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CONSISTENT RELATIVIZATION OF SU(6) FOR TWO-PARTICLE REACTIONS

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CONSISTENT RELATIVIZATION OF SU(6) FOR TWO-PARTICLE REACTIONS

Up to now nobody have succeeded in finding such a relativistic version of $SU(6)$ which would be self-consistent, i.e. would commute with the free equations of motion and would give reasonable restrictions on reaction amplitudes. The only progress in this trend was the $SU_W(6)$ group for colinear processes/1/.

Here we propose a group $SU_x(6)$ isomorphic to $SU(6)$ which satisfies the mentioned requirements and is applicable to two-particle reactions without colinearity limitations.

As an example we consider here quarks. The corresponding transformations for them are

$$\delta\psi(p) = [\tfrac{i}{2}\omega^a\lambda_a + e^i_\mu(p)\gamma_\mu\gamma_5(a_i + a^a_i\lambda_a)]\psi(p), \qquad (1)$$

were $e^i_\mu(p)$ are three vectors orthogonal to momentum p and to each other, $(e^i p) = 0$, $(e^i e^j) = \delta_{ij}$, λ_a are 8 Gell-Mann matrices, and ω^a, a_i, and a^a_i are transformation parameters. These transformations commute with the free Dirac equation. Vectors $e_\mu(p)$ can be expressed in many ways in terms of momenta of particles involved in a reaction. The following choice seems to be the most natural for two-particle reactions

$$e^1_\mu(p) = N_1\left(p_\mu - \frac{p^2}{(Pp)}P_\mu\right),\ e^2_\mu = N_2\,\epsilon_{\mu\nu\lambda\rho}\,p_{1\nu}\,p_{2\lambda}\,p_{3\rho},\ e^3_\mu(p) = N_3\,\epsilon_{\mu\nu\lambda\rho}\,e^1_\nu e^2_\lambda\,p_\rho,$$

$N_i(p=0)=1.$

where $P_\mu = p_{1\mu} + p_{2\mu} = p_{3\mu} + p_{4\mu}$ is the total momentum and N_1, N_2, and N_3 are normalization factors. Without loss of relativistic content it is convenient to pass to the center of mass system and to two component spinors $\phi(\vec{p})$. Then (1) takes the form

$$\delta\phi(\vec{p}) = \{\tfrac{i}{2}\omega^a\lambda_a + i(a_1 + a^a_1\lambda_a)\frac{(\vec{\sigma}\vec{p})}{|\vec{p}|} + i(a_2 + a^a_2\lambda_a)(\vec{\sigma}\vec{n}) + i(a_3 + a^a_3\lambda_a)\frac{(\vec{\sigma}[\vec{n}\vec{p}])}{|\vec{p}|}\}\phi(\vec{p}), \qquad (3)$$

where $\vec{n}$ is the unit normal to the reaction plane. It is interesting that the operator $ie^1_\mu(p)\gamma_\mu\gamma_5$ turns into the helicity operator $\frac{(\vec{\sigma}\vec{p})}{|\vec{p}|}$. That means the

x/ In /2/ this group arose as a subgroup of the infinite-parameter group considered there, the group $SU_W(6)$ is a particular case of $SU_x(6)$.

conservation of total helicity for invariant amplitudes.

3. For any momentum the transformations (3) are isomorphic to the usual momentum-independent $SU(6)$ transformations

$$\delta\phi'(\vec{p}) = \{\tfrac{1}{2}\omega^{a}\lambda_{a} + i(a_{k} + a_{k}^{a}\lambda_{a})\sigma_{k}\}\phi'(\vec{p}). \qquad (4)$$

Really, the transformations (3) and (4) are connected by similarity transformation

$$\phi'(\vec{p}) = S(\vec{p})\phi(\vec{p}), \qquad (5)$$

where the matrix $S(\vec{p})$ produces spin rotation, which transform $\frac{(\vec{\sigma}\vec{p})}{|\vec{p}|}$ into σ_x, $\vec{\sigma}\vec{n}$ into σ_y, and $\frac{(\vec{\sigma}[\vec{n}\vec{p}])}{|\vec{p}|}$ into σ_z but does not affect the momenta. We can choose the $S(\vec{p})$ in the form

$$S(\vec{p}) = e^{\frac{i}{2}\vec{\omega}\vec{\sigma}}, \qquad \vec{\omega} = -\vec{n}\arcsin\frac{p_z}{|\vec{p}|}. \qquad (5)$$

Therefore, the quarks with different momentum transform according to different, but equivalent representations of the group $SU(6)$. The similarity transformations (5) equate them.

