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CALCULATION OF NUCLEON-PHONON INTERACTION EFFECTS IN A SIMPLE NUCLEAR MODEL

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I. Introduction

In paper[2] a method was presented for solving nucleon-phonon interaction problems. This method is connected neither with the construction of series in powers of an interaction constant nor the diagonalization of a high order matrix and, what is more important, allows to expect that some reasonable results will be obtained in the most interesting cases, namely when the interaction is rather strong and the distance between two non-perturbed levels is small in comparison with the collective excitation energy.

The results of exact calculation of the energy spectrum in a simple model[2] are presented below. Exactness of approximate methods formulated in the same paper is discussed. Giving these results we also hope that some of nucleon-phonon interaction effects will be, perhaps, typical for some other models. For instance, the appearance of an approximately equidistant spectrum of states under the condition that a harmonic collective degree of freedom fails formally to be extracted is simply due to a quantum mechanical level repulsion and therefore not related to a certain model.

Now let us remind briefly the main results of paper[2]. It concerns a simple system with the Hamiltonian:

$$H = E_1 \sum_{m=-j}^{j} a_{1m}^{+} a_{1m} + E_2 \sum_{m=-j}^{j} a_{2m}^{+} a_{2m} + \hbar\omega\, b^{+} b + \gamma \sum_{m=-j}^{j} (a_{1m}^{+} a_{2m} + a_{2m}^{+} a_{1m})(b^{+} + b),$$

where $b^{+}(b)$ are the pseudoscalar phonon creation (destruction) operators. The single-fermion states alone were studied whose wave function was written in the form

$$\Psi_m = a_{1m}^{+} F(b^{+})|0> + a_{2m}^{+} \Phi(b^{+})|0>$$

and the Schrödinger equation was:

$$\begin{aligned} \{E_1 + \hbar\omega\, b^{+} b\} F(b^{+})|0> + \gamma (b^{+} + b)\Phi(b^{+})|0> &= E F(b^{+})|0> \\ \{E_2 + \hbar\omega\, b^{+} b\} \Phi(b^{+})|0> + \gamma (b^{+} + b) F(b^{+})|0> &= E \Phi(b^{+})|0> \end{aligned} \qquad (1)$$

with the normalization condition

$$<0|F(b)\, F(b^{+})|0> + <0|\Phi(b)\Phi(b^{+})|0> = 1 \qquad (2)$$

($F(b^{+})$ and $\Phi(b^{+})$ are consequently $\sum_{n=0}^{\infty} f_n (b^{+})^n$ and $\sum_{n=0}^{\infty} \phi_n (b^{+})^n$ and the normalization condition can be written as

$$\sum_{n=0} n!(|f_n|^2 + |\phi_n|^2 = 1).$$

It was shown that to find a solution for eq. (1) it is enough to solve the following system of differential equations

$$\left\{ E_1 + \hbar\omega\, z \frac{d}{dz} \right\} F(z) + \gamma \left(z + \frac{d}{dz} \right) \Phi(z) = E F(z)$$

$$\left\{ E_2 + \hbar\omega\, z \frac{d}{dz} \right\} \Phi(z) + \gamma \left(z + \frac{d}{dz} \right) F(z) = E \Phi(z) \qquad (3)$$

which can be derived from eq. (1) by means of a formal substitution:

$$b^+ \to z \qquad b \to \frac{d}{dz}$$

$$F(b^+)|0\rangle \to F(z) \qquad \sum_{n=0}^{\infty} f_n (b^+)^n \to \sum_{n=0}^{\infty} f_n z^n$$

$$\Phi(b^+)|0\rangle \to \Phi(z) \qquad \sum_{n=0}^{\infty} \phi_n (b^+)^n \to \sum_{n=0}^{\infty} \phi_n z^n .$$

This procedure presents a simplified version of the Fock functional method[1].
The normalization condition (2) selects only the entire solutions of the system (3).

