problem was reduced to the method of the least squares and solved by means of the linearization. Using the given average field energy levels and the interaction constant G , the correlation function C , chemical potential λ , root-mean-square fluctuation of the number of particles $\sqrt{\Delta n^2}$ and the energy of the system $\mathcal{E}$ have been calculated within the accuracy of four-six signs after comma.

Pairing Energies and Correlation Functions

To define the value of G we calculate pairing energies of neutrons P_N and protons P_Z using the formula:

$$P_N = 2E(Z, N-1) - E(Z,N) - E(Z, N-2). \tag{2}$$

The calculated values of pairing energies of the neutron system at $G_N = 0.020\hbar\mathring{\omega}_0$ and $G_N = 0.024\hbar\mathring{\omega}_0$ and the proton system at $G_Z = 0.024\hbar\mathring{\omega}_0$ and $G_Z = 0.028\hbar\mathring{\omega}_0$ and also the corresponding experimental data/8,9/ are shown in Fig. 2 and 3. By comparing the calculated values of pairing energies with experimental data we find average values of the pairing interaction constants $G_N = 0.024\hbar\mathring{\omega}_0$, $G_Z = 0.026\hbar\mathring{\omega}_0$. Note the value of pairing energy decreases with increasing deformation that is connected with the weakening of the role of residual interactions when the deformation increases.

Fixing the scheme of the single-particle levels on the basis of the analysis of experimental data on spectra of odd-mass nuclei with account of the influence of superfluidity and defining from pairing energies the constants G_N and G_Z we exclude in this way any ambiguity from subsequent calculations.

We investigate the behaviour of the correlation function $C = G\sum_s u_s v_s$ and chemical potential λ which corresponds to the ground states of even-even and odd-mass nuclei. The behaviour of the chemical potential λ depending on the degree of filling up the shell for systems with odd number of neutrons is giving in Fig. 4. The value of λ fluctuates near the energy of the Fermi surface level, these deflections being about 1 MeV. The fluctuations of the difference $|\lambda - E_F|$ for excited states become still more. The deflection of λ from E_F into excited states of even systems turns out to be especially great.

The behaviour of the functions C for the ground states of even and odd systems is given in Fig.5 and 6. Note the values of C for the ground states of odd systems depend strongly on the behaviour of single-particle levels nearest to the Fermi surface energy. One can see from the scheme of single-particle neutron levels (Fig. 1) that in the region of deformations $\delta = 0.26 - 0.36$ the difference of energies between the levels N = 97 and N = 99 reaches 1.4 MeV, that leads to a sharp decrease of the quantity C for the corresponding odd-mass nuclei. The values of C for the ground states of even systems are less sensible to the behaviour of the single-particle levels. The values of C for odd systems (2n – 1) are on the average less by (20% – 30%) than those for even (2n) systems, that agrees with the evaluations in/11/. Thus the appearence of one quasiparticle leads to a considerable reduction of the superfluidity. It should be noted the values of C decrease as a rule with increasing deformation which testifies to the fact that the role of pairing correlations is reduced when δ increases. The function C for singlequasi-particle states of odd systems has a minimum value for the ground state and increases for excited states as the excitation energy increases, approaching the value of C for the ground state of the corresponding even system (see Table 1). In the case of the even system the function C for two-quasi-particle excited states decreases by 30 or more percent, sometimes it vanishes. The value of this correlation function increases with the excitation energy increase.

Single– Particle Levels of Odd-Mass Nuclei

On the basis of the superfluid nucleus model we calculate the spectrum of single-particle levels for both odd - N and odd- Z nuclei in the region under investigation. As an example we give in Fig. 7 the calculated and experimental levels Dy^{161} and Lu^{175}. Note that the excitation energy values calculated by us agree better with experimental data in comparison with those in the Nilsson schemes. However, because the levels of odd-mass nuclei depend strongly on the behaviour of average field levels it is difficult to expect to recieve a detailed agreement with experimental data. We investigate therefore the densities of single-particle levels. The average density of neutron levels for $99 \leq N \leq 109$ is found to be equal to 3.3 levels per 1 MeV, and the density of experimental levels averaged in the same way is 3.1 levels per 1 MeV. The average density of the calculated proton levels for $63 < Z < 73$ is equal to 3.6 levels per 1 MeV. The corresponding experimental density is 3.4 levels per 1 MeV. The average densities of the calculated proton and neutron levels is large r than those of corresponding Nilsson schemes by a factor of 1.7. The calculation carried out are in agreement with the investigations of the single-particle level density in/13/.

