JOINT INSTITUTE FOR NUCLEAR RESEARCH

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E-456

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ЖЭТФ, 1960, т 38, в 6, с 1872- 1878.

Dubna 1960

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Submitted to JETP

ABSTRACT

It is shown that the invariance of the universal V–A Fermi weak interaction with respect to the Fierz transformation forbids the μ^- meson being captured by the proton in the triplet S state. It also forbids the μ^- meson being captured by the proton in the singlet S state with the emission of one photon. It is shown further that the state of two fermions or that of two anti-fermions produced in all weak interaction processes can be described by one wave function, if the effects of other interactions involving these two particles are neglected. In particular, the general functions describing the polarization correlation are given. The asymmetry in the angular distribution of the neutron emitted by a nucleus after capturing a μ -meson is discussed.

I

Zeldovich, Gerstein[1] and Chou, Maevsky[2] have shown that the universal Fermi weak interaction proposed by Feynman and Gell-Mann[3], on the one hand, and by Sudarshan and Marshak[4], on the other, forbids the μ -meson being captured by the proton in the triplet S state. It was later pointed out by Dye et al[5], that this interaction also forbids the radiative capture of the μ -meson by the proton with the emission of one photon in the singlet S state. It is interesting to investigate whether the vanishing of the S -matrix elements describing these processes are due to the operation of some law of symmetry. As the consequences of symmetries are usually quite general, its investigation might be useful to the test of the universality of the V–A Fermi weak interaction. On the other hand, the symmetry of the weak interaction is usually weakened by the strong interaction, such investigation might be also useful to the study of the renormalization effects of the strong interactions on the weak interaction.

It is shown in the next section that the state of two fermions or that of two anti-fermions produced in all weak interaction processes can be described by one wave function if the effects of other interactions involving these two particles are neglected. In their centre of mass system these two particles have zero total angular momentum. It gives immediately a rigorous proof of the results of [1] and [5], in which the non-relativistic approximation was used and only the contributions of the first non-zero perturbation approximation were calculated. In the third section, the functions describing the polarization correlation between two fermions or that of two anti-fermions produced in any process induced by the universal Fermi weak interaction of various possible forms as proposed in [3] are given, if other interactions involving these two particles are neglected. Thus the measurement of the polarization correlation can either be used as a test of the universality of the V–A Fermi weak interaction, or to yield information on the renormalization effect of the strong interaction. The polarization of the photon emitted in a weak interaction process is also discussed in connection with the two component spinor theory proposed in [3]. In the last section it is pointed out, that the relativistic effect can induce asymmetry in the angular distribution of the neutron emitted by a nucleus after capturing a μ -meson, even if the renormalization effect and the effect of the final state interaction are neglected.

II

It is well known, that the universal Fermi weak interaction

$$H_i = (A,B)(C,D) \equiv \frac{G}{\sqrt{2}}\, \bar{\psi}_A \gamma_\mu (1+\gamma_5)\psi_B \cdot \bar{\psi}_C \gamma_\mu (1+\gamma_5)\psi_D \tag{1}$$

proposed in /3/ and /4/ has interesting properties with respect to a transformation studied by Fierz/6/

$$\psi_B \rightleftarrows \psi_D \tag{2}$$

ψ_A, ψ_B, ψ_C and ψ_D are fields describing respectively four kinds of ½ spin particles A,B,C and D. If they are treated as anticommuting quantities we have

$$(A, B)(C, D) = (A, D)(C, B). \tag{3}$$

On the other hand, if they are commuting quantities we have

$$(A, B)(C,D) = -(A,D)(C, B). \tag{4}$$

The first order perturbation approximation of the S -matrix element describing the capture of the μ -meson by the proton is

$$\frac{-iG}{\sqrt{2}} \int \bar{\psi}_N \gamma_\mu (1+\gamma_5)\psi_p\, \bar{\psi}_\nu \gamma_\mu (1+\gamma_5)\psi_\mu \, dx. \tag{5}$$

If the μ mesic atom is in the triplet S state with the total spin parallel to the Z -axis and if the velocity in the K orbit is neglected, then we have:

$$\psi_p \times \psi_\mu = N \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} e^{-iEt} \tag{6}$$

where N is the amplitude of the wave function when the coordinates of the proton and those of the μ -meson coincide, E is the energy of the μ -mesic atom. From (4) and (6) follows immediately that the expression (5) must be zero, and thus confirms the results of /1/ and /2/.

