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Recently many sum rules for neutrino reactions, electroproduction and isovector nucleon form factors have been obtained on the basis of current algebra and unsubtracted dispersion relations for the amplitudes[1-5].

On the other hand, it has been shown by L.D.Solovlev that the sum rules corresponding to trivial commutators ($[A,B]=0$) can be deduced from some assumptions about the high-energy behaviour of the amplitudes and one-dimensional unsubtracted dispersion relations for it[6].

The generalization of this method with the account of $SU(3)$-symmetry has been applied to derive sum rules for strong interactions[7].

In this note we would like to show that it is possible to get the sum rule for electroproduction and the Cabibbo-Radicati relation with the only assumption about the definite high-energy behaviour of the virtual isovector amplitude of the Compton scattering on the proton without any postulated algebra.

These relations have been previously obtained from the nontrivial commutators[5].

Consider the amplitude

$$T^{(+,-)}_{\mu,\nu} = i\int d^4x\, e^{-iq_1x} \langle p_2 | T(j^{(+)}_{\mu}(x), j^{(-)}_{\nu}(0)) | p_1 \rangle \quad , \qquad (1)$$

where $j^{(\pm)}_{\mu}$ are the isovector components of vector current density; p_1 and p_2 are the initial and final proton momenta, $p_1^2 = p_2^2 = M^2$, q_1 , and $q_2 = p_1 - p_2 + q_1$ are the photon momenta.

In the following we put:

$$q_1^2 = q_2^2 = q^2 \quad .$$

The amplitude $T_{\mu\nu}$ has the form:

$$T^{(+,-)}_{\mu\nu} = \bar{u}(p_2) M^{(+,-)}_{\mu\nu} u(p_1) \quad . \qquad (2)$$

The quantity $M_{\mu\nu}$ can be expanded with the account of time reversal and gauge invariance as follows/5/ :

$$\epsilon_2^{\mu} M_{\mu\nu}^{(+,-)} \epsilon_1^{\nu} = \sum_{j=1}^{12} H_{(j)}^{(+,-)}(\nu, t, q^2) I^{(j)} \quad , \qquad (3)$$

where the gauge invariant quantities $I^{(j)}$ are defined as in ref./5/. We need only the second one $H_{(2)}$. The corresponding invariant $I_{(2)}$ has the form:

$$I_{(2)} = [\epsilon_2 \cdot P - \frac{\nu M}{(q_1 \cdot q_2)} q_1 \cdot \epsilon_2][\epsilon_1 \cdot P - \frac{\nu M}{(q_1 \cdot q_2)} q_2 \cdot \epsilon_1] \quad , \qquad (4)$$

where ϵ_1^{μ} , ϵ_2^{ν} are photon polarization vectors and

$$\nu = \frac{q_1 \cdot P}{M} = \frac{q_2 \cdot P}{M} \quad ,$$

$$t = (q_1 - q_2)^2 \quad , \qquad P = \frac{p_1 + p_2}{2} \quad .$$

The optical theorem reads:

$$\frac{e^2}{2} \sum_{(s)} \sum_{(\alpha)} \operatorname{Im} [\bar{u}^s(p) \epsilon_{(\alpha)}^{\mu} M_{\mu\nu} \epsilon_{(\alpha)}^{\nu} u^s(p)]_{t=0} = \sqrt{\nu^2 - q^2}\,\Big|_{t=0} \sigma_{tot} \quad , \qquad (5)$$

where σ_{tot} is the total cross section for the virtual photoproduction on the proton.

After averaging over spin indices and summing over photon polarization we obtain from eq.(5)

$$\operatorname{Im} H_{(2)}(\nu, t=0, q^2) = \frac{2q^2}{e^2 M^2 (\nu^2 - q^2)^{1/2}} (\sigma_T + \sigma_L) \quad , \qquad (6)$$

where $\sigma_L = 0$ for $q^2 = 0$.