II. APPROXIMATE METHODS

1. In order to solve the equations when interaction is rather strong $(\gamma/\hbar\omega \geq 1)$ and the distance between two nonperturbed levels small $(E_2 - E_1 = 2\varkappa \ll \hbar\omega)$ the approximation of strong coupling was worked out. It consists in expanding an exact solution in the eigenfunctions of the Hamiltonian with $\epsilon = 0$. For the expansion coefficients the following equation was obtained:

$$C_n (n - \gamma^2 - E) = \sigma \epsilon\, e^{-2\gamma^2} \sum_{n'} (-)^{n'} C_{n'} (2\gamma)^{n'-n} L_n^{n'-n} (4\gamma^2)$$

ϵ – distance between nonperturbed levels, $\sigma = \pm 1$, γ – interaction constant, $L_n^{n'-n}$ – joined Laguerre polynomial, $\hbar\omega = 1$. Expanding E and C_n in the power series of ϵ we have

$$E_{n_0} = n_0 - \gamma^2 - \sum_{k=1}^{\infty} E_{n_0}^{(k)}$$

$$E_{n_0}^{(k)} = -\sigma \epsilon\, e^{-2\gamma^2} \sum_{n=0}^{\infty} (-)^n C_n^{(k-1)} (2\gamma)^{n-n_0} L_{n_0}^{n-n_0} (4\gamma^2)$$

$$C_{n \neq n_0}^{(k)} = \frac{1}{n - n_0} \sum_{e=1}^{k} C_n^{(k-e)} E_n^{(e)} + \frac{\sigma \epsilon\, e^{-2\gamma^2}}{n - n_0} \sum_{n'} (-)^{n'} C_{n'}^{(k-1)} (2\gamma)^{n'-n} L_n^{n'-n} (4\gamma^2)$$

$$C_{n_0} = 1 \qquad C_n^{(0)} = \delta_{n n_0}$$

For example

$$E_{n_0}^{(1)} = -\sigma \epsilon\, e^{-2\gamma^2} (-)^{n_0} L_{n_0} (4\gamma^2)$$

$$E_{n_0}^{(2)} = -\epsilon^2 e^{-4\gamma^2} (-)^{n_0} \sum_{n \neq n_0} (-)^n \frac{L_0^{n-n} (4\gamma^2)\; L_{n_0}^{n-n_0} (4\gamma^2)}{n - n_0}$$

2. When $2\epsilon \approx 1$ and γ is small a resonant formula can be used

$$E^{(0)}_{n_0} = n_0 - \tfrac{1}{2} \pm \sqrt{(\epsilon - \tfrac{1}{2})^2 + n_0 \gamma^2}$$

with the correction which is equal to

$$E^{(2)}_{n_0} = 2\gamma^2 \left\{ \frac{\pm \gamma \sqrt{n_0} - 2}{(n_0 + 3)(4 \pm 4\gamma\sqrt{n_0} - 2\gamma^2)} + \frac{(\pm\gamma\sqrt{n_0} + 2)\ \theta(n_0 - 2)}{(n_0 + 1)(4 \pm 4\gamma\sqrt{n_0} + 2\gamma^2)} \right\}$$

when $2\epsilon = 1$

$$\theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

3. Finally when $\gamma \ll \epsilon$ we can use the perturbation method:

$$E^{(0)}_{n_0} = n_0 + \sigma\epsilon$$

$$E^{(2)}_{n_0} = \frac{\sigma\gamma^2}{2\epsilon - \sigma} + \frac{4\sigma\epsilon\gamma^2}{4\epsilon^2 - 1}\, n_0$$

III. NUMERICAL SOLUTION

In order to examine how all these approximations work the problem of eigenvalues was solved numerically. It was naturally to use the fact that series $\sum_{n=0}^{\infty} n!(f_n^2 + \phi_n^2)$ is convergent only when E is equal to an eigenvalue of the system. For this it was necessary to take into account the limiting but rather big number of terms in order that the series would not practically depend on the values of separate terms and find their sum in an energy interval where the eigenvalue was supposed to be present.

A point in which this sum is minimal should be regarded as an eigenvalue. It can be found within any accuracy needed.