Thus, as in the transuranic region/2/, the density of low-energy levels agrees with experiment and is about twice as large as the density of levels in the Nilsson scheme. Note the effect of increasing the level density is connected with superfluid properties of ground and excited states. The necessary level density can be obtained by no changes of single-particle levels in the model of independent particles.

Even-Even Nucleus Spectra and Evaluation of Calculation Precision

The most interesting and hopeful is the application of methods based on the superfluid nucleus model to the analysis of even-even nucleus spectra. In[2] it has been hown that in the excited state $|K,K+1\rangle$ of the even system where one quasi-particle is on the level K and another of the level K + 1 (K denotes the final filled up level for G=0 and K + 1 denotes the subsequent level with higher energy etc.) the superfluidity of the system decreases strongly and sometimes is reduced to zero. This is connected with the fact that for the correlated pairs the level K and K+1 are blocked and, therefore, in states available for pairs a large gap appears. Since below this gap the number of states is equal to that of particles and from the energy point of view it is not advantageous for pairs to occupy K+2 and higher levels because of a great loss of kinetic energy, then in the $|K,K+1\rangle$ state the superfluidity is very small. In connection with this the energy of the system in the $|K,K+1\rangle$ state decreases and as a rule the energy difference between the $|K,K+1\rangle$ and ground states is less than the value of energy gap 2 C being the correlation function of the ground state.

We illustrate these considerations on the example $^{102}_{70}Yb^{172}$ whose calculated characteristics for excited states are given in Table II. In the excited proton states $|K,K+1\rangle$ and $|K,K+2\rangle$ et al the superfluidity is absent, i.e. C=0 while in the ground state $C_0 = 0.12\,\hbar\mathring{\omega}_0$ = 0.85 MeV. In higher energy states the superfluidity increases approaching the value of the ground state. In the neutron system the superfluid properties are weakened considerably in the $|K,K+1\rangle$ state, because $C = 0.034\,\hbar\mathring{\omega}_0$ while in the ground state $C_0 = 0.126\,\hbar\mathring{\omega}_0 \approx 0.93$ MeV. Now we compare the calculated spectrum Yb^{172} with experimental data[14], which have been obtained in studying the decay Lu^{172} with the configuration: neutrons $\{1/2-[521]\downarrow\}$, protons $\{7/2+[404]\downarrow\}$, i.e. I = 4, π^- , K= 4. The excited 3^+ 3 1172 KeV state in the spectrum Yb^{172} is undoubtedly a single-particle one. It can be a proton state of the type $\{7/2+[404]\downarrow\} - \{1/2+[411]\downarrow\}$ as well as a neutron one $\{1/2-[521]\downarrow\} + \{5/2-[512]\uparrow\}$. The 3^+3 level is an example of the $|K,K+1\rangle$ state and its energy is lower than that of the gap, since the neutron gap $2C_0 = 1.86$ MeV and the proton one $2C_0 = 1.7$ MeV. Because the superfluid properties of the system in the $|K,K+1\rangle$ state is decreased then its moment of inertia in this state must increase. The experiment confirms this fact since we have $\frac{\hbar^2}{2J} = 13$ for the ground state and $\frac{\hbar^2}{2J} = 11$ for the $|K,K+1\rangle$ state.

Note the moment of inertia of the system which is in the excited state depends on superfluid properties of both the specified and other states. A sharp decrease of the magnitude of the correlation function C for this state does not lead therefore to a considerable change of the moment of inertia of the excited-state system by comparing with that of the ground- and other excited-state-system.

Note that those 0^+ states, where both quasi-particles being on the same level are calculated with smaller accuracy than other states. The conservation of number of particles on the average leads to difficulties which are concentrated in these states. Among two-quasi-particle states there is one redundant state but the ground and 0^+ states are not ortogonal. Indeed, in the case Er^{166} for $\delta = 0.31$ $G_Z = 0.028\,\hbar\mathring{\omega}_0$ and $G_N = 0.024\,\hbar\mathring{\omega}_0$ the evaluations of the non-ortogonality lead to the following results

a) proton states

$\langle K-1, K-1 | 0 \rangle = 0.30$ $\langle K+1, K+1 | K-1, K-1 \rangle = 0.10$
$\langle K, K | 0 \rangle = 0.39$ $\langle K, K | K-1, K-1 \rangle = 0.02$
$\langle K+1, K+1 | 0 \rangle = 0.38$ $\langle K+1, K+1 | K, K \rangle = 0.13$

b) neutron states

$\langle K-1, K-1 | 0 \rangle = 0.08$ $\langle K+1, K+1 | K-1, K-1 \rangle = 0.15$
$\langle K, K | 0 \rangle = 0.12$ $\langle K, K | K-1, K-1 \rangle = 0$
$\langle K+1, K+1 | 0 \rangle = 0.10$ $\langle K+1, K+1 | K, K \rangle = -0.23$

where $| 0 \rangle$ is the ground state.