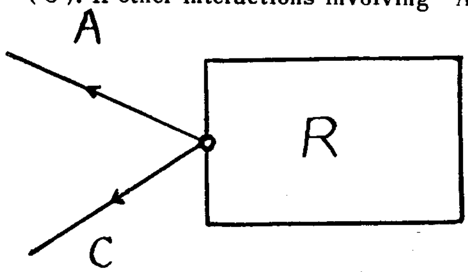
For the convenience of further investigations, it is useful to subject (1) at first to a charge conjugation transformation,

$$H_i = -\frac{G}{\sqrt{2}}\, \bar{\psi}_{\bar{B}} \gamma_\mu (1-\gamma_5)\psi_{\bar{A}}\, \bar{\psi}_C \gamma_\mu (1+\gamma_5)\psi_D \tag{7}$$

where the fields are treated as anti-commuting quantities, $\bar{A}$ and $\bar{B}$ denote the anti-particles of A and B. Thus $\psi_{\bar{A}}$ is an operator which can annihilate an anti-particle $\bar{A}$ or create a particle A. After a Fierz transformation (7) becomes:

$$H_i = \sqrt{2}\, G\, \bar{\psi}_C (1-\gamma_5)\psi_{\bar{A}}\, \bar{\psi}_{\bar{B}} (1+\gamma_5)\psi_D \tag{8}$$

Let us study a process, in which two fermions A and C are emitted through the operation of (8). If other interactions involving A and C are neglected, the Feynman diagramm



can be schematically represented by Fig. 1. The corresponding S-matrix element in the momentum representation is of the following form:

$$\langle \vec{p}_a, \vec{S}_a; \vec{p}_c, \vec{S}_c; \lambda | S | i \rangle = \bar{u}(\vec{p}_c, \vec{S}_c)(1-\gamma_5)\, v(-\vec{p}_a, -\vec{S}_a)\cdot R \qquad (9)$$

where $\vec{p}_a$, $\vec{S}_a$ and $\vec{p}_c$, $\vec{S}_c$ are the momenta and the spins of A and of C, respectively. λ represents all the remaining quantum numbers chosen to label the final state. $|i\rangle$ is the initial state. u, v and $(1 - \gamma_5)$ represent respectively the external lines and a part of the vertex shown in Fig. 1. u is the wave function of the positive energy state, while v is that of the negative energy state. R represents all the remaining parts of the diagramm, which are shown as a block in the figure. The final state is then:

$$|f\rangle = |\vec{p}_a, \vec{S}_a; \vec{p}_c, \vec{S}_c; \lambda\rangle\langle \vec{p}_a, \vec{S}_a; \vec{p}_c, \vec{S}_c; \lambda | S | i \rangle . \qquad (10)$$

In particular, when the momentum-energy $p_a + p_c$ transferred to the partial system composed of A and C is given, the wave function W of this partial system is

$$W(\vec{p}_a, \vec{S}_a; \vec{p}_c, \vec{S}_c) = \left\{ \bar{u}(\vec{p}_c, \vec{S}_c)(1-\gamma_5)\, v(-\vec{p}_a, -\vec{S}_a) \right\} \times u(\vec{p}_a, \vec{S}_a)\, u(\vec{p}_c, \vec{S}_c) \qquad (11)$$

which is independent of R . Thus the wave function of the partial system composed of A and C is independent of what else is happening in the process, if the momentum-energy transferred to the partial system is given and if other interactions involving A and C are neglected.

The wave function W is easily calculated. When W is written down in the center of mass system of A and C, it is found that the total inherent angular momentum of the partial system is equal to zero. It is a superposition of ${}^1S_0$ and ${}^3P_0$ states. If one of the particles is a neutrino, the ${}^1S_0$ and the ${}^3P_0$ states have equal weights. However, if the velocities of A and C are both small, W is essentially a ${}^1S_0$ state, the relative amplitude of the ${}^3P_0$ state is only of the order v , which is the speed of A or that of C . The speed of the light is taken as unity. The result is immediately understandable from the form of the expression (8). It is convenient to represent $\bar{\psi}_c(1-\gamma_5)\psi_{\bar{A}}$ by a single factor ϕ and to regard A and C as forming one single system. (8) becomes then:

$$H_i = \sqrt{2}\, G\, \phi\, \bar{\psi}_{\bar{B}}(1+\gamma_5)\psi_D \qquad (12)$$

ϕ behaves like a scalar with respect to the proper Lorentz transformation. Thus the internal angular momentum of the system composed of A and C emitted through the operation of (12) must be zero.