For direct calculations it was convenient to pass to the linear combinations of functions $F(z)$ and $\Phi(z)$:

$$X(z) = F(z) + \Phi(z)$$

$$Y(z) = F(z) - \Phi(z) = \sigma X(-z).$$

Then eq. (3) is reduced to:

$$\left\{(z+\gamma)\frac{d}{dz} - E + \gamma z\right\} X(z) = \sigma\epsilon X(-z). \tag{4}$$

Substituting $X(z)$ as a series $X(z) = \sum_{n=0}^{\infty} x_n z_n$ in (4) we obtain

$$x_{n+1} = \frac{1}{\gamma(n+1)} \left[(E - n)\, x_n - \gamma\, x_{n-1} + \sigma\epsilon (-)^n x_n \right] \tag{5}$$

and

$$\sum_{n=0}^{\infty} n!\,(f_n^2 + \phi_n^2) = \sum_{n=0}^{\infty} n!\, x_n^2. \tag{6}$$

Formulae (5) and (6) were calculated and with their help eigenvalues E_0, E_1 and E_2 found for a set of parameters of (and).

The results obtained are represented by curves in Figs. 1-10.

IY. COMPARISON OF EXACT AND APPROXIMATE RESULTS

Now we pass to the analysis of the results obtained. At first we shall try to reveal separate regularities in an exact solution and then compare them with those obtained by approximate methods.

First of all let us have a look at Fig. 1. When $\gamma=0$ the energy levels are there represented by straight lines, corrections appearing when interaction is very weak by dotted lines.

There are two kinds of the corrections: the levels of the same symmetry stopped to cross each other (it is known from a theorem of quantum mechanics that such levels must repulse each other),

2. the indentical displacement of all the levels occurs when $\epsilon = 0$.

Results presented in Fig. 2 ($\gamma = 0{,}2$) are similar to those of Fig. 1, but here the repulsion between levels is stronger. With increasing γ the repulsion between pairs of levels 0^- and $1^-, 1^+$ and $2^+, 3^-$ (not shown) and 2^- becomes stronger and the picture changes: at first level 2^- becomes lower than level 2^+ (Fig. 4) and then level 1^+ also becomes lower than level 1^- (Fig. 5). This take place under the condition $\epsilon \ll 1$. A further repulsion between level 2^- and a higher level when $\epsilon \approx 1.6$ and between levels 1^- and 2^- when $\epsilon \approx 0.5$ results in that levels 2^- and 2^+ are at first bringing together (Fig. 7) and then level 2^- again becomes higher than level 2^+. It is interesting to notice that with increasing γ strips of pairs of levels 0^- and $0^+, 1^-$ and $1^+, 2^-$ and 2^+ become more and more similar to each other (Fig. 8 and Fig. 9). The larger the γ and the lower the level energy the narrower the strips. Notice that the smaller the γ, the larger the ϵ, the closer the levels of the same symmetry. The two last regularities can easily be seen in Fig. 2. Levels 2^- and 1^- are rather close to each other when $\epsilon = 1{,}45$ (see also Fig. 3) and levels 1^+ and 2^+ are already separated by much greater energy interval when $\epsilon = 0.5$.

Besides, when $\epsilon = 0.5$ we have $E_{1^-} - E_{0^-} < E_{2^+} - E_{1^+}$. One can prove the first regularity by comparing the distance between levels 2^- and 1^- when $\epsilon = 1.45$ (Fig. 2), when $\epsilon = 1.25$ (Fig. 5) and when $\epsilon = 1.1$ (Fig. 5). Though at the same time a decrease of ϵ is seen.

Now compare the exact results with those obtained by approximate methods. Results of a resonant approximation are given in Fig. 4 and 6. It can be noted that when $\gamma = 0.2$ the above results are in very good agreement with the positions of the levels. When $\gamma = 0.6$ a deviation of the exact results is considerable (although the corrections introduced improve agreement with the exact results). This means that the resonant formulae are useful only for small values of γ. It is necessary to remind that the perturbation method does not work when $\epsilon \approx 0.5$ even if the value of γ is rather small. But between points of a maximum rapprochement one can use the perturbation method. Results of calculations in the strong coupling approximation (the corrections up to the second power of ϵ were taken into account) are presented in Figs. 4 ($\gamma = 0.4$) and 8 ($\gamma = 1$).

It can be seen that when $\gamma = 0.4$ the strong coupling approximation is satisfactory up to $\epsilon \approx 0.5$. When $\gamma = 1$ it works already with $\epsilon \approx 1$. It means that the range of application of the approximation is extended with increasing γ.