Using the formulae given in /1/ we evaluate the error which is due to the conservation of number of particles on the average. Values of root-mean square fluctuation of number of particles $\sqrt{\overline{\Delta n^2}}$ calculated are given in Table I and II . The relative magnitude of the fluctuation $\sqrt{\overline{\Delta n^2}}/2\Omega$ (Ω - is the number of summed levels) changes rather strongly in transition from ground states to excited ones but it does not exceed 6 percent. Thus, the accuracy of our calculations is restricted not to the conservation of number of particles on the average but generally to the accuracy with which single-particle levels of the self-consistent field are known.

In conclusion we are pleased to acknowledge N.N. Bogolubov, K.L. Gromov, B.S. Dzelepov, L.K. Peker for highly fruitful discussions of the work.

Table I

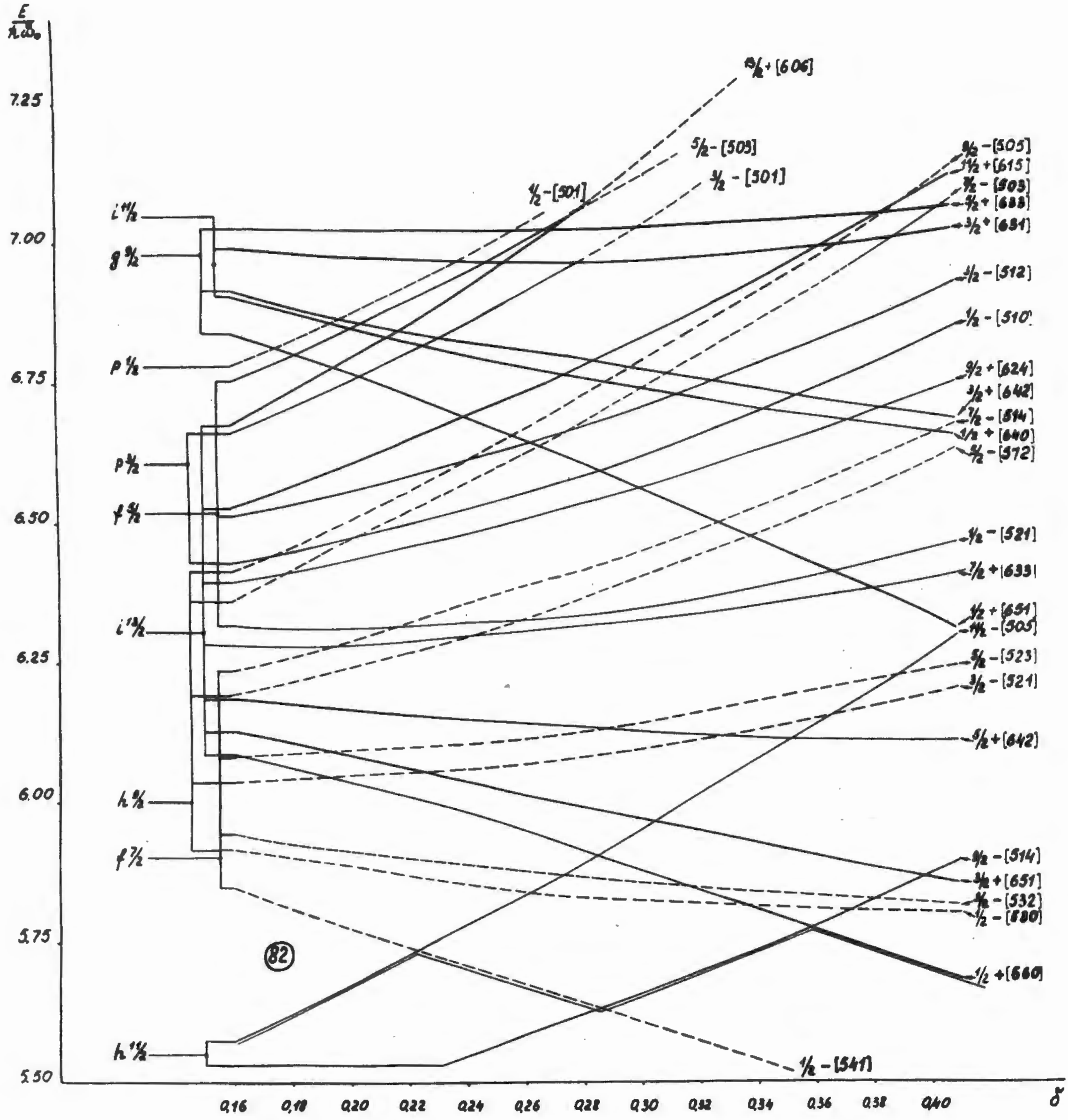
Characteristics of ground and excited states of odd-mass nuclei

