ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОБЪЕДИНЕННЫЙ ИНСТИТУТ ЯДЕРНЫХ ИССЛЕДОВАНИЙ

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ON SHAPES OF ALLOWED BETA SPECTRA

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1. Small Deviations in Beta Spectra

One would think that the foundations of beta decay theory are rather well established since the Universal Fermi Interaction explains a great lot of phenomena. There remain, however, some experimental facts which should be carefully analysed in order to account for details of the theory. The most important among them seem to be the small deviations which have been observed in beta spectra [1, 2]. The Fermi-Kurie plots for electron and positron spectra have exhibited deviations from linearity, which could not be explained by means of Fierz interference terms since there exists now a strong experimental support (as well as even more strong theoretical reasons, too) that these terms are vanishing. Furthermore, taking Fierz interference terms would imply opposite direction of the mentioned deviations for electron and positron spectra, whereas experimentally it was found by Langer and his collaborators that in both cases there is an excess of low-energy β -particles. In view of the slightly strange consequences presented in this paper and following from the assumption of Langer 's results it may be pointed as necessary to reinvestigate experimentally the problem in order to obtain either a support for our hypotheses or to give them up.

The following decays were investigated experimentally:

1) $^{32}P \xrightarrow{\beta^-} {}^{32}S$ $(1^+ \to 0^+)$, lg ft = 7.9, allowed G-T transition;
2) $^{114}In \xrightarrow{\beta^-} {}^{114}Sn$ $(1^+ \to 0^+)$, lg ft = 4.5, -"- -"- ;
3) $^{22}Na \xrightarrow{\beta^+} {}^{22}Ne$ $(3^+ \to 2^+)$, lg ft = 7.4, -"- -"- ;
4) $^{90}Y \xrightarrow{\beta^-} {}^{90}Zr$ $(2^- \to 0^+)$, lg ft = 8.0, first forbidden unique transition.

The analysis of experimental spectrum shapes resulted in introducing an empirical multiplicative correction term: $(1 + \frac{b}{E})$ to yield a linear Fermi-Kurie plot, of the same type for $\beta^{\mp}$ -spectra, with the values of b lying in the interval from 0.2 to 0.4 (in our unit system in which $c = \hbar = m_e = 1$). It was shown by the present author [3] that spectrum shape of the desired type may be obtained under a simple assumption of semiphenomenological nature (addition of corrections of first order in the gradient of lepton fields). It is, however, necessary to fit into a regular theory, without any ad hoc assumptions.

II. Numerical Estimations in the Framework of the Conventional Theory

It is well known that usually in unique beta transitions we have to deal with only one decisive energy independent nuclear matrix element $(M_{GT}$ or $B_{ij})$ which plays the role of a multiplicative factor in constructing the Fermi-Kurie plot. With respect to the transitions mentioned, this is approximately true only in the case of the first forbidden decay of ^{90}Y and in the allowed decay of ^{114}In, whereas the two other allowed G-T decays exhibit too high values of lg ft which is an argument in favour of the suppression of the usually dominant matrix element. Thus a series of additional correction terms, usually regarded as being of no importance, may be of comparable magnitude, changing even the resulting spectrum shape. In the conventional treatment of the (V-A) interaction there are following additional terms for allowed G - T transitions:

$$\int \gamma_5 \vec{r}\,, \quad \int \vec{\alpha}\times\vec{r}\,, \quad \int \vec{\sigma} r^2\,, \quad \int (\vec{\sigma}\cdot\vec{r})\vec{r}\,.$$

It was shown [4] that in ordinary decays (in which lg ft does not exceed 4) these corrections are vanishingly small. This result may be adapted to the decay of ^{114}In but the two other decays (with matrix elements less by two orders of magnitude) escape this treatment and need a more thorough analysis.