Fig. 8 also shows results given by the perturbation method. It can be seen that there is a whole region of values of ϵ lying to the right of the point $\epsilon = 1$ where neither the strong coupling approximation nor the perturbation method can be used.

But in this region the behaviour of the levels has an interesting peculiarity.

Fig. 10 shows how the difference in energy of levels close to each other depends on ϵ. These differences are very close to each other when $1.5 \leq \epsilon \leq 3.5$ and weakly depend on ϵ i.e. the perturbation spectrum of the system is approximately equidistant. This means that in this region of variation of ϵ inharmonic effects are practically absent and the interaction leads only to a renormalization of both the phonon frequency and the fermion energy.

References

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2. И.Н. Михайлов, Р.В. Джолос. Препринты ОИЯИ Р-2567 (1966). Acta Physica Polonica (in print).

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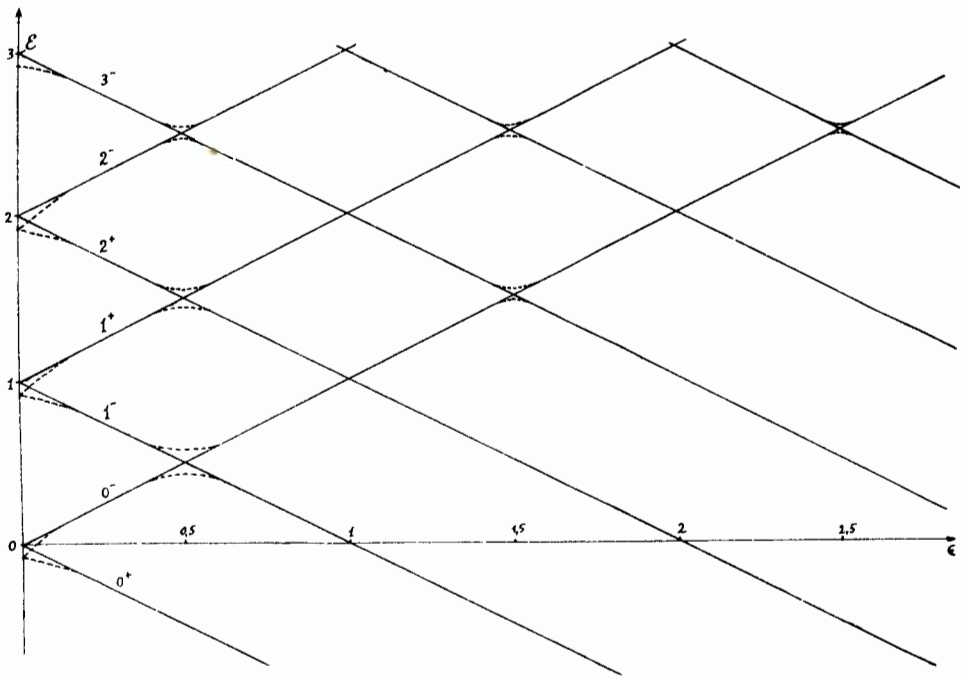

Fig. 1. Continuous curves are energies of states when $\gamma = 0$. Dotted curves are corrections when $\gamma \ll 1$.