	State of the system		C in units $\hbar \overset{\circ}{\omega}_0$	$\sqrt{\overline{\Delta n^2}}$	$E_\rho - \lambda$ in units $\hbar \overset{\circ}{\omega}_0$
N =95, δ =0,30 G =0,024 $\hbar \overset{\circ}{\omega}_0$	K+3	11/2 – [505]	0,091	2,01	– 0,088
	K+2	3/2 + [651]	0,090	2,00	– 0,017
	K+1	3/2 – [521]	0,077	1,77	– 0,004
	K	5/2 + [642]	0,074	1,71	+ 0,010
	K–1	5/2 – [523]	0,076	1,75	+ 0,016
	K–2	7/2 + [633]	0,094	2,08	+ 0,033
	K–3	1/2 – [521]	0,095	2,09	+ 0,034
Z =71 δ =0,28 G =0,028 $\hbar \overset{\circ}{\omega}_0$	K+3	1/2 – [541]	0,101	–	+ 0,123
	K+2	9/2 – [514]	0,089	–	+ 0,118
	K+1	5/2 + [402]	0,080	–	+ 0,113
	K	7/2 + [404]	0,068	–	+ 0,106
	K–1	1/2 + [411]	0,104	–	+ 0,067
	K–2	7/2 – [523]	0,106	–	+ 0,065
	K–3	3/2 + [411]	0,110	–	+ 0,065

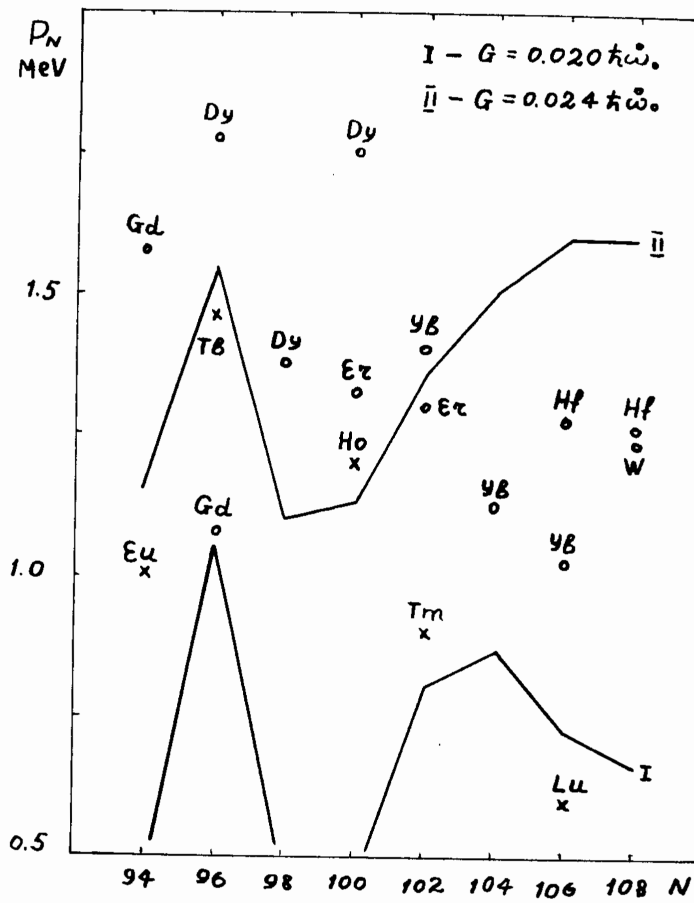
Table II

Spectrum ${}_{70}Yb^{172}_{102}$ $1\hbar\mathring{\omega}_0 = 7.373$ MeV.