From the well known relation, valid for allowed transitions :

$$|C_F M_F|^2 + |C_{GT} M_{GT}|^2 = \frac{1}{ft}\cdot\frac{2\pi^3 \ln 2\cdot \hbar^7}{m_e^5 c^4} \qquad (1)$$

let us estimate the Gamow-Teller matrix element (with the value $C_{GT} = C_A$ given in ref.[5]). This matrix element comes to be of the order of $7\cdot 10^{-3}$ for ^{32}P and $12.5\cdot 10^{-3}$ for ^{22}Na. We use the standard formula for spectrum shape:

$$\begin{aligned} N(E)\,dE = \mathcal{F}_0\cdot pEq^2 \Big\{ & (|C_A|^2 + |C_A'|^2)\Big[\left|\int\vec{\sigma}\right|^2 - \tfrac{2}{3}\,\mathrm{Re}\Big\{\Big(\int\vec{\sigma}\Big)^*\Big(\int\vec{\sigma} r^2\Big)\Big\}(E_0-E)\Big(\tfrac{E_0}{2} - \tfrac{5}{6}E + \\ & + \tfrac{1}{3E} - \xi\Big) - \tfrac{4}{9}\,\mathrm{Re}\Big\{\Big(\int\vec{\sigma}\Big)^*\Big(\int(\vec{\sigma}\cdot\vec{r})\vec{r}\Big)\Big\}(E_0-E)\Big(E-\tfrac{1}{E}+3\xi\Big) + \tfrac{2}{3}\,\mathrm{Im}\Big\{\Big(\int\vec{\sigma}\Big)^*\Big(\int\gamma_5\vec{r}\Big)\Big\}\Big(E_0-\tfrac{1}{E}+3\xi\Big)\Big] \qquad (2) \\ & + \tfrac{2}{3}\,\mathrm{Re}\,(C_A^* C_V + C_A'^* C_V')\cdot \mathrm{Re}\Big[\Big(\int\vec{\sigma}\Big)^*\Big(\int\vec{\alpha}\times\vec{r}\Big)\Big]\cdot\Big(E_0 - 2E + \tfrac{1}{E} - 3\xi\Big)\Big\}\,dE \end{aligned}$$

and the approximate form of the nuclear matrix elements given in ref.[6] based on the one-particle model. According to the large ft-value we assume that the matrix element $\int\vec{\sigma}$ is approximately one percent of its value for ordinary favoured transitions. Thus the ratios $\left|\int\gamma_5\vec{r}\right| / \left|\int\vec{\sigma}\right|$ etc. are multiplied by a factor of the order of 10^2. Nevertheless, we do not obtain simply a spectrum proportional to $(1+\frac{b}{E})$, since the terms with $\int\vec{\sigma} r^2$, $\int(\vec{\sigma}\cdot\vec{r})\vec{r}$ and $\int\vec{\alpha}\times\vec{r}$ include an evident energy dependence of another type (terms proportional to E and E^2) which was hitherto unobserved. Only in one very special case we obtain the spectrum of the desired empirical shape for ^{32}P. This happens if we take into account only the additional matrix element $\int\gamma_5\vec{r} \approx 0.0009\cdot 10^2 \int\vec{\sigma}$; we obtain $b \sim 0.29$. Even under this rather artificial assumption of considering only one correction term no agreement with experiment can be achieved in the case of the positron decay of ^{22}Na, since the coefficient b does not exceed 0.05 for the ratio $\left|\int\gamma_5\vec{r}\right| / \left|\int\vec{\sigma}\right|$ from the reasonable interval of values $(0 \div 0.1)$.

The explanation of deviations in beta spectra in the framework of conventional corrections seems thus to be impossible even in the case of the λ-forbidden transitions. It seems to be a mere accident that for one possible value of $\int \gamma_5 \vec{r}$ we obtained the empirical correction for ^{32}P, since similar estimations fail to yield even approximately the correction factor for the positron decay of ^{22}Na. There arises the problem: how to explain the deviations on a solid basis, not by assuming ad hoc some strange cancellations of matrix elements, and without the need to introduce a different explanation for each experimental case separately. Besides, it is of interest to mention that there is also another phenomenon in β - decay, which cannot be explained theoretically. Experimental values of the longitudinal polarization of β -particles were namely $\sim$ 10% lower than the theoretical value $-v/c$[7]. A reinvestigation of the problem made by the same authors has led them to the conclusion that polarization has been systematically underestimated, yet there remain deviations from theory of the order of $\sim$ 5%. This problem seems to be closely connected with the former and perhaps they may be simultaneously solved.