4. One can easily construct invariant amplitudes using the spinors $\phi'(\vec{p})$, which transform according to the usual $SU(6)$ transformations (4). Therefore, in terms of ϕ' in c.m.s. we can construct invariant amplitudes according to the usual $SU(6)$ prescriptions.

So, the quark-quark scattering amplitude can be written as

$$A(\phi_4'^{+}\phi_2')(\phi_3'^{+}\phi_1') + B(\phi_4'^{+}\phi_1')(\phi_3'^{+}\phi_2'); \quad \phi_i' = \phi'(\vec{p}_i), \qquad (6)$$

where A and B are arbitrary form factors, depending on the energy and angle of scattering θ. In terms of usual spinors this amplitude is more complicated, e.g.

$$(\phi_4'^{+}\phi_2')(\phi_3'^{+}\phi_1') = \cos^2\frac{\theta}{2}(\phi_4^{+}\phi_2)(\phi_3^{+}\phi_1) - \frac{i}{2}\sin\theta[(\phi_4^{+}(\vec{\sigma}\vec{n})\phi_2)(\phi_3^{+}\phi_1) +$$

$$+ (\phi_4^{+}\phi_2)(\phi_3^{+}(\vec{\sigma}\vec{n})\phi_1)] - \sin^2\frac{\theta}{2}(\phi_4^{+}(\vec{\sigma}\vec{n})\phi_2)(\phi_3^{+}(\vec{\sigma}\vec{n})\phi_1).$$

It is convenient to work in terms of "primed" spinors. One can deal analogously with other multiplets (35, 56 etc).[x] Using appropriate spin rotations one can also pass to new ("primed") quantities which are transformed according to the conventional $SU(6)$. Then the amplitudes are constructed according to usual $SU(6)$ rules.

[x] The transformations of 35-, 56-and 189-plets can be found in [2] (see § 5 and Appendix 4).

6. Cross sections summed up over the spin states are independent of the matrices $S(\vec{p})$. Really, $\sum_s \phi'(s)\phi'^{+}(s) = S\sum_s\phi(s)\phi^{+}(s)S^{-1} = 1$. Therefore we draw an important conclusion that for the cross sections summed up over spin states all consequences are independent of the choice of vectors e_μ^i. These consequences are such as if we would construct cross sections according to $SU(6)$ prescriptions ignorizing the non-zero value of momenta. The correctness of the choice of e_μ^i can be checked only by means of polarization effects.

7. The problem of elastic unitarity does not arise, because the group obtained is the internal symmetry group which commutes with the free equation. So, in the simple case of the unitary singlet-quarks scattering [3] the amplitude is $A(\cos\frac{\theta}{2} - i(\vec{\sigma}\vec{n})\sin\frac{\theta}{2})$. This means that we have for phase shifts $\delta_{\ell+1}^{-} = \delta_{\ell}^{+}$, i.e. at the given angular momentum j phase-shifts are degenerated in the orbital momentum ℓ.

8. At the same time, the group $SU_x(6)$ obtained can be used only at very high energies when mass differences in a multiplet became non essential (by analogy with the isotopic group which works unless the energy is of the order of the electromagnetic mass differences). At moderate energies $SU_x(6)$ can strongly contradict experiment.

9. Thus, we obtain the dynamical (like $SU_w(6)$) relativistic group $SU_x(6)$ which commutes with the equation of motion, is self-consistent and isomorphic to $SU(6)$. Consequences and predictions of this group will be considered later on.

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