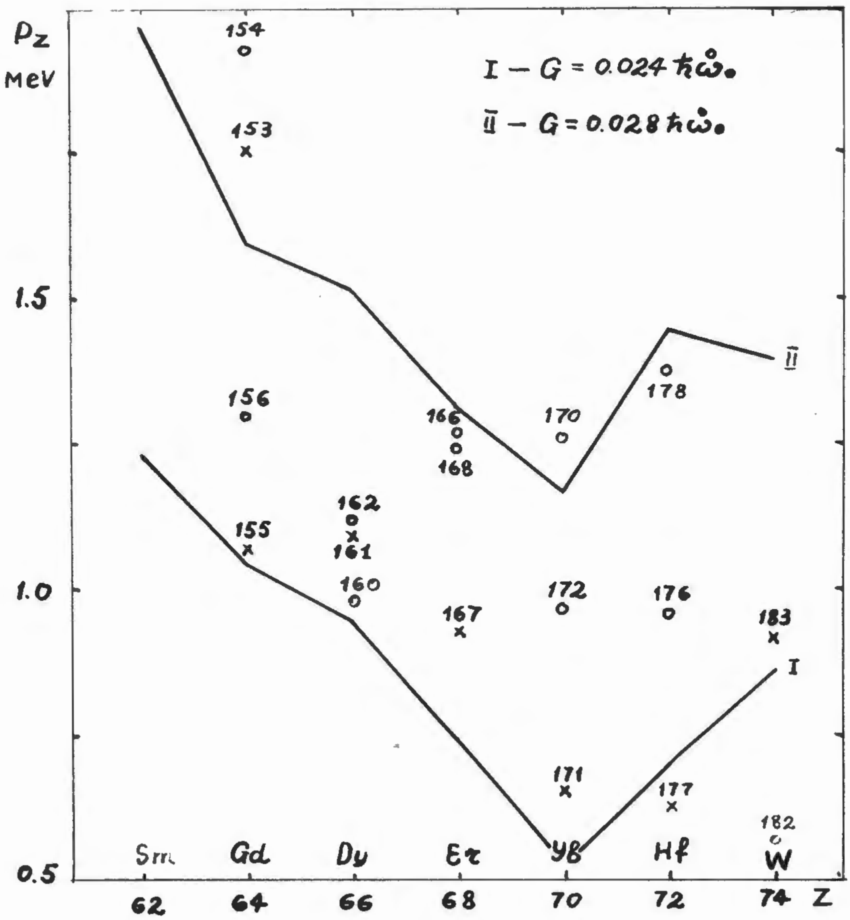
$\delta = 0{,}29$	$G_Z = 0{,}028\,\hbar\mathring{\omega}_0$		Proton levels			$\delta = 0{,}29$	$G_N = 0{,}024\,\hbar\mathring{\omega}_0$		Neutron levels		
state	C ($\hbar\mathring{\omega}_0$)	$\sqrt{\overline{\Delta n^2}}$	$E_F-\lambda$ ($\hbar\mathring{\omega}_0$)	$K\pi$	E (MeV)	state	C ($\hbar\mathring{\omega}_0$)	$\sqrt{\overline{\Delta n^2}}$	$E_F-\lambda$ ($\hbar\mathring{\omega}_0$)	$K\pi$	E (MeV)
$\|0\rangle$	0,109	2,00	−0,084	0+ 0	0	$\|0\rangle$	0,108	2,33	0	0+	0
$\|K,K+1\rangle$	0	0	−0,130	3+,(4+)	1,54	$\|K,K+1\rangle$	0,034	0,87	−0,023	(2+),3+	0,98
$\|K,K+2\rangle$	0	0	−0,114	(2+),3+	1,69	$\|K+1,K+1\rangle$	0,053	1,45	+0,013	0+	1,08
$\|K-1,K+1\rangle$	0	0	−0,124	(o−),7−	1,76	$\|K-1,K+1\rangle$	0,046	1,30	−0,032	1−,(6−)	1,11
$\|K-1,K+2\rangle$	0	0	−0,102	1−,(6−)	1,91	$\|K,K\rangle$	0,057	1,49	−0,050	0+	1,17
$\|K,K+3\rangle$	0	0	−0,101	(4−),5−	1,91	$\|K,K+2\rangle$	0,054	1,41	−0,002	3+,(4+)	1,25
$\|K,K\rangle$	0,084	1,82	−0,165	0+	2,29	$\|K-1,K\rangle$	0,061	1,59	−0,052	(3−),4−	1,30
$\|K+1,K+1\rangle$	0,084	1,77	0	0+	2,40	$\|K-1,K+2\rangle$	0,058	1,54	−0,011	(0−),7−	1,33
$\|K-2,K+1\rangle$	0,019	0,40	−0,099	(2+9,5+	2,50	$\|K+1,K+2\rangle$	0,061	1,62	+0,019	(1+),6+	1,38
$\|K-1,K\rangle$	0,086	1,84	−0,165	(3−),4−	2,50	$\|K-1,K-1\rangle$	0,065	1,67	−0,054	0+	1,42
$\|K+1,K+2\rangle$	0,086	1,79	0	(1+),6+	2,55	$\|K+2,K+2\rangle$	0,067	1,74	+0,025	0+	1,66
$\|K+2,K+2\rangle$	0,087	1,81	0	0+	2,68	$\|K,K+3\rangle$	0,064	1,61	+0,005	(4−),5−	2,03
$\|K-1,K-1\rangle$	0,088	1,87	−0,166	0+	2,70	$\|K+1,K+3\rangle$	0,068	1,73	+0,023	2−,(7−)	2,16