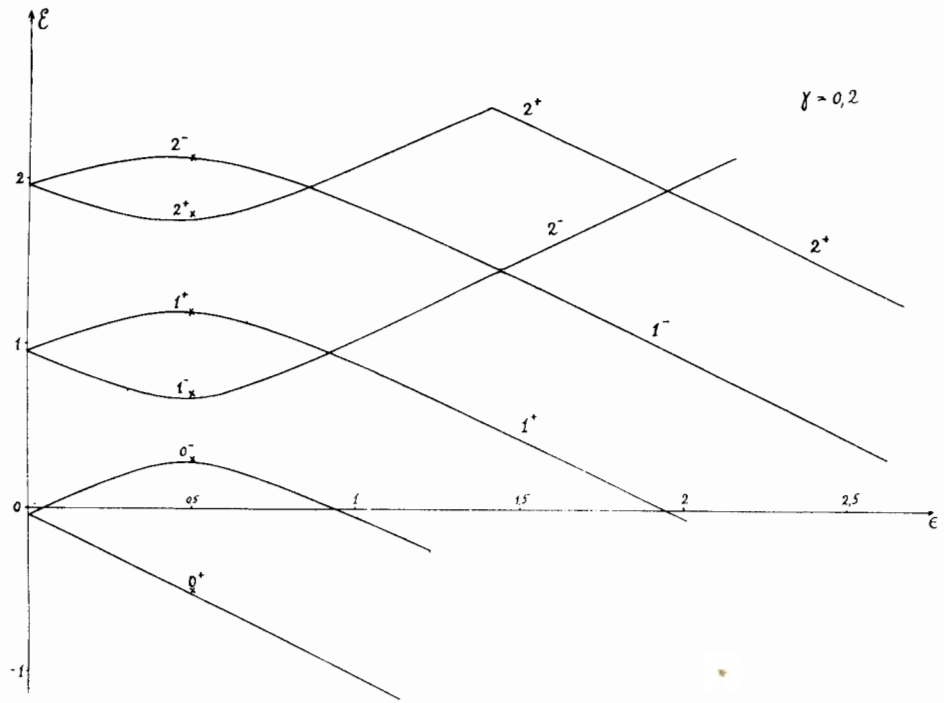

Fig. 2. Energies of states when $\gamma = 0.2$. Results of calculations obtained by means of resonant formula are marked by crosses.

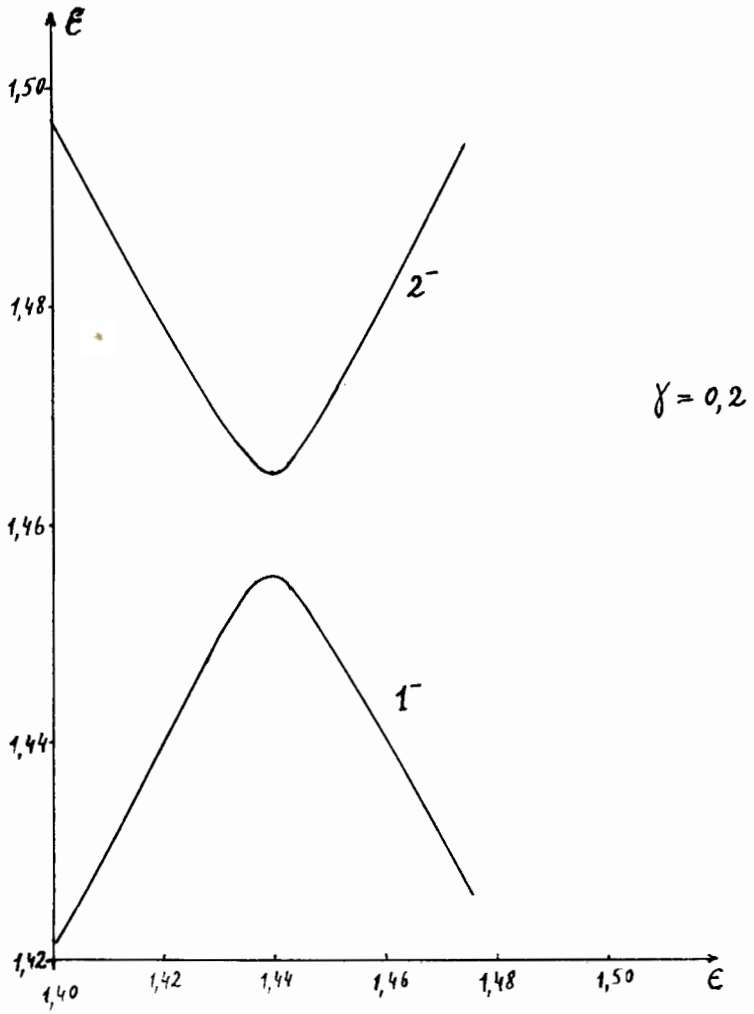

Fig. 3. Energies of states 1^- and 2^- when $\gamma = 0.2$.