The results of /1/ /2/ and /5/ are now easily understandable. The capture of the μ -meson by the proton in the ^3S state is rigorously forbidden due to the law of conservation of angular momentum. As the radiative transition from a $J=0$ state to another $J=0$ state with the emission of one photon is impossible, the radiative capture of the μ meson by the proton in the ^1S state with the emission of one photon is also rigorously forbidden.

III

Since the wave function W is independent of what else is happening in the process, it can be applied in a quite wide field. For the sake of illustration, the polarization correlation of A and C is discussed. Table 1 contains the relative probabilities for various polarization states, which are easily obtained from the wave function W.

Table 1

$\vec{S}_a, \vec{p}_a$	$\vec{S}_c, \vec{p}_c$	Relative probability
parallel	*parallel*	$\frac{1}{2}(1-v_a)(1-v_c)(1-\cos\theta_{ac})$
parallel	*anti-parallel*	$\frac{1}{2}(1-v_a)(1+v_c)(1+\cos\theta_{ac})$
anti-parallel	*parallel*	$\frac{1}{2}(1+v_a)(1-v_c)(1+\cos\theta_{ac})$
anti-parallel	*anti-parallel*	$\frac{1}{2}(1+v_a)(1+v_c)(1-\cos\theta_{ac})$.

v_a and v_c are the magnitudes of the velocities of A and C respectively. θ_{ac} is the angle between $\vec{p}_a$ and $\vec{p}_c$. The functions in Table 1 described the polarization correlation of fermions emitted in all weak interaction processes, provided other interactions involving these fermions can be neglected. For example, let us investigate the neutrino-neutron polarization correlation in the radiative capture of the μ - meson by the proton. Let A denote the neutrino, while C represent the neutron, then it is evident from Table 1, that the neutrino must be lefthanded, while the degree of the longitudinal polarization of the neutron is:

$$\frac{\cos\theta - v}{1 - v\cos\theta} \qquad (13)$$

v is here the neutron velocity, and θ is the angle between the neutrino and the neutron momenta. Table 1 can be applied as well to the μ meson decay, β decay of nucleons and hyperons ... etc.

Since the theory is invariant with respect to the time reversal, Table I can also be applied to the discussion of the dependence of the transition probability on the polarization state of the fermions in the initial state. As an example it can be applied to the discussion of the dependence of the capture rate of the μ -meson on the hyperfine structure of the μ -mesic atom. As a first orientation, it can be assumed that only $^1S_0$ state of the μ meson and the proton contribute to the capture. To include in finer details, the contribution of the $^3P_0$ state has also to be taken into account.

As the V–A Fermi interaction is invariant with respect to the combined reflection, the universal functions which describe the polarization correlation between the anti-fermions can be obtained from Table I by the following substitution:

$$a \rightarrow \bar{a}\,, \qquad c \rightarrow \bar{c} \qquad (14)$$

$$\text{parallel} \rightleftarrows \text{anti-parallel}$$

where indices $\bar{a}$ and $\bar{c}$ are used to specify quantities referring to the anti-fermions $\bar{A}$ and $\bar{C}$. Violations of the laws described by these functions either question the universality of the V–A Fermi weak interaction theory or point to the effect of other interactions, such as the effect of the renormalization.

As pointed out in /3/ , the interaction Hamiltonian

$$H_i = \frac{G}{\sqrt{2}} \bar{\psi}_A \gamma_\mu (1-\gamma_5) \psi_B \bar{\psi}_C \gamma_\mu (1-\gamma_5) \psi_D \qquad (15)$$

as well as the interaction Hamiltonian

$$H_i = \frac{G}{\sqrt{2}} \bar{\psi}_A \gamma_\mu (1-\gamma_5) \psi_B \bar{\psi}_C \gamma_\mu (1+\gamma_5) \psi_D \qquad (16)$$

can be a priori with equal justifications be chosen as the universal form of the weak interaction. In order to decide, which of the interaction Hamiltonians (1), (15) and (16) is the correct one, it is necessary to investigate what different consequences these interaction Hamiltonians lead to. It is therefore useful to point out, that (1), (15) and (16) lead to different polarization correlations.