In the following, two tentative explanations of the presented deviations will be given, both being only a first step of a more detailed analysis to be performed subsequently, and both needing a further support. One of these alternative explanations makes the deviations observed due to possible G-nonconserving interactions, whereas the second proves to draw some conclusions from the hypothesis of an intermediary chiral boson.

III. G-nonconserving Interactions

In order to explain the deviations in β-spectra, let us present the most general transition matrix element for a nucleon decay in the notation of Weinberg[8]:

$$M = [\bar{u}_e \gamma_\lambda (1+\gamma_5) u_\nu][\bar{u}_p (f_V \gamma_\lambda + g_V \sigma_{\lambda\mu} k^\mu + i h_V k_\lambda) u_n] +$$
$$+ [\bar{u}_e i\gamma_5 \gamma_\lambda (1+\gamma_5) u_\nu][\bar{u}_p (f_A i \gamma_5 \gamma_\lambda + g_A k_\lambda \gamma_5 + i h_A \sigma_{\lambda\mu} \gamma_5 k^\mu) u_n] \qquad (3)$$

The effect of strong interaction is described by means of 6 form factors f_A, f_V etc., which are functions of $k^2 = k^\lambda k_\lambda$, k_λ being the 4-momentum transfer. If time reversal holds and final state interactions are neglected, the form factors in (3) are real.

The primary weak interactions have been divided by Weinberg[8] into two classes according to the G-transformation* properties of the strongly interacting currents. It was shown that for nucleon processes the terms with h_V and h_A can arise only from the second-class interactions which are absent in the Feynman-Gell-Mann theory. If these interactions would be present, this would break the deep relation between the weak interactions and isotopic spin, suggested by Feynman and Gell-Mann. In the following we

* The G-transformation is defined as the product of charge symmetry and charge conjugation.

shall see that there exists a test for the existence of such interactions.

Let us consider the β -decay of complex nuclei assuming that the main contribution to the nuclear matrix elements comes from the structure of the nucleon, rather than of the nucleus*.

The relativistic form of the S-matrix element is given in Table I. We apply the Foldy-Wouthuysen transformation and the nucleon operators are taken in the nonrelativistic form as in ref [9]. In Table I we have 4-component nucleon spinors, whereas in Table II, where the nonrelativistic form of the S-matrix element is given, we have two-component nucleon spinors and $\vec{\sigma}$ is a Pauli matrix in nucleon space.

In the further treatment the dependence of the form factors upon the four-momentum transfer will be disregarded for a specific transition and they will be treated as constants (which may be, however, different in different decays). A rough guess would give the magnitude of the g and h constants at least about one order of magnitude lower than the magnitude of f. We shall therefore disregard those terms in Table II, in which the g and h constants are multiplied by $\frac{1}{2M}$ (M is here the nucleon mass in units of electron mass).

In order to obtain all desired formulae it is sufficient to adapt the results of paper [9] after a suitable change of notation**.

The problem of relative signs for β^- and β^+ -transitions may be easily solved if Weinberg's Theorem [8] is taken into account. From this theorem a change of the sign of h_V and h_A in interference terms follows for β^+ -decay and the final formulae for spectrum shape are given below:

a) for Fermi transitions ($0 \rightarrow 0$):

$$\mathcal{N}_0(E)dE = \mathcal{F}_0(\pm Z, E)\, p E q^2 \left|\int 1\right|^2 \Big\{ |f_V|^2 \pm 2\,\mathrm{Re}(f_V h_V^*) E_0 + \left[2\,\mathrm{Re}(f_V h_V^*) \pm \tfrac{1}{M}|f_V|^2\right]\left(E_0 \pm 2\xi - \tfrac{1}{E}\right)\Big\}\, dE \qquad (4)$$

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*This means that we consider only the decay of physical nucleon inside the nucleus and we neglect the possibility of exchange or cooperative effects in complex nuclei. Since for high Z Coulomb effects destroy G-invariance, both first- and second-class interactions are retained.

