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# DISPERSION SUM RULES FOR STRONG AND ELECTROMAGNETIC INTERACTIONS

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## 1. Introduction

Recently much attention has been devoted to the current algebra. An important tool for deriving results from the current algebra is the dispersion technique of Fubini, Furlan and Rossetti /1/.

An alternative approach, based on the usual one-dimensional dispersion relations, has been developed by L.D.Soloviev /2,3/. Some results reported in these papers have also been obtained earlier from the current algebra. These results are similar to those which are obtained from trivial commutators $[A,B]=0$.

This approach assumes the existence of local meson fields. These local fields cannot be introduced in composite models of the elementary particles, for example, in the quark model.

It is more natural to begin with the local vector and axial currents in the composite particles models. In this paper we propose further development of the dispersion approach without postulating any current algebra. Let us consider the local operators $\tilde{j}_a = \mathrm{div} A_a$ which have the same quantum numbers and the transformational properties as the pseudoscalar mesons. For such quantities as

$$F(k)=\int dx\, e^{-ikx}\Theta(x_0)\langle A|[\tilde{j}_a(x),\tilde{j}_\beta(0)]|B\rangle \tag{1.1}$$

$$\Phi(k)=\int dx\, e^{-ikx}\,\Theta(x_0)\langle A|[\tilde{j}_a(x),V_e(0)]|B\rangle\ , \tag{1.2}$$

where $V_e(x)$ is electromagnetic current, the dispersion relations (d.r.) can be proved for negative and sufficiently small positive $K^2$.

The number of subtractions in the d.r. is determined by the dynamics of the interaction. We shall suppose that the d.r. for (1.1),(1.2) as also the corresponding scattering and virtual photoproduction amplitudes with the real $\pi$-mesons have the same number of subtractions.

In order to pass to the physical amplitudes we may use the pole approximation for the matrix elements of the operators $\tilde{j}_\alpha(x)$

$$<A|\tilde{j}_\alpha(0)|B> = \frac{C_\alpha}{m_\alpha^2 - q^2} T_{A\to B+\alpha} \quad , \tag{1.3}$$

where $T_{A\to B+\pi}$ is the vertex function for the real $\pi$ -meson.

Further we shall consider concrete processes.

## 2. Dispersion Sum Rules for Strong Interaction

Let us consider the dispersion relation for the quantity

$$F(q) = \int dx\, e^{-iqx}\, \Theta(x_0) <A|[\tilde{j}_\alpha(x) \tilde{j}_\beta(0)]|B> \quad , \tag{2.1}$$

where $\tilde{j}_\alpha(x)$ is the divergence of the axial current which has all quantum numbers of the $\pi$ -meson.

The quantity $F(q)$ has an isotopic structure and transformational properties of the $\pi - N$ scattering amplitude and can be presented in the form

$$F(q) = \bar{U}(\tilde{A} + \hat{Q}\tilde{B})U \quad , \tag{2.2}$$

where $\hat{Q} = \gamma \cdot [q + \frac{p_1 - p_2}{2}]$, and $p_1, p_2$ are the baryon momenta. The amplitude $\tilde{B}$ corresponds to the spin-flip $\pi - N$ -scattering amplitude. For this reason it is natural to assume that the dispersion relations for $\tilde{B}$ and $E\tilde{B}$ have no subtractions.

Thus we get the following sum rule:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, \tilde{B}(E, \vec{p}^{\,2})\, dE = 0 \quad , \tag{2.3}$$

where $E = q_0$ and $\vec{p}$ is the momentum of the nucleon in the Breit coordinate system.

The isotopic structure of $\tilde{B}$ can be written in the form

$$\tilde{B} = \delta_{\alpha\beta} \tilde{B}_{odd} + \frac{1}{2}[\tau_\alpha \tau_\beta] \tilde{B}^{(-)} \quad . \tag{2.4}$$

The sum rule (2.3) is nontrivial only for the amplitude $\tilde{B}_{odd}$ which is an odd function of E. Writting the sum rule (2.3) for $\tilde{B}_{odd}$ in terms of the usual Mandelstam variables $s$ and $t$ and picking out the one-nucleon term we get



$$\tilde{g}_{\pi NN}(q_1^2) \tilde{g}_{\pi NN}(q_2^2) + \frac{1}{\pi} \int_{(M+\mu)^2}^{\infty} \mathrm{Im}\, \tilde{B}_{odd}(s,t)\, ds = 0 \quad , \tag{2.5}$$

where $\tilde{g}_{\pi NN}(q^2)$ is defined by

$$<N(p_2)|\tilde{j}_\alpha(0)|N(p_1)> = i\tilde{g}_{\pi NN}((p_1 - p_2)^2) \bar{U}(p_2) \gamma_5 \tau_\alpha U(p_1) \; . \tag{2.6}$$

In calculating $\mathrm{Im}\, \tilde{B}_{odd}$ we shall take into account only the contribution of $N^*_{3/2}$ (1236) .

