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DISPERSION SUM RULES AND SU(3) SYMMETRY

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References:

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In recent papers [1-3] a method have been developed to obtain sum rules from the dispersion relations without any current algebra. In papers [1,2] sum rules have been obtained from the dispersion relations for the scattering and photoproduction amplitudes. This approach suppose the existence of local meson fields, which cannot be introduced in composite models of elementary particles. It was suggested in [3] to consider the dispersion relations for quatities constructed from the local vector and axial-vector currents.

Further we shall use instead of the non-local meson current a divergence of the local axial-vector current with the same quantum numbers and pole approximation for the matrix elements of divergence of the axial-vector current.

An important assumption in deriving the dispersion sum rules is the number of subtractions in the dispersion relations for the definite amplitudes.

Assuming that for the amplitude $f(s,t)$ the dispersion relations

$$f(s,t)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{Im\, f(s',t)\,ds'}{s'-s}\,, \quad s\cdot f(s,t)=\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{Im\, f(s',t)\,s'ds'}{s'-s} \tag{1}$$

are valid, we get the following sum rule

$$\int_{-\infty}^{+\infty} ds\, Im\, f(s,t)=0\,. \tag{2}$$

In [1-3] the sum rules of the type (2) have been used to obtain the relations between coupling constants and magnetic moments of nucleon, which are in excellent agreement with experiment.

Notice that in deriving these relations the assumption that the sum rule is saturated by the nucleon and isobare is used.

A consistent analysis of the approximations made in using the dispersion sum rules is to be based on the SU(3) symmetry, which must be valid in the limit of the equal masses in the unitary multiplets.

In this paper we consider the dispersion sum rules for the scattering and photoproduction of the pseudoscalar meson octet on the baryon octet in the framework of the SU(3) symmetry, taking into account only octet and decuplet in the intermediate octets and discuss the rule of the additional intermediate states and the subtractions in the dispersion relations (1). Let us consider the quantity

$$\mathcal{M} = \int dx\, e^{iqx}\, \theta(x_0) \langle p' | [\tilde{j}_\alpha(x), \tilde{j}_\beta(0)] | p \rangle \qquad (3)$$

where $\dot{\tilde{j}}_\alpha(x)$ is the divergence of the axial-current, which has the all transformational properties of the meson-baryon scattering amplitude and can be presented in the form

$$\mathcal{M} = \bar{u}\,(A + \hat{Q}\cdot B)\,u \;, \qquad \hat{Q} = \frac{1}{2}(\hat{q}_1 + \hat{q}_2)$$

The unitary structure of the spin-flip amplitude B is

$$B(s,t) = \sum_{j=0}^{3} I_i^{(\pm)} \cdot B_i^{(\pm)}(s,t) \qquad (4)$$

where

$$I_1^{(\pm)} = Sp\,(\bar{B}P_2P_1B) \pm Sp\,(\bar{B}P_1P_2B)$$

$$I_2^{(\pm)} = Sp\,(\bar{B}BP_1P_2) \pm Sp\,(\bar{B}BP_2P_1)$$

$$I_3^{(\pm)} = Sp\,(\bar{B}P_2BP_1) \pm Sp\,(\bar{B}P_1BP_2)$$

$$I_0^{(\pm)} = Sp\,(\bar{B}P_2)\cdot Sp\,(BP_1) \pm Sp\,(\bar{B}P_1)\cdot Sp\,(BP_2)$$

and $$\sum_{i=1}^{3} I_i^{(+)} = I_0^{(+)} + Sp(\bar{G}G)\cdot Sp(P_2P_1)$$

Supposing that the dispersion relations (1) are valid for amplitudes $B_i^{(\pm)}$ we obtain:

$$\int_{-\infty}^{+\infty} ds\, \mathrm{Im}\, B_i^{(\pm)}(s,t) = 0 \tag{5}$$

From the crossing symmetry

$$B_i^{(\pm)}(s,t) = \mp B_i^{(\pm)}(u,t) \tag{6}$$

and the sum rule (5) is trivial for the amplitudes $B_i^{(-)}(s,t)$.

Taking into account in the intermediate states only the baryon octet and decuplet and using the Goldberger-Treiman relations we obtain the following relations for the coupling constants:

$$(d+f)^2 = \frac{4}{3} g_*^2 \cdot R \tag{7a}$$

$$(d-f)^2 = \frac{1}{3} g_*^2 \cdot R \tag{7b}$$

$$2(d^2-f^2) = \frac{4}{3} g_*^2 \cdot R \tag{7c}$$

$$\frac{4}{3} d^2 = \frac{3}{2} g_*^2 \cdot R \tag{7d}$$

where $$R = \frac{4m^2}{3} - \frac{[M^2+m^2-\mu^2][(M-m)^2-\mu^2]}{6M^2}$$

and the coupling constants are normalized by

$$g_{\pi NN} = d+f \quad , \quad g_{\pi NN^*} = g_*$$

From the first there relations in (7) we find

$$d = 3f \,, \quad d^2 = \frac{3}{4} g_*^2 \cdot R \tag{8}$$

From the SU(6) symmetry

$$d = \frac{3}{2} f \,, \quad d^2 = \frac{3}{4} g_*^2 \cdot R\,(M = m,\ t = \mu = 0) \tag{9}$$

From (8) we obtain the isobar widths which are in good agreement with the experiment.