F i g. 1. The Nilsson scheme for the eigenvalues of neutrons
$82 < N < 126$



F i g. 2. Calculated (continuous curves) and experimental/8,9/ values of neutron pairing energies (0-are even-even nuclei, x-are nuclei with odd A).



F i g. 3. Calculated (continuous curves) and experimental[8,9] values of proton pairing energies (0-are even-even nuclei, x-are nuclei with odd A).

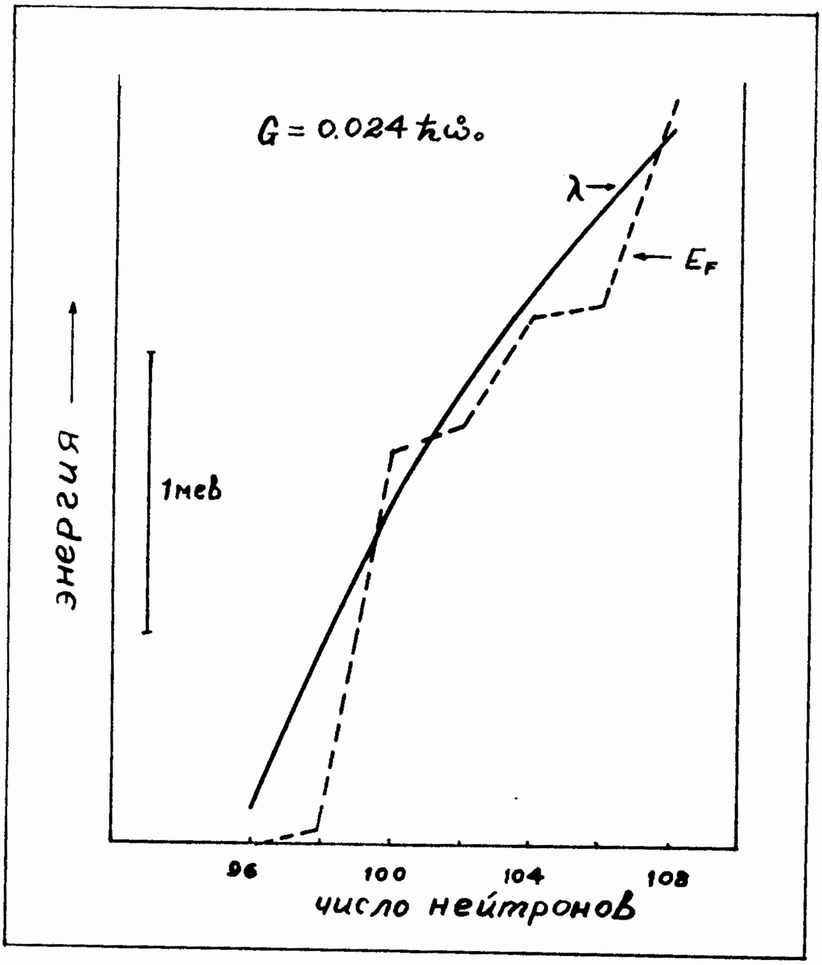
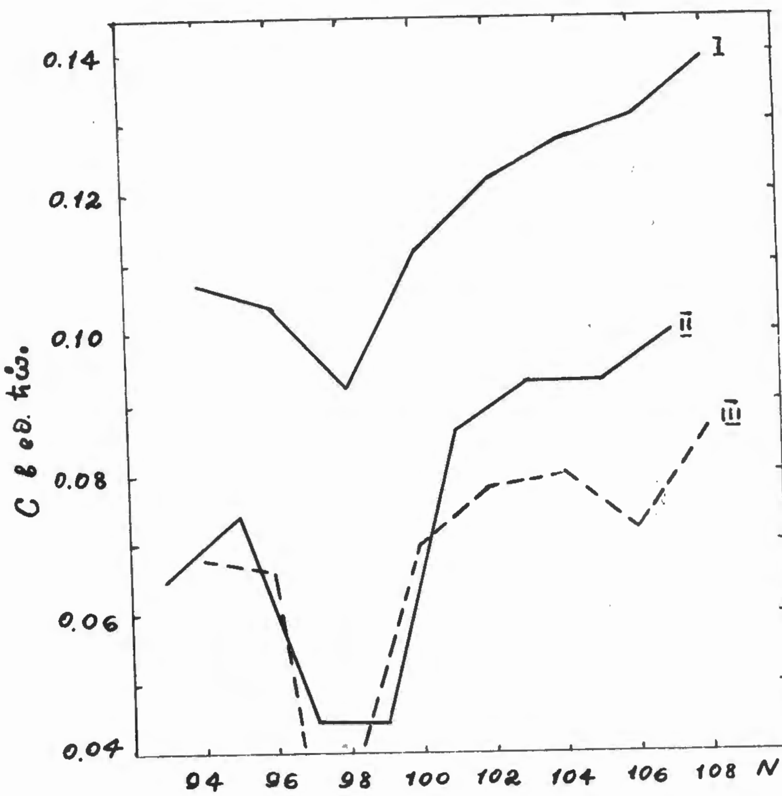
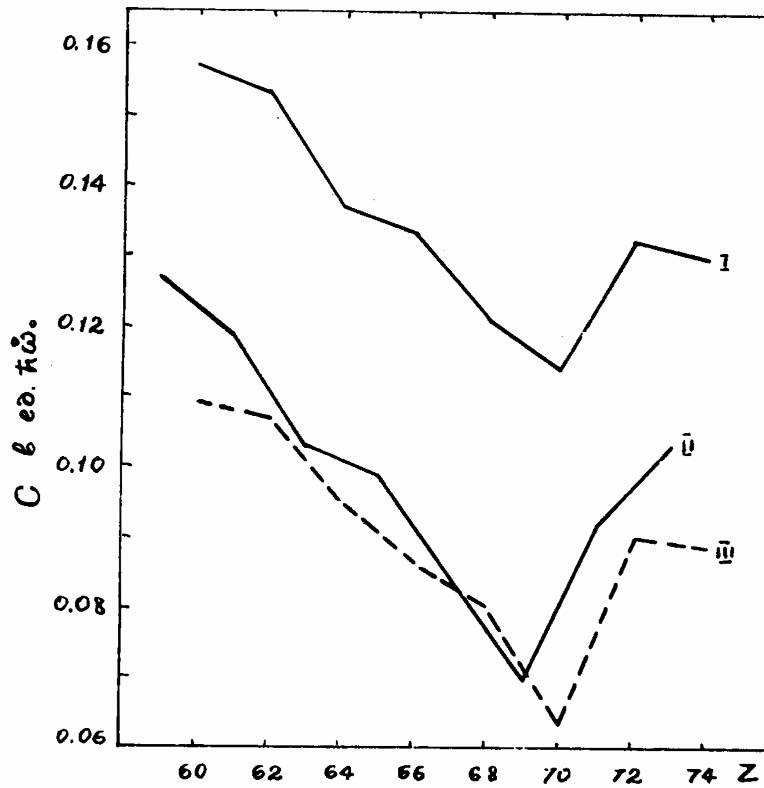


Fig. 4. Behaviour of the Fermi surface energy E_F and of the chemical potential λ for ground states of the even neutron system.



F i g. 5. Behaviour of the correlation function C of the neutron system depending on the number of neutrons N in the nucleus.

