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ADLER-WEISBERGER RELATION AND DISPERSION SUM RULES IN THE THEORY OF STRONG INTERACTIONS

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§ 1. Introduction

Difficulties arizing in the theory of strong interactions led to revisions of the principal ideas of quantum field theory. There were even doubts about the validity of its application to strong interactions /1/. Recently the situation has still more aggravated because the strongly interacting particles are now thought of as not elementary and their interaction is assumed to be nonlocal. In this connection a question arises: which local quantities can be preserved in the field theory and how is it possible to obtain information on strong interactions out of these quantities.

In formulating the local quantum field theory the fundamental quantities are the radiation operators or currents /2/ whose matrix elements are straight forwardly connected with the vertex functions and amplitudes of different processes.

The local properties of the currents and the microcausality condition allow us to prove the dispersion relations for the amplitudes of different processes.

In the conventional quantum field theory to each elementary particle there correspond its fields and currents.

Applying quantum field theory to strong interactions one should take into account that the strongly interacting particles, the number of which is increasing, cannot be considered elementary in the true sense of this word.

At the present time a great deal of attention is being paid to the quark model in which all the strongly interacting particles-hadrons are build up out of three basic particles-quarks. From the point of view of quantum field theory only these basic particles can, apparently, be called elementary, and only to them one can ascribe their fields and currents.

Out of the presently known particles only leptons can be apparently considered elementary. According to the low of the leptonic charge conservation, leptons interact with hadrons only in pairs: $(e\tilde{\nu}_e)$ and $(\mu\tilde{\nu}_\mu)$ in weak interactions and $(e\tilde{e})$, $(\mu\tilde{\mu})$ in electromagnetic interactions.

Thus, out of the local currents one can build up vector and axial currents which describe weak and electromagnetic interactions in the lowest order of the coupling constant.

The hypothesis of isotopic invariance permits one to ascribe to these currents certain properties under isotopic transformations. So, the electromagnetic current is given the properties of an isovector and an isoscalar, while the weak current is given the properties of an isovector for the decays conserving strangeness an isospinor for the decays violating strangeness. If we believe in unitary symmetry or the quark model we are able to generalize the electromagnetic and weak currents to the nonet of vector and to the nonet of the axial local currents. In view that the weak and electromagnetic interaction constants are small one can say that the vector and axial currents describe the reaction of the composite system, which the strongly interacting particles comprise, on the weak external perturbation, yielding indirect information about strong interactions. For the quantities concerned with the local vector and axial currents, quantum field theory allows one to prove dispersion relations.

The effective tool for extracting information about strong interactions is the Goldberger-Treiman relation or PCAC - hypothesis.

Some authors formulate the PCAC - hypothesis as an operator equality

$$\operatorname{div} A^{\pi}(x) = c\,\phi_{\pi}(x) \tag{1.1}$$

which establishes the proportionality of the pion field and the axial current divergence with the quantum numbers of the pion.

If the pion is a composite particle then such an operator equality is meaningless, since in this case the pion cannot be described by the local field $\phi_{\pi}(x)$. Therefore it is more reasonable, from the standpoint of the composite model, to formulate the PCAC - hypothesis in the pole approximation on the basis of the unsubtracted dispersion relations in the virtual momentum.

One can attempt to generalize the Goldberger-Treiman relation to the axial currents having the quantum numbers of K -mesons. It should be expected, however, that such relations will have a lower accuracy because of the large K- meson mass.

To obtain information about strong interactions on the basis of the currents introduced above, Gell-Mann postulated, as a dynamical principle, the equal-time commutation relations, i.e. a certain algebra of these currents.

Fubini, Furlan and Rossetti suggested to combine the algebra of currents and dispersion relations in order to obtain the so-called "sum rules" interconnecting the constants of different processes/4/.

One of us (N.B.) drew attention to the fact that to obtain a number of relations derived usually from the algebra of currents it suffices to use only the local properties of the currents from which the dispersion relations follow. The dynamics is contained in the assignment of the number of subtractions in these dispersion relations (the same number which was used by Fubini, Furlan and Rossetti in combining the algebra and dispersion relations).

The algebraic properties of currents turn out to be not essential. This is quite in accord with the idea that in quantum field theory the dynamics is fully determined by a set of local currents and by a chain of quasi local operators/2/.

Starting from this idea, on the basis of the ordinary dispersion relations with certain restrictions on the number of subtractions, some relations have been obtained/5-9/ which connected the coupling constants and the magnetic moments of baryons, including the well-known Cabibbo-Radicati relation.