(15) can be obtained from (1) by a space reflection. All results obtained from (1) can be transcribed into those corresponding to (15) by means of a space reflection. In particular, scalar quantities will remain thereby unchanged, while pseudoscalar quantities will change their sign. In order to distinguish between (1) and (15), pseudoscalar quantities have to be measured. The polarization correlation between the fermions in the process caused by (15) can be obtained from Table 1 by the following substitution:

$$\text{parallel} \rightleftarrows \text{anti-parallel} \qquad (17)$$

The polarization between anti-fermions can be obtained in a similar way. After a charge conjugation transformation (16) can be written as:

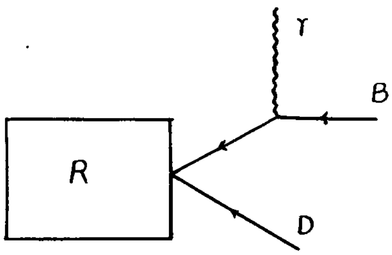
$$-\frac{G}{\sqrt{2}}\,\bar{\psi}_{\bar{B}}\,\gamma_\mu(1+\gamma_5)\psi_{\bar{A}}\,\bar{\psi}_c\,\gamma_\mu(1+\gamma_5)\psi_n \tag{18}$$

Thus the polarization correlation resulted from (16) can be obtained from Table 1 by the substitution

$$a \rightarrow \bar{b} \tag{19}$$

Thus (16) leads to a universal polarization correlation between a fermion and an anti-fermion.

Interactions formed of two component spinors as proposed in /3/ also lead to interesting consequences with respect to the polarization state of the photon emitted by a fermion in the weak interaction process, if the fermion is initially at rest. Take for example the process described by the diagramm shown in Fig. 2. The vertex represents the V–A Fermi weak interaction. A γ quantum is emitted by the particle B . The corresponding S matrix element is:



$$R\cdot\gamma_\mu(1+\gamma_5)\,\frac{i(\hat{p}_b-\hat{k})-m_b}{(p_b-k)^2+m_b^2}\cdot\frac{\hat{e}^*}{\sqrt{2\omega}}\,u(\vec{S}_b,\vec{p}_b) \tag{20}$$

R represents the whole diagramm except the parts starting from the external line B and ending at the weak interaction vertex. The meanings of the other factors and notations are obvious. We have:

$$\left\{i(\hat{p}_b-\hat{k})-m_b\right\}\hat{e}^* = \hat{e}^*\left\{i\hat{k}-i\hat{p}_b-m_b\right\} \tag{21}$$

since B is initially at rest. As

$$\left\{i\hat{p}_b+m_b\right\}u(\vec{S}_b,\vec{p}_b)=0 \tag{22}$$

the expression (20) becomes:

$$\frac{i R\cdot\gamma_\mu(1+\gamma_5)\hat{e}^*\hat{k}\;u(\vec{S}_b,\vec{p}_b)}{\left\{(p_b-k)^2+m_b^2\right\}\sqrt{2\omega}}\,. \tag{23}$$

It is easily shown that for the left-hand polarized photon

$$\hat{e}^*\hat{k} = -\frac{1}{\sqrt{2}}(1-\gamma_5)\,\vec{e}^{\,*}\cdot\vec{\sigma}\cdot\omega\,. \tag{24}$$

It follows from (23) and (24) that the emission of the left-hand polarized photon by B is forbidden, a result first pointed out in /5/ and by Manacher and Wolfenstein/7/. It is easily seen that if the weak interaction is described by (15) , the emission of the right-handed photon by B at rest is forbidden. Therefore, the measurement of the polarization of the photon emitted by B at rest offers a possible way of deciding, whether the interaction Hamiltonian is of the form (1) or of form (15).

IV

As an example of the application of the results of the previous sections, the angular distribution of the neutron emitted by a nucleus, which has captured a μ meson is discussed. It was shown by Dolinsky and Blokhintsev[8], that the neutron emitted by an unpolarized nucleus capturing a polarized μ -meson and that emitted by a polarized nucleus capturing an unpolarized μ -meson both have an isotropic distribution, if the weak interaction leading to the capture is of the V–A type and if the effects of the renormalization and the final state interaction are neglected. The non-relativistic approximation is used in their calculation. In particular the velocity of the proton is assumed to be very small, the small components of the proton wave function are neglected. Their results are now immediately understandable. In their centre of mass system, the $\mu^{\cdot}$ meson can only be captured by the proton, when they are in a $J = 0$ state. It is therefore evident, that the angular distribution of the emitted neutron must be isotropic, irrespective of whether the nucleus and the μ - meson are polarized or not, so long as the μ -meson and the proton in the nucleus are regarded as at rest before the capture.