**In Alaga's formulae (1) ... (9) we do the following substitutions (with the new constants on the right side):

a) for Fermi transitions:

$$g_2^2 \rightarrow |f_V + h_V E_0|^2$$

$$\frac{1}{M} g_2^2 \rightarrow 2\,\mathrm{Re}\left[(f_V + h_V E_0)\left(\frac{1}{2M} f_V + h_V\right)^*\right]$$

b) for Gamow-Teller transitions :

$$g_4^2 \rightarrow |f_A - h_A E_0|^2$$

$$\frac{1}{M} g_4^2 \rightarrow 2\,\mathrm{Re}\left[(f_A - h_A E_0)\left(\frac{1}{2M} f_A + h_A\right)^*\right]$$

$$\frac{1}{M} g_2 g_4 \rightarrow 2\,\mathrm{Re}\left[(f_A - h_A E_0)\left(\frac{1}{2M} f_V + g_V\right)^*\right]$$

Three other constants g_1 , g_3 and g_5 have to be put equal to zero. If the S, T and P interactions will be once more of interest, the formulae presented in [9] can be also adapted to take into account the additional terms for tensor interaction which are given by Weinberg[8].

b) for – Gamow-Teller transitions (Δ J = 1):

$$\mathcal{N}_1(E)\,dE = \mathcal{F}_0(\pm Z, E)\, pEq_\nu^2 \,|\int\vec{\sigma}|^2 \Big\{ |f_A|^2 \mp 2\,\mathrm{Re}(f_A h_A^*) E_0 + \\ + \tfrac{1}{3}\big[2\,\mathrm{Re}(f_A h_A^*) \pm \tfrac{1}{M}|f_A|^2\big](E_0 \pm 2\zeta - \tfrac{1}{E}) + \\ + \tfrac{1}{3}\cdot 2\,\mathrm{Re}(\tfrac{1}{M} f_A f_V^* + 2 f_A g_V^*)(E_0 - 2E \mp 2\zeta + \tfrac{1}{E})\Big\}\, dE \qquad (5)$$

The upper signs are for β^--decay, the lower ones - for β^+-decay. The spectrum shape for a Δ J = 0 (not 0→0) transition is given by the sum of the expressions (4) and (5). In the same manner as the formulae for spectrum shapes one may obtain angular correlation formulae from the expressions (3) and (8) in ref. 9/.

One easily finds that Langer's empirical shape correction factor may be easily derived from (4) and (5) . In order to simplify our expression let us assume for the time being that the constants f_V etc. may be considered as real constants, and that the following estimation is correct:

$$|h_V| \gg \tfrac{1}{M}|f_V|$$

$$|h_A| \gg \mathrm{Max}\left(\tfrac{1}{M}|f_A|,\ \tfrac{1}{M}|f_V|,\ |g_V|\right) \qquad (6)$$

It follows that a part of terms appearing in (4) and (5) may be omitted and that the spectrum shape is proportional to $pEq^2\,(1 + \frac{b}{E})$, where

a) for Fermi transitions :
$$b_F = \frac{-2x_F}{1 + 2x_F(E_0 \pm E_0 \pm 2\zeta)}$$
$$x_F = \frac{h_V}{f_V}$$

b) for Gamow-Teller transitions:
$$b_{GT} = \frac{-2x_{GT}}{3 + 2x_{GT}(E_0 \mp 3E_0 \pm 2\zeta)}$$
$$x_{GT} = \frac{h_A}{f_A}$$

The upper (lower) signs are for β^-(β^+)- decay.

The result obtained in this way is important though a little strange: there is some indication that in β-decay we have an admixture of the G-nonconserving interaction of the order of ten percent. For

$x_{GT} = -0.125$, i.e. $h = -1/8\, f_A$ * we obtain $b \sim 0.12$ (for ^{22}Na and ^{32}P) and $b \sim 0.4$ (^{114}In), which is consistent with experimental values. The explanation of Langer's results in this way is still, however, no satisfactory argument in favour of the existence of G-nonconserving interaction and we must wait with final conclusions until more experimental data are available. Especially spectrum shapes for pure G-T and Fermi emitters should be carefully investigated in order to obtain further information about possible deviations.