From the crossing symmetry of the T amplitude we get the following properties:

$$\mathrm{Re}\ H^{(+,-)}_{(2)}(\nu) = \mathrm{Re}\ H^{(-,+)}_{(2)}(-\nu) \quad ,$$

$$\mathrm{Im}\ H^{(+,-)}_{(2)}(\nu) = -\mathrm{Im}\ H^{(-,+)}_{(2)}(-\nu) \quad . \tag{7}$$

Consider now the difference of the virtual and real forward amplitudes $H_{(2)}$:

$$\Phi^{(+,-)}_{(2)}(\nu,q^2) \equiv H^{(+,-)}_{(2)}(\nu,q^2,t=0) - H^{(+,-)}_{(2)}(\nu,q^2=0,t=0) \quad . \tag{8}$$

We assume that the function $\Phi^{(+,-)}_{(2)}(\nu,q^2)$ for small $|q^2|$ has the high-energy behaviour which enables us write down the unsubtracted dispersion relations both for $\Phi^{(+,-)}_{(2)}(\nu)$ and $\nu\cdot\Phi^{(+,-)}_{(2)}(\nu)$ in the variable ν */.

From this assumption and crossing relations (7) it follows that:

$$\int_{-\infty}^{\infty} \mathrm{Im}\ \Phi^{(+,-)}_{(2)}(\nu,q^2)\,d\nu = \int_{\nu_0}^{\infty} \mathrm{Im}\ [\Phi^{(+,-)}_{(2)}(\nu,q^2) - \Phi^{(-,+)}_{(2)}(\nu,q^2)]\,d\nu = 0 \quad .$$

The difference of the amplitudes in eq.(9) can be expressed through the amplitudes $H^{(1)}_{(2)}$ and $H^{(3)}_{(2)}$ with the total isotopic spin $I=\frac{1}{2}$ and $I=\frac{3}{2}$:

$$H^{(+,-)}_{(2)} - H^{(-,+)}_{(2)} = \frac{4}{3}[H^{(1)}_{(2)} - H^{(3)}_{(2)}] \quad . \tag{10}$$

Separating the one-particle contribution [5] in eq.(9) and using the optical theorem (6) and the relation (10) we get

$$\frac{M^2}{2\pi}\int_{\nu_0}^{\infty} \mathrm{Im}\,[H^{(+,-)}_{(2)}(\nu,q^2,t=0) - H^{(-,+)}_{(2)}(\nu,q^2,t=0)]\,d\nu =$$

$$= [F^{v}_{1}(q^2)]^2 - \frac{q^2}{4M^2}[F^{v}_{2}(q^2)]^2 - \frac{2q^2}{\pi e^2}\int_{\frac{(M+\mu)^2-M^2}{2M}}^{\infty} \times (2\sigma^{\frac{1}{2}}_{v} - \sigma^{\frac{3}{2}}_{v})\frac{d\nu}{\sqrt{\nu^2-q^2}}\Big|_{t=0} \tag{11}$$

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*/ It is worth noting that just the same assumption for the amplitude $H_2(\nu)$ leads to contradiction.

where $F^{v}_{1,2}(q^2)$ are the isovector electric and magnetic form factors of the nucleon, respectively, $\sigma^{1/2}_{v}$, $\sigma^{3/2}_{v}$ are the total cross sections for the isovector photoproduction on protons of the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ states, μ is the pion mass.