However, the relation (7d) contradicts the three previous ones.

Before discussing this inconsistency we consider analogous relations for the virtual photoproduction of the pseudoscalar meson octet on the baryon octet.

In this case we consider the sum rules for a amplitude which corresponds to the structure $\bar{u}\, \sigma_{\mu\nu} F_{\mu\nu} \hat{k} \gamma_5 u$.

Taking into account only the contributions of the baryon octet and decuplet we obtain the following relations:

$$(d+f)(\mu_D' + \mu_F') = \frac{3}{9M} g_* \cdot c_* \cdot R' \tag{10a}$$

$$(d-f)(\mu_D' - \mu_F') = \frac{2}{9M} g_* \cdot c_* \cdot R' \tag{10b}$$

$$2(d \cdot \mu_D' - f \cdot \mu_F') = \frac{8}{9M} g_* \cdot c_* \cdot R' \tag{10c}$$

$$\frac{4}{3} d \mu_D' = \frac{1}{M} g_* \cdot c_* \cdot R' \tag{10d}$$

where

$$R' = \left[m^2 + \frac{mM}{2} + \frac{M^3}{2M} + \frac{3\mu^2}{2} + \frac{m\mu^2}{2M} + 3(kq) \right]$$

$$\mu_N^{(S)} = \frac{1}{2}\left(\mu_F' - \frac{1}{3}\mu_D'\right); \quad \mu_N^{(V)} = \frac{1}{2}\left(\mu_F' + \mu_D'\right) \tag{11}$$

$$\mu_{N^* \to N\gamma} = \frac{2\sqrt{2}}{3} c_*$$

From the first three relations of (10) using (8) we find

$$\mu'_D = 3\mu'_F \tag{12}$$

and hence $\mu'^{(s)}_N = 0$.

The relation (10d) as in the meson-scattering case, contradicts the previous ones.

Thus, the sum rules for the scattering and photoproduction of the meson octet on the baryon one with limitation by the contributions of the baryon octet and decuplet in intermediate states give rise to eight homogeneous relations for the fourth quantities $\left(\frac{d}{f}\right)$, $\left(\frac{d}{g_*}\right)$, $\left(\frac{\mu'_D}{\mu'_F}\right)$, $\left(\frac{\mu'_D}{\varepsilon_*}\right)$

Among these eight equations sic ones have a consistent solution which is given by (8) and (12) and the remaining ones contradict the former and can be satisfied only by the trivial solution.

This circumstance can be possibly explained by that we confine ourselves to the octet and decuplet of baryons in intermediate states.

The contradictions arising here can be demonstrated by the following example.

For $\eta\Lambda$ - scattering only the Λ- baryon can appear the intermediate state in our approximation and the sum rule for this process is satisfied only by the trivial solution $g_{\eta\Lambda\Lambda} = 0$.

One may note that conflicting relations correspond to the only unitary structure to which the baryon singlet can contribute. However, at present there is no known baryon singlet which would be able to improve the situation.

Another possible way out from this situation is a modification of dispersion relations (1) by adding subtractions. For the case when an amplitude has the asymptotic behaviour of the type

$$f(s,t) \to \frac{c(t)}{s}, \quad s \to \infty \tag{13}$$

the sum rule (12) reduces to the following form

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} ds\, \mathrm{Im}\, f(s,t) + c(t) = 0. \tag{14}$$

If one assumes that the asymptotics of the spin-flip amplitude of meson-baryon scattering is deminated by the exchange of unitary singlet and octet in the t-chennel we obtain, instead of (7) the following relations:

$$(d+f)^2 = \frac{4}{3} g_*^2 \cdot R + \left(a - \frac{1}{3} a_D + a_F\right) \tag{15a}$$

$$(d-f)^2 = \frac{1}{3} g_*^2 \cdot R + \left(a - \frac{1}{3} a_D - a_F\right) \tag{15b}$$

$$2\left(d^2 - f^2\right) = \frac{4}{3} g_*^2 \cdot R + \left(a - \frac{4}{3} a_D\right) \tag{15c}$$

$$\frac{4}{3} d^2 = \frac{3}{2} g_*^2 \cdot R + \left(a - \frac{4}{3} a_D\right) \tag{15d}$$

where a, a_D, a_F - are constants determining the asymptotics. From the third and fourth relations of (15) of the amplitude it follows

$$3f^2 - d^2 = \frac{1}{4} g_*^2 \cdot R$$

Using the additional relation which follows from (7a–c): $d^2 = \frac{3}{4} g_*^2 \cdot R$ (SU(6) symmetry gives also this relation) we get:

$$d/f = 3/2, \quad a_D/a_F = 3/2, \quad a = a_F. \tag{16}$$

We can consider the exchange of nonet in the t-channel as one of the possible cases. For this case $\mathcal{Q} = 2\eta_F$ and we derive $\frac{d}{f} = 1.506$ which is close to the SU(6)results.

This, the addition of subtractions to the dispersion relation for the spin-flip amplitude of meson- baryon scattering allows to get a consistent system of relations in the frame work of SU(6) symmetry.

In conclusion we would like to stres that for deriving the dispersion sum rules (2) and (14) no algebra is needed. The essential point of our approach is the dispersion relations for the commutator of local currents. The dynamics of processes is determined by the number of subtractions in dispersion relations and by set of intermediate states which saturate the dispersion sum rule.

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