The matrix element of transition to $N^*$ has the following form

$$<N(p_2)|\tilde{j}_\alpha(0)|N^*(p_1)> = i\sqrt{2}\, \tilde{g}_{\pi N^* N} \bar{\Psi} \Psi_{\mu,\alpha} (p_1 - p_2)_\mu \; . \tag{2.7}$$

We obtain

$$\tilde{g}_{\pi NN}(q_1^2) \tilde{g}_{\pi NN}(q_2^2) = \frac{2}{9} \tilde{g}_{\pi N^* N}(q_1^2) \tilde{g}_{\pi N^* N}(q_2^2) \{-3t + \frac{2m}{M}(m+M)^2 + (q_1^2 + q_2^2)(2 - \frac{m}{M}) - \frac{1}{M^2}(M^2 - m^2 + q_1^2)(M^2 - m^2 + q_2^2)\} \, . \tag{2.8}$$

The right-hand side of equation (2.8) is dependent on $t = (q_1 - q_2)^2$ whereas the left-hand side is dependent on $q_1^2$ and $q_2^2$ only. The explanation is that in deriving the sum rule (2.8) we confine ourselves to the contribution of $N^*$ instead of an infinite set of states.

As the functions $\tilde{g}(q^2)$ are fast decreasing with $q^2 \to -\infty$ the equation (2.8) in limit $q^2 \to -\infty$ is trivially satisfied. Apparently, the eq. (2.8) has the most accuracy with zero values of the transfer moments.

For zero values of quantities $q_1^2, q_2^2$ and $t$ the dispersion relations definitely can be proved and we obtain

$$\tilde{g}^2_{\pi NN}(0) = \frac{2}{9}(1 + \frac{m}{M})^2 (4mM - m^2 - M^2) \tilde{g}^2_{\pi N^* N}(0) \; . \tag{2.9}$$

The limitation to zero momentum is useful when one calculates the coupling constants with the help of the quark model.

Using the Goldberger-Treiman relations we can get the following equation for the coupling constants with the real $\pi$ -mesons:

$$g^2_{\pi NN} = \frac{2}{9}(1 + \frac{m}{M})^2 (4mM - m^2 - M^2) g^2_{\pi N^* N} \; . \tag{2.10}$$

In addition to nucleon and $N^*$ other resonances, for instance, $N^1_{1/2}$(1518)

and the resonances with negative parity, can appear in the sum rule (2.8).

However, as the equation (2.10) is satisfied with great accuracy, the contributions of the other resonances, apparently compensate each other.

## 3. The Sum Rules for the Magnetic Moments

Let us consider the dispersion relations for

$$\Phi_{\bullet}(q) = \int e^{-iqx}\,\Theta(x_0)\langle A|[\tilde{j}_a(x), V_{\bullet}(0)]|B\rangle d^4x \quad , \tag{3.1}$$

where $\tilde{j}_a(x)$ is the divergence of the axial current and has the quantum number of the $\pi$ - meson. $\Phi_{\bullet}(q)$ has an isotopic structure and the transformational properties of the virtual photoproduction amplitude of the real $\pi$-mesons [3,4].

We shall suppose that the scalar amplitude $\tilde{\Phi}_6$ which corresponds to the structure $\bar{U}\gamma_5\hat{k}(\hat{k}\gamma_{\bullet}-\gamma_{\bullet}\hat{k})U$ decrease and for $\tilde{\Phi}_6$ and $E\tilde{\Phi}_6$ the dispersion relations without subtractions can be written. From these assumptions the sum rule follows:

$$\int_{-\infty}^{\infty} ds\, \mathrm{Im}\,\tilde{\Phi}_6(s,t,k^2,q^2) = 0 \quad . \tag{3.2}$$