Up to now, only the Adler and Weisberger relation/10/

$$1 = g_A^2 + \frac{f_\pi^2}{\pi} \int_\mu^\infty \frac{k\,d\omega}{\omega^2} [\sigma_{\pi^- p} - \sigma_{\pi^+ p}] , \qquad (2)$$

which determines the renormalization of the axial constant of the weak interaction, has been the monopoly of the algebra of currents.

The aim of the present paper is to obtain relation (1), on the basis of ordinary dispersion relations. We also use the additivity principle in the quark model for the zero-angle scattering amplitudes near the threshold/11,12/.

§2. Adler-Weisberger Relation and Additivity Principle

Consider the quantity

$$T_{\alpha\beta} = \int dx\, e^{-iqx}\, \theta(x_0) < p_2 | [\tilde{j}_\alpha(x), \tilde{j}_\beta(0)] | p_1 > , \qquad (2.1)$$

where $\tilde{j}_\alpha(x) = \operatorname{div} A^\alpha(x)$, $\alpha = 1, 2, 3$ stands for the isotopic index.

The local operator $\tilde{j}_\alpha(x)$ has all the transformation properties and the quantum numbers of the pion current. The locality of the currents $\tilde{j}_\alpha(x)$ makes it possible to prove, for (2.1), the dispersion relations in the variable

$\omega = \frac{1}{2m}(q, p_1 + p_2)$ when the values of q^2 are negative or sufficiently small positive. The quantity $T_{\alpha\beta}$ has the structure of the πN-scattering amplitude and in the case corresponding to the forward scattering in the lab.system ($\vec{p}_1 = \vec{p}_2 = 0$) can be represented as

$$T_{\alpha\beta} = \delta_{\alpha\beta}(A^{(+)} + \omega B^{(+)}) + \tfrac{1}{2}[\tau_\alpha \tau_\beta](A^{(-)} + \omega B^{(-)}), \qquad (2.2)$$

where the functions $A^{(\pm)}$ and $B^{(\pm)}$ have the following symmetry properties

$$A^{(\pm)}(-\omega) = + A^{(\pm)}(\omega),$$
$$B^{(\pm)}(-\omega) = \mp B^{(\pm)}(\omega). \qquad (2.3)$$

Let us determine the function $f^{(-)}(\omega, q^2)$ related to the crossing antisymmetric part of $T_{\alpha\beta}$:

$$\omega f^{(-)}(q^2, \omega) = A^{(-)} + \omega B^{(-)}. \qquad (2.4)$$

It follows from the symmetry properties (2.3) that $f^{(-)}(\omega, q^2)$ is an even function of ω and finite at $\omega = 0$. We suppose that for the function $f^{(-)}(\omega, q^2)$ the dispersion relation without subtractions holds

$$f^{(-)}(\omega, q^2) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\mathrm{Im} f^{(-)}}{\omega' - \omega}\, d\omega' = f^{(-)}_{pole}(\omega, q^2) + \frac{2}{\pi}\int_{\mu}^{\infty} \frac{\mathrm{Im} f^{(-)}}{\omega'^2 - \omega^2}\omega' d\omega'. \qquad (2.5)$$

Using the definition of the matrix element of the current $\tilde{j}_\alpha(x)$ between the single-nucleon states

$$\langle p_2 | \tilde{j}_\alpha(0) | p_1 \rangle = 2 m g_A D(q^2)\bar{u}\gamma_5 u, \qquad (2.6)$$

where $D(0) = 1$, the pole term, singled out in dispersion relation (2.5), can be written down as

$$f^{(-)}_{pole}(\omega, q^2) = - g_A^2 \frac{4m^2 q^3 D^2(q^2)}{q^4 - 4m^2\omega^2}. \qquad (2.7)$$

Making use of the Goldberger-Treiman relation and of the optical theorem we are able to connect the imaginary part of the function $f^{(-)}$ above the threshold $\omega \geq \mu$ with the total cross sections of $\pi^{\pm}$-meson scattering on protons

$$\operatorname{Im} f^{(-)}(\omega, q^2) = \left(\frac{\mu^2 f_\pi}{\mu^2 - q^2}\right)^2 \frac{k}{2\omega}\left[\sigma_{\pi^- p} - \sigma_{\pi^+ p}\right]. \qquad (2.8)$$

Note that the function $f^{(-)}(\omega, q^2)$ should be considered as an analytical function of two complex variables q^2 and ω. In the language of πN-scattering q^2 is the mass of the incident pion, whereas ω is its lab.energy. These two quantities are related by

$$q^2 = \omega^2 - \vec{q}^{\,2} = \omega^2 (1 - \beta^2), \qquad (2.9)$$

where β is the velocity of the incident pion.