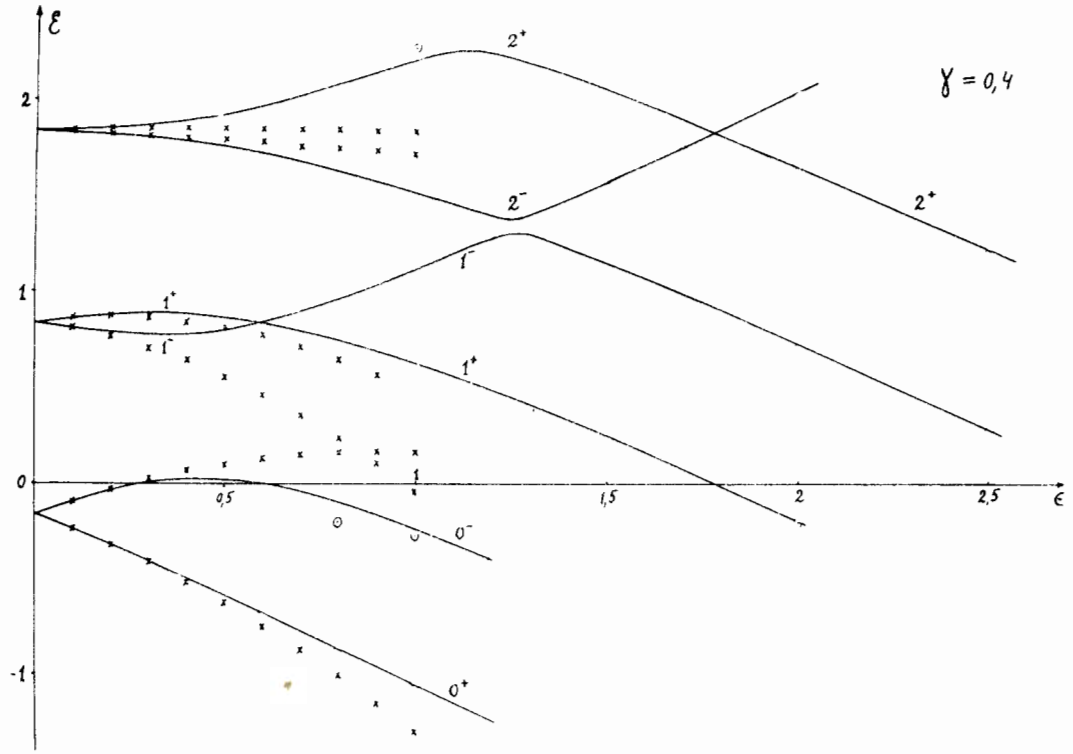

Fig. 4. Continuous curves are energies of states when $\gamma = 0.4$. Results of the approximation of strong coupling are marked by crosses.