IV. The Hypothesis of the Intermediate Singlet Boson in Beta Decay

In a series of papers by Tanikawa and his collaborators [10/11/12/] there has been proposed a two-stage process for the Fermi interaction. This interaction should be namely transmitted by some Bose fields with zero spin having only renormalizable interactions with the Fermi particles. The Yukawa model of beta decay was modified in the way that the Bose field had the source consisting of a nucleon and a lepton. Similar modifications have been done with respect to other decays. The effective four - fermion interaction in β -decay is of the experimentally well established (V - A) form :

$$H_F = g^2 \bar{\psi}_p \gamma_\mu (1+\gamma_5) \psi_n \bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_\nu + \text{H.c.} \qquad (7)$$

This interaction results by eliminating the B-field from the basic Hamiltonian:

$$H_S = g\left[\bar{\psi}_p (1-\gamma_5) \psi_e^c B + \bar{\psi}_n (1-\gamma_5) \psi_\nu^c B\right] + \text{H.c.} \qquad (8)$$

where B is a complex scalar neutral field with the same chirality as its multiplier in the Hamiltonian H_S. ψ^c is the charge conjugate spinor $C\bar{\psi}$.

Some time ago there have been proposed many elegant methods and principles to obtain the desired U.F.I. of (V - A) form [13/14/15/]. None of them, however, could give the experimental ratio between the axial vector and vector parts of the Universal Fermi Interaction. It was always -1 instead of the experimental value - 1.25 for C_A/C_V. One possible objection which may arise against the presented method to introduce the (V A) interaction is the same which stands against other elegant principles from which this interaction may be deduced: it likewise gives the ratio C_A/C_V equal to -1 instead of -1.25.

The intermediary boson hypothesis ought not to be regarded, however, as a mere artificial complication of the present situation. One argument in favour of this hypothesis is the fact that it explains in a straightforward manner the deviations in beta spectra. In the energy spectrum of β-particles, besides the usual factor $F_0 E p q^2$ (in the standard β-decay theory notation) we obtain the factor

* The agreement of the theory with experiment for $|h_A|/|f_A| \sim 0,1$ may be regarded as a post hoc justification of the estimation (6).

$$\phi = (M_B^2 - M^2 + 2ME)^{-2} \qquad (9)$$

from the intermediate meson propagator. Here M_B, M and E are the meson mass, nucleon mass and electron energy, respectively. For a suitable value of the meson mass we can obtain a spectrum shape of the desired type. There will be an excess of low energy β -particles of both signs and the deviations in the Fermi-Kurie plot may agree with a high accuracy with the deviations observed experimentally [1/ 2/]. In order to illustrate this thesis we give below a table of the values of the quantity

$$\vartheta = \sqrt{\frac{N(E)}{pE\mathfrak{F}_0}} \cdot q^{-1} \qquad (10)$$

which after multiplying by $q = (E_0 - E)$ should give the Fermi-Kurie plot. The values are given for a β-transition in which the maximum β -particle energy E_0 is equal to 5 (in units of $m \; c^2$). This magnitude of E_0 is of the order appearing in the experiments of Lanaer [1/2/] where it was e.g. 4.9 for ^{114}In and 4.34 for ^{32}P. The numerical values in Table III are given with the accuracy to a common multiplicative factor which was arranged in the way as to yield unity for the spectrum end.

It may be easily concluded from Table III that a possible existence of an intermediary boson of mass $\sim$ 1900 m_e may account in a sufficient manner for the observed deviations in beta spectra.

V. Final Remarks

The alternative explanations above of the deviations in beta spectra do not pretend to exhaust all the theoretical possibilities. It was the author's intention to confine attention only to those which seemed the simplest in the present stage when scarce experimental data do not allow to distinguish between them.