From eq. (11) it follows:

$$\frac{M^2}{2\pi}\int_{\nu_0}^{\infty} \mathrm{Im}\,[H^{(+,-)}_{(2)}(\nu, q^2=0, t=0) - H^{(-,+)}_{(2)}(\nu, q^2=0, t=0)]\, d\nu = 1 \quad . \tag{12}$$

The sum rule (9) then takes on the form:

$$\frac{M^2}{2\pi}\int_{\nu_0}^{\infty} \mathrm{Im}\,[H^{(+,-)}_{(2)}(\nu, q^2, t=0) - H^{(-,+)}_{(2)}(\nu, q^2, t=0)]\, d\nu = 1 \quad . \tag{13}$$

This is the same rule which has been previously obtained from the current algebra. Substituting expression (11) into eq. (13) we obtain the sum rule for electroproduction /4,5/

$$[F^{v}_{1}(q^2)]^2 - \frac{q^2}{4M^2}[F^{v}_{2}(q^2)]^2 - \frac{2q^2}{\pi e^2}\int_{a}^{\infty}(2\sigma^{\frac{1}{2}}_{v} - \sigma^{\frac{3}{2}}_{v})\frac{d\nu}{\sqrt{\nu^2 - q^2}} = 1, \tag{14}$$

where $a = \frac{(M+\mu)^2 - M^2}{2M}$.

Now we take the derivative of eq.(14) with respect to q^2 and put $q^2 = 0$. We use the definition of mean square isovector nucleon radius:

$$\frac{dF^{v}_{1}(q^2)}{d[q^2]}\Big|_{q^2=0} = \frac{1}{6}\langle r^2_{v}\rangle = \frac{1}{6}[\langle r^2_{p}\rangle - \langle r^2_{n}\rangle] \quad ,$$

and the normalization conditions:

$$F_1^v(0) = 1 \quad , \quad F_2^v(0) = \mu'(p) - \mu(n) \quad ,$$

where $\mu'(p)$ and $\mu(n)$ are the anomalous magnetic moments of the proton and neutron, respectively. Then we get the well-known Cabibbo-Radicati relation /2/:

$$\left[\frac{\mu'(p) - \mu(n)}{2M} \right]^2 + \frac{2}{\pi e^2} \int_a^\infty (2\sigma_v^{1/2} - \sigma_v^{3/2}) \frac{d\nu}{\nu} = \frac{1}{3} [\langle r_p^2 \rangle - \langle r_n^2 \rangle] \quad . \tag{15}$$

Note that the Cabibbo-Radicati relation can be also obtained from the assumption about the unsubtracted dispersion relations both for the quantities $\frac{\partial H_{(2)}(\nu, q^2, t=0)}{\partial [q^2]} \Big|_{q^2=0}$ and $\nu \cdot \frac{\partial H_{(2)}(\nu, q^2, t=0)}{\partial [q^2]} \Big|_{q^2=0}$.

In fact, in this case we have the sum rule

$$\int_{-\infty}^{\infty} \mathrm{Im} \, \frac{\partial H_{(2)}^{(+,-)}(q^2, \nu, t=0)}{\partial [q^2]} \Big|_{q^2=0} d\nu = 0 \quad . \tag{16}$$

Using expression (11) we immediately obtain relation (15) from eq. (16). The sum rule (16) can also be cast into the form

$$\frac{M^2}{2\pi} \int_{-\infty}^{\infty} \mathrm{Im} \, H_{(2)}^{(+,-)}(q^2, \nu, t=0) \, d\nu = C(q^2) \quad , \tag{17}$$

where $C(q^2)$ is an arbitrary function, provided

$$\frac{dC(q^2)}{d[q^2]} \Big|_{q^2=0} = 0 \quad .$$

Comparing now eqs. (17) and (13) we see that the sum rule (13) is a special case of the sum rule (17) with $C(q^2) = 1$ Thus, the sum rule (16) or (17) and consequently the Cabibbo-Radicati relation has no direct connection with current algebra. The same method can be applied to derive the sum rules from the other invariant amplitudes.

In conclusion we wish to stress that the validity of the sum rule for electroproduction and the Cabibbo-Radicati relation depends only on the validity of assumed asymptotic behaviour and thus unsubtracted dispersion relation for the amplitudes. This approach, in principle, allows us to understand which of the sum rules previously obtained from current algebra actually do not require the validity of the current algebra.

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