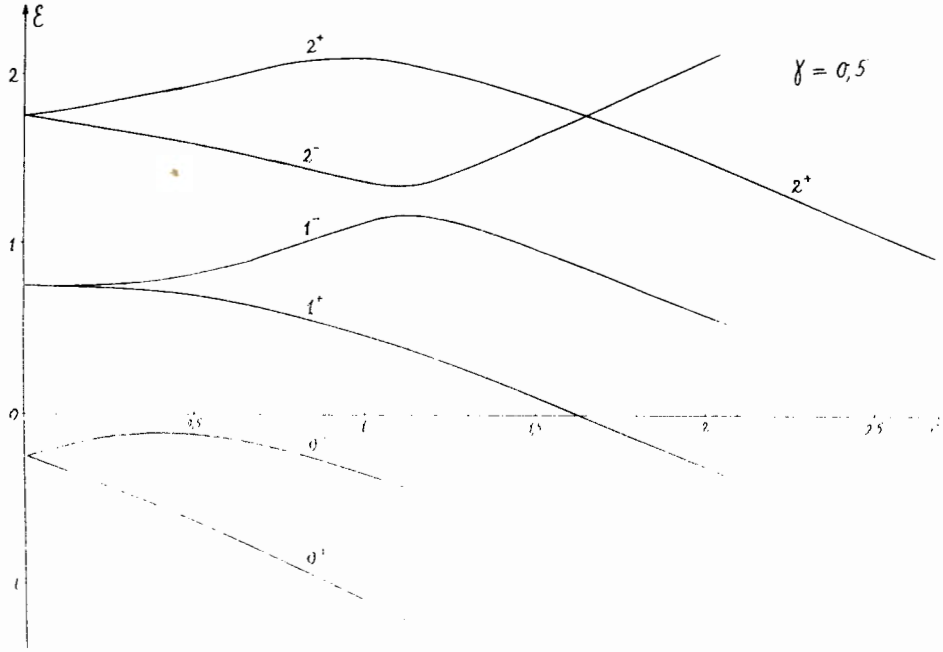

Fig. 5. Energies of states when $\gamma = 0.5$

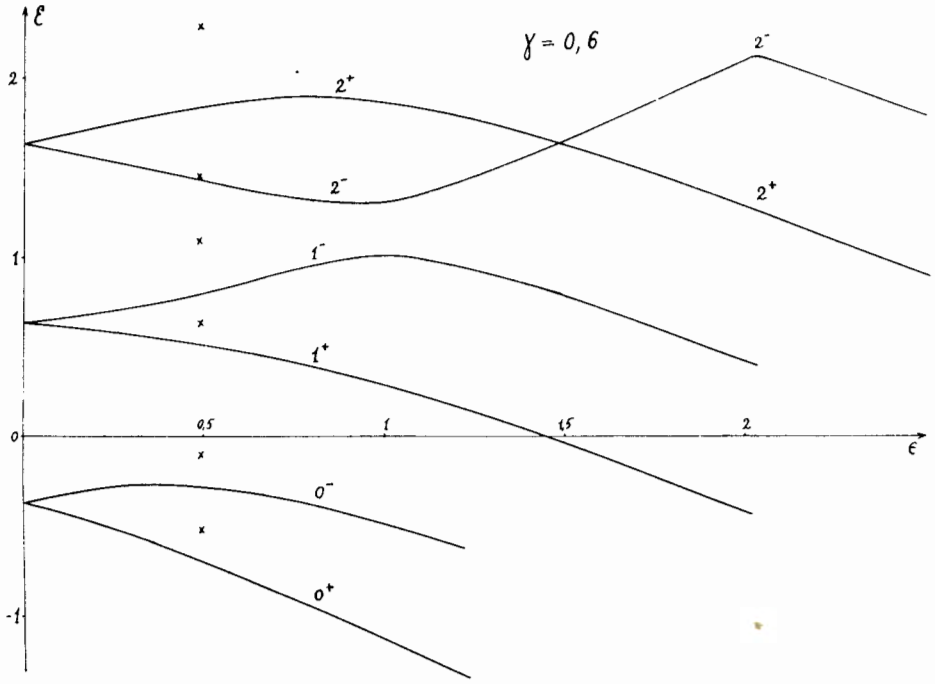

Fig. 6. Energies of states when $\gamma = 0.6$. Results of calculations by resonant formula are marked by crosses.

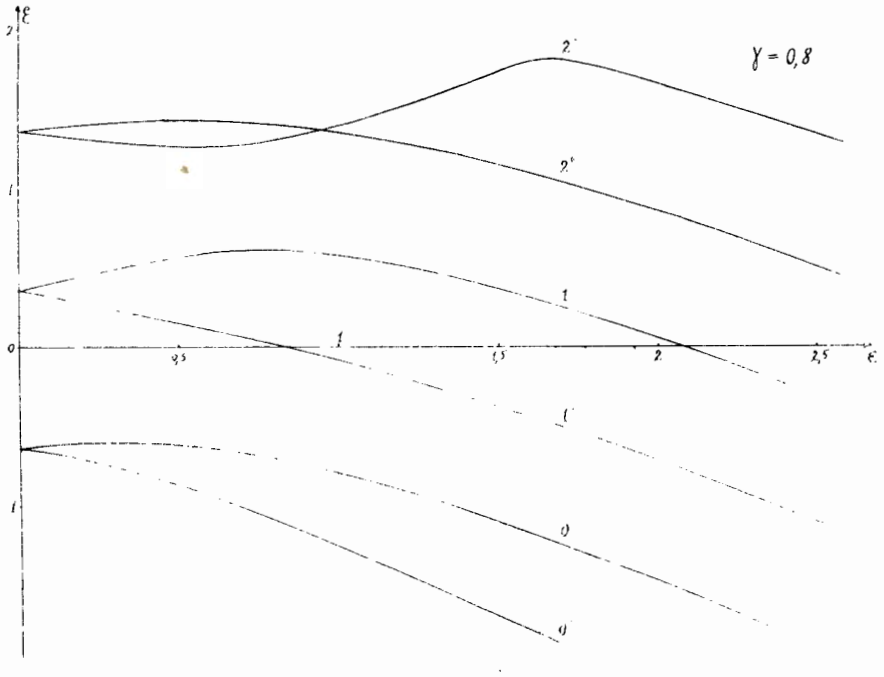

Fig. 7. Energies of states when $\gamma = 0.8$.