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Table I

Relativistic Form of the S - matrix Element for Nucleon Decay

	Lepton covariant	General form of the nucleon covariant for pure Fermi transitions	General form of the nucleon covariant for pure G-T transitions
V	$L(1) = [u_e^+ (1+\gamma_5) u_\nu]$	$[u_p^+ (f_V + g_V \vec{P} \cdot \beta\vec{\alpha} + h_V E_0 \beta)\, u_n]$	$[u_p^+ (g_V \vec{P} \cdot \beta\vec{\alpha})\, u_n]$
	$L(\vec{\alpha}) = [u_e^+ \vec{\alpha} (1+\gamma_5) u_\nu]$	$[u_p^+ (-f_V \vec{\alpha} - g_V E_0 \beta\vec{\alpha} - h_V \vec{P} \beta)\, u_n]$	$[u_p^+ (-f_V \vec{\alpha} - g_V E_0 \beta\vec{\alpha} + i g_V \vec{P} \times \beta\vec{\sigma})\, u_n]$
A	$L(\vec{\sigma}) = [u_e^+ \vec{\sigma} (1+\gamma_5) u_\nu]$	$[u_p^+ (i h_A \vec{P} \times \beta\vec{\alpha})\, u_n]$	$[u_p^+ (f_A \vec{\sigma} - g_A \vec{P} \beta\gamma_5 - h_A E_0 \beta\vec{\sigma} + i h_A \vec{P} \times \beta\vec{\alpha})\, u_n]$
	$L(\gamma_5) = [u_e^+ \gamma_5 (1+\gamma_5) u_\nu]$	———	$[u_p^+ (-f_A \gamma_5 - g_A E_0 \beta\gamma_5 - h_A \vec{P} \cdot \beta\vec{\sigma})\, u_n]$
	Nonrelativistic form of the nucleon operators	$\vec{\alpha} = \frac{1}{2M} \vec{P}$ $\beta = 1$	$\vec{\alpha} = -\frac{i}{2M} \vec{P} \times \vec{\sigma}$ $\beta = 1$ $\gamma_5 = \frac{1}{2M} \vec{P} \cdot \vec{\sigma}$

Table II

Nonrelativistic Form of the S-matrix Element for Nucleon Decay

	Lepton covariant	Nonrelativistic form of the nucleon covariant for pure Fermi transitions	pure G-T transitions
V	$L(1)$	$\{u_p^+[f_V + h_V E_o + \underline{\frac{1}{2M} g_V \lvert\vec{P}\rvert^2}]u_n\}$	————
	$L(\vec{\alpha})$	$-\{u_p^+[(\frac{1}{2M}f_V + h_V + \underline{\frac{1}{2M} g_V E_o})\vec{P}]u_n\}$	$i\{u_p^+[(\frac{1}{2M}f_V + g_V + \underline{\frac{1}{2M} g_V E_o})(\vec{P}\times\vec{\sigma})]u_n\}$
A	$L(\vec{\sigma})$	————	$\{u_p^+[(f_A - h_A E_o - \underline{\frac{1}{2M} h_A}\lvert\vec{P}\rvert^2)\vec{\sigma} + \underline{\frac{1}{2M}(-g_A + h_A)(\vec{P}\cdot\vec{\sigma})\vec{P}}]u_n\}$
	$L(\gamma_5)$	————	$-\{u_p^+[(\frac{1}{2M}f_A + h_A + \underline{\frac{1}{2M} g_A E_o})(\vec{P}\cdot\vec{\sigma})]u_n\}$

The terms underlined are second order correction terms discarded in further considerations.

Table III

Values of the Quantity $\backepsilon$ for a Hypothetical β- Decay with $E_o = 5$

E	$\backepsilon$ from the formula with the empirical correction $(1+\frac{b}{E})$		$\backepsilon$ from the formula with the theoretical correction factor (9) for the boson mass M_B			
	$b = 0.2$	$b = 0.3$	$1875\ m_e$	$1900 m_e$	$1925 m_e$	$1950 m_e$
1	1.07	1.11	1.11	1.06	1.045	1.035
2	1.03	1.04	1.08	1.045	1.035	1.025
3	1.02	1.02	1.05	1.03	1.02	1.015
4	1.005	1.01	1.025	1.02	1.01	1.01
5	1	1	1	1	1	1