After picking out the one-nucleon term it takes the following form:

$$\tilde{g}_{\pi NN}(q^2)F_{\mu}^{(v,s)}(k^2) + \frac{2}{\pi}\int_{(M+\mu)^2}^{\infty} \mathrm{Im}\,\tilde{\Phi}_6^{(v,s)}(s,t,q^2,k^2)\,ds = 0 \quad , \tag{3.3}$$

where $F^{(v,s)}(k^2)$ are the isovector and the isoscalar magnetic form factors of the nucleon. In the integral (3.3) only the contribution from the resonances $N^{*}_{3/2}(1236)$ and $N^{1}_{1/2}(1518)$ is taken into account. With this approximation the sum rule (3.3) takes the following form for $k^2=0$

$$\tilde{g}_{\pi NN}(q^2)(\mu_p-\mu_n-1) + \tilde{g}_{\pi N'N}(q^2)(\mu_{pp'}-\mu_{nn'}) = \frac{8}{9}g_v^{*}\frac{\tilde{g}_{\pi NN}(q^2)}{M}\Big[m^2+\frac{Mm}{2}+\frac{m^3}{2M} - \\ -\frac{3q^2}{2}-\frac{mq^2}{2M}-3(kq)\Big] \quad , \tag{3.5}$$

$$\tilde{g}_{\pi NN}(q^2)(\mu_p+\mu_n-1) + \tilde{g}_{\pi N'N}(q^2)(\mu_{pp'}+\mu_{n'n}) = 0 \quad ,$$

where $\mu_{NN'}$ and $g_v^{*}$ are the magnetic moments of the transitions $N' \to N+\gamma$ and $N^{*} \to N+\gamma$. Going to the limit of zero transfer momentum and repeating the discussion of § 2 we obtain equations which connect the magnetic moments and the coupling constants with the real $\pi$ -mesons:

$$g_{\pi NN}(\mu_p-\mu_n-1) + g_{\pi N'N}(\mu_{pp'}-\mu_{nn'}) = \frac{8}{9}g_v^{*}\frac{g_{\pi N^{*}N}}{M}\Big(m^2+\frac{Mm}{2}+\frac{m^3}{2M}\Big) \quad , \tag{3.6}$$

$$g_{\pi NN}(\mu_p+\mu_n-1) + g_{\pi N'N}(\mu_{pp'}+\mu_{nn'}) = 0 \quad . \tag{3.7}$$

There is no experimental data about the magnetic moments of the transition $N^1 \to N+\gamma$ and we cannot check the relations (3.6),(3.7), directly.

## 4. The Transition Magnetic Moments in the Quark Model

We can estimate the constants $\mu_{NN'}$ in the sum rules (3.9),(3.10) using the quark model.

In the quark model the electromagnetic interaction and the interaction with the $\pi$ -mesons can be phenomenologically described by the effective hamiltonians

$$H_{\bullet} = \frac{1}{2M_q}\vec{H}\sum_i(\vec{\sigma}e)_i\, e^{ikx_i} \quad , \tag{4.1}$$

$$H_{\pi} = \frac{q_0}{2M_q}\, i\vec{k}\,\pi_a\sum_i(\vec{\sigma}\tau_a)_i\, e^{ikx_i} \quad , \tag{4.2}$$

where $M_q$ is the quark mass, $q_0$ is the coupling constants of the $\pi$ - mesons with the quarks. The matrix elements of the operators (4.1),(4.2) between the nucleons are given by the following formulas:

$$\langle N|H_{\pi}|N\rangle = \frac{g_{\pi NN}}{2m_N}\, i\vec{k}\,\pi_a\,\vec{\sigma}\tau_a \;, \qquad g_{\pi NN} = 5g_0 \quad , \tag{4.3}$$

$$\langle N|H_{\bullet}|N\rangle = \frac{e}{2m_N}\vec{H}\vec{\sigma}\mu_N \;, \qquad \mu_p = 3 \quad \mu_n = -2 \quad , \tag{4.4}$$

The transition matrix elements are determined by the SU(6) structure of the wave function $N^*$. We shall use the relativistic composite model of the higher resonances developed in /5/.

According to this paper the higher baryon resonances are considered as bound states of the three quarks in the p -wave. The relativistic wave function together with the negative parity resonances describes the positive parity resonances. The wave function of these states have the structure of the SU(6) group 70-plet. After the calculation of the matrix elements from (4.1) and (4.2) we get

$$g_{\pi n \tilde{n}} = \frac{4a}{5} g_{\pi n' n} ,$$

$$\mu_{pp'} = \frac{3am}{m_{N'}} \qquad \mu_{nn'} = -\frac{am}{m_{N'}} . \tag{4.5}$$

From the experimental decay width $\Gamma_{N' \to N+\pi} = 140 \pm 20$ MeV we find $\frac{g^2_{\pi N' N}}{4\pi} = 1,5 \pm 0,2$ .

Finally we obtain for the isoscalar magnetic moment

$$\mu_p + \mu_n = 0,85 \pm 0,03 . \tag{4.6}$$

The calculations show that the sum rule for the isovector magnetic moment is in a satisfactory agreement with the experimental data.

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