I - even-even system for $G = 0.024\ \hbar\omega_0$
II - odd system for $G = 0.024\ \hbar\omega_0$
III - even-even system for $G = 0.020\ \hbar\omega_0$



F i g. 6. Behaviour of the correlation function C of the proton system depending on the number of proton Z in the nucleus

I - even-even system for G = 0.028 $\hbar\omega_0$
II - odd system for G = 0.028 $\hbar\omega_0$
III - even-even system for G = 0.024 $\hbar\omega_0$

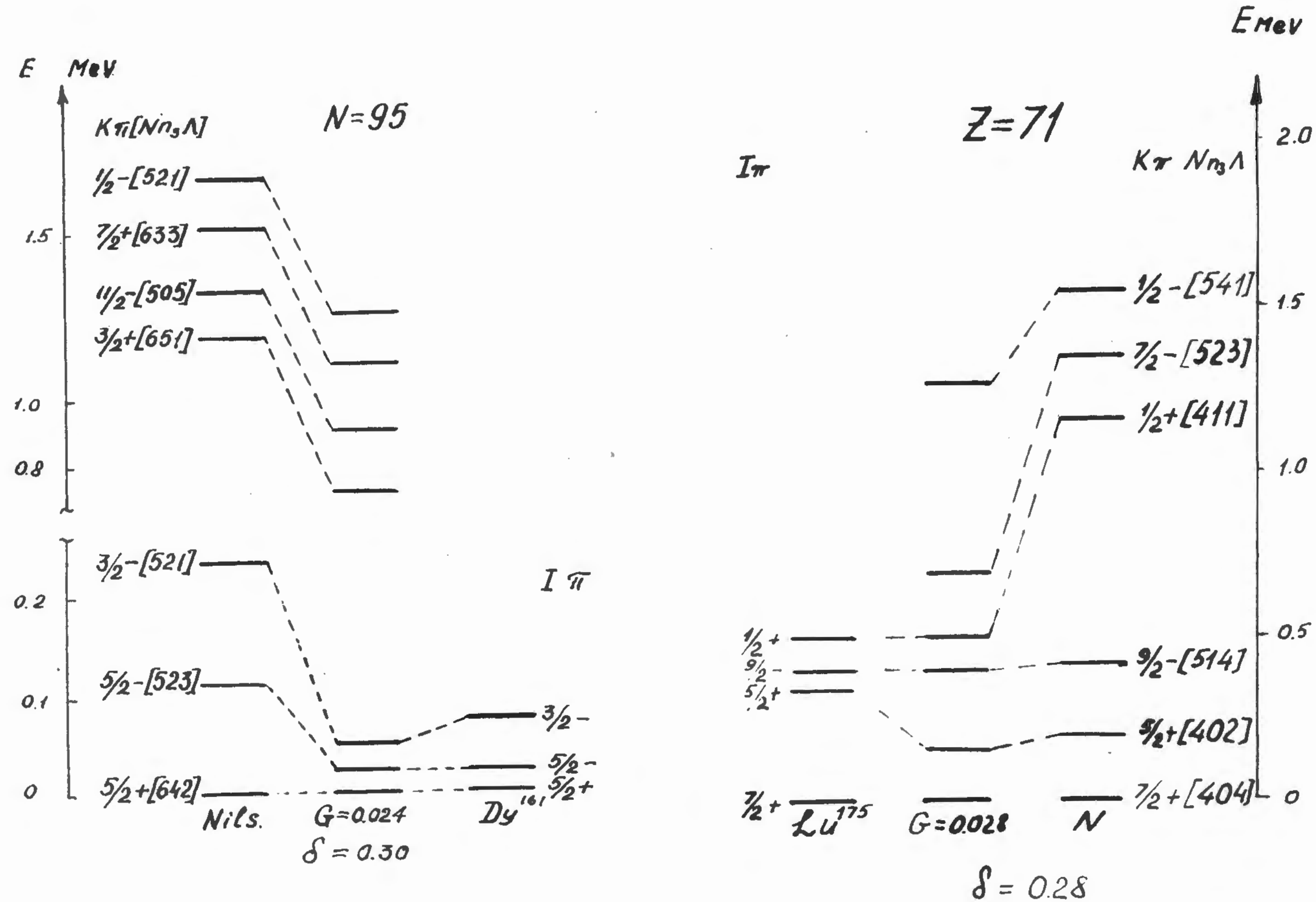


Fig. 7. Single-particle levels of odd-mass nuclei.
Notations: Nils. -spectrum according to the Nilsson scheme; in the middle there is spectrum according to the superfluid nucleus model; X^A – is an experiment (without rotation band), G is given in units $\hbar\overset{\circ}{\omega}_0 = 41\ A^{1/3}$ MeV.

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