Further we will consider the dispersion relation for the function $f^{(-)}$ when the value of β is fixed. This dispersion relation assumes the form (2.10) if use is made of (2.7) and (2.8)

$$f^{(-)}(q^2, \gamma) = -g_A^2 \frac{4m^2 D^2(q^2)}{q^2 - 4m^2\gamma^2} + \left(\frac{\mu^2 f_\pi}{\mu^2 - q^2}\right)^2 \frac{1}{\pi}\int_\mu^\infty \frac{k' d\omega'}{\omega'^2 - \omega^2}\left[\sigma_{\pi^- p} - \sigma_{\pi^+ p}\right], \qquad (2.10)$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ is the Lorentz factor.

Going over, (in 2.10), to the zero mass and velocity limit $(q^2 = 0,\ \gamma = 1)$, we obtain

$$f^{(-)}(0,1) = g_A^2 + \frac{f_\pi^2}{\pi}\int_\mu^\infty \frac{k\, d\omega}{\omega^2}\left[\sigma_{\pi^- p} - \sigma_{\pi^+ p}\right]. \qquad (2.11)$$

In order to get the sum rule determining the renormalization of the axial constant for weak interaction, g_A, we formulate the additivity principle: the quantity $[r_\alpha r_\beta] f^{(-)}(0,1)$ is additively composed of the contributions from the quarks forming the nucleon

$$[r_\alpha r_\beta] f^{(-)}(0,1) = \sum_{i=1}^{3} [r_\alpha r_\beta]_i f_i^{(-)}(0,1). \qquad (2.12)$$

An account of only one Born term in the quark amplitude gives

$$f_i^{(-)}(0,1) = (g_A^2) \quad \text{quarks}. \qquad (2.13)$$

Fixing (V - A) -variant of the weak interaction for free quarks we shall suppose that the proper axial constant for the interacting quarks is not renormalized

$$(g_A^2)_{quark} = 1. \qquad (2.14)$$

This is equivalent to the assumption that the virtual effects are suppressed in the quark interaction involving soft massless pseudophotons. It should be expected in this case that in the quark amplitude near the threshold the Born term dominates. As a result, we obtain the well-known Adler-Weisberger relation

$$1 = g_A^2 + \frac{f_\pi^2}{\pi} \int_\mu^\infty \frac{k\,d\omega}{\omega^2} [\sigma_{\pi^- p} - \sigma_{\pi^+ p}] . \qquad (2.15)$$

§3. Discussion

We have shown that the consideration of dispersion relations without subtractions for the quantity related to the divergences of the local axial currents allows one to obtain the Adler-Weisberger relation under the following assumptions:

1) The additivity principle for the scattering amplitude of the massless pseudoscalar particle near the threshold.

2) The proper axial constant of the weak interaction of quarks is not renormalized.

3) The assumption that the virtual effects are suppressed in the quark interaction with the massless pseudophoton in the zero momentum limit.

Assumptions (2) and (3) are closely connected and consistent with the general speculations of the composite model for elementary particles. According to this model, the basic particles-quarks have no structure of their own and their interaction determines the structure of the composite particles.

The validity of assumptions (2) and (3) is likely to be accounted for the large quark mass. Whereas for the nucleon the relative contribution of the virtual effects to the axial constant is of the order of $\frac{\mu}{m} = 0.15$, then for the quark $\frac{\mu}{M} = 0$ at $M \to \infty$.

In conclusion we would like to point out an independent possibility of checking the assumptions made in the work. Using the Goldberger-Treiman relation, it is possible to connect the function $f^{(-)}(0,1)$ with the corresponding amplitude of πN- scattering $f^{(-)}_{\pi N}(0)$, at zero pion energy. Assuming approximately that $f^{(-)}_{\pi N}(0) \cong f^{(-)}_{\pi N}(\mu)$ and having in view the above principles we are able to connect the weak decay constant of charged pions f_π with the S-wave lengths of πN-scattering

$$\frac{1}{f_\pi^2} = \frac{4\pi}{3\mu^2}\left(1 + \frac{\mu}{m}\right)(a_1 - a_3). \qquad (3.1)$$

Using $(a_1 - a_3) = 0.28$ we obtain $f_\pi = 0.85 \pm 0.15\,\mu$, in a good agreement with experiment.

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