The above results will be modified by two effects, namely: 1) the effects of the strong interaction, in particular, the effects of the renormalization and the final state interaction; 2) the relativistic effect due to the motion of the proton. It is shown in[8], that an appreciable anisotropy would appear, if the effects of the strong interaction are taken into account. The important terms in their expression for the asymmetry parameter are proportional to

$$\frac{E_\nu}{m_N} \simeq \frac{m_\mu}{m_N} \qquad (25)$$

where E_ν is the energy of the emitted neutrino, m_μ and m_N are the mass of the μ meson and that of the nucleon respectively. Due to the constructive interference of the weak magnetism term and the pseudoscalar term, the numerical coefficient before the expression (25) is equal to 4 , which is quite large.

By using Table 1, it can be shown easily, that the relativistic effect can also give rise to an asymmetry in the angular distribution, which is proportional to

$$\frac{m_\pi}{m_N} \qquad (26)$$

m_π is the mass of the π -meson, which is also roughly the magnitude of the momentum of the nucleon in the nucleus. As is already stated above, the neutron is emitted isotropically in a system, in which the center of mass of the μ -meson and the proton is at rest. In a system, in which the centre of mass is moving, the neutron is emitted preferentially in the forward direction. Let us investigate the case, in which the proton capturing the μ -meson is in the $S_{\frac{1}{2}}$ state. The μ -meson is assumed to be at rest and completely polarized in the direction of the Z -axis. The proton, and therefore the centre of mass of the proton and the μ -meson, both move isotropically in all direction. Since the capture probability is strongly spin and velocity dependent as shown by Table 1, the relativistic effect easily leads to asymmetry in the angular distribution of the neutron.

For the sake of illustration, the following two situations are analysed in detail:

1) The proton moves in the direction of the Z -axis.

2) The proton moves in the direction opposite to the Z -axis.

The indices α and c in Table 1 are now understood to label the proton and the μ -meson respectively. If the spin of the proton is parallel to the spin of the μ -meson, the capture probability must be zero according to Table 1. If the spin of the proton is anti-parallel to the spin of the μ -meson, the capture probability in the situation 1) is, according to Table 1, proportional to

$$(1 + v) \tag{27}$$

while the capture probability in the situation 2) is proportional to:

$$(1 - v) \tag{28}$$

v is here the magnitude of the velocity of the proton. Thus the proton has a large probability of capturing the μ -meson, if its direction of motion is parallel to the spin of the μ -meson. More neutrons are therefore emitted along the direction of the polarization of the μ -meson. The asymmetry is obviously proportional to:

$$v \simeq \frac{m_\pi}{m_N} .$$

The above result is confirmed by a calculation, in which the wave function of the proton is assumed to be that of a particle in a spherical box, while the wave function of the neutron is assumed to be that of the plane wave.

The above discussion shows, that the effect of the renormalization is as usual blurred here by the relativistic effect. Incidently, the effect of the renormalization and the relativistic effect work against each other in this case, as the effect of the renormalization causes more neutrons to be emitted in the direction opposite to that of the spin of the μ -meson according to [8]. The interaction of the final state such as the spin orbital interaction, makes the situation still more complicated. It is interesting to note, that the experiment of Astbury et al.[9] on S^{32} gives a negative asymmetry parameter, while the experiment of Baker and Rubbia[10] on the magnesium gives a positive asymmetry parameter. It might be useful to perform experiments on nucleus which has a simple structure, such as Ne^{20}. Since the nucleons in the outermost shell are in the 2 s state, the treatment of the spin orbital interaction in the final state is easier.

It has been shown experimentally[11], that the β -decay interaction is mainly of the V–A type. It is interesting to see whether the weak interaction leading to the μ -capture is also of the V–A type. Telegdi[12] showed, that the μ -capture interaction cannot be of the form (16). However, the experiment which he cited does not prove that the μ -capture interaction is necessarily of the V–A type, viz. of the form (1). Since the quantity measured in the experiment is a scalar, it can also be explained by an interaction of the form (15). In order to pin down the μ -capture interaction as of the V–A type, a pseudoscalar quantity has yet to be measured, such as the longitudinal polarization of the neutron emitted during the capture.

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Received on January 3, 1960.