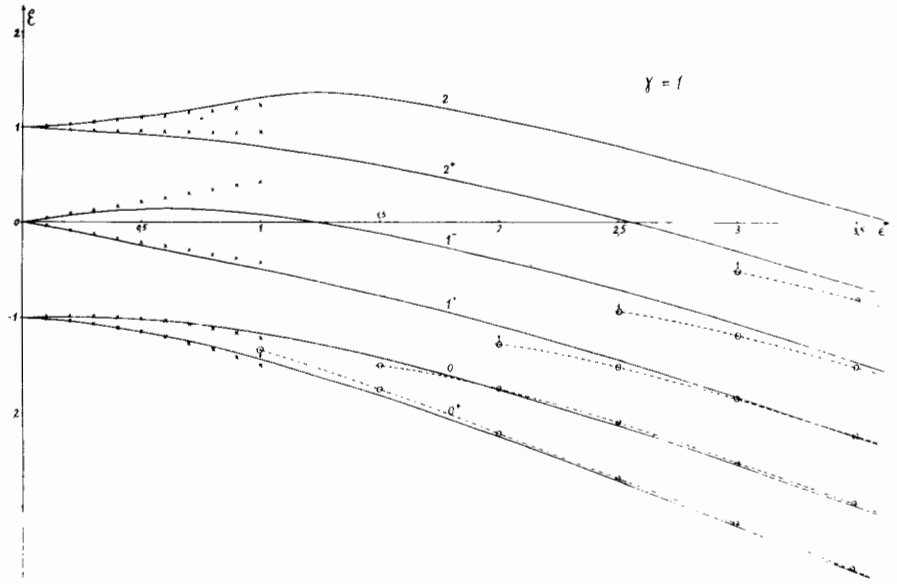

Fig. 8. Continuous curves are energies of states when $\gamma = 1.0$. Results of the approximation of strong coupling are marked by crosses. Dotted curves are results of the perturbation method.

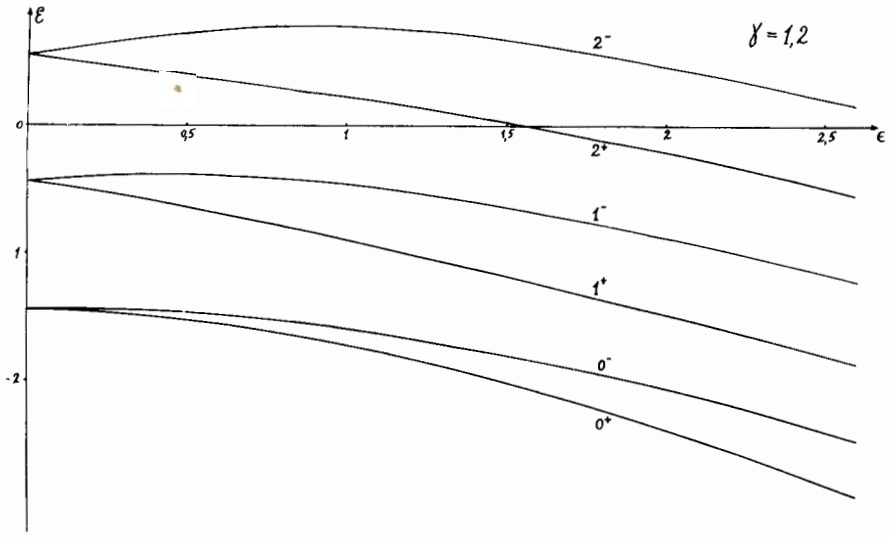

Fig. 9. Energies of states when $\gamma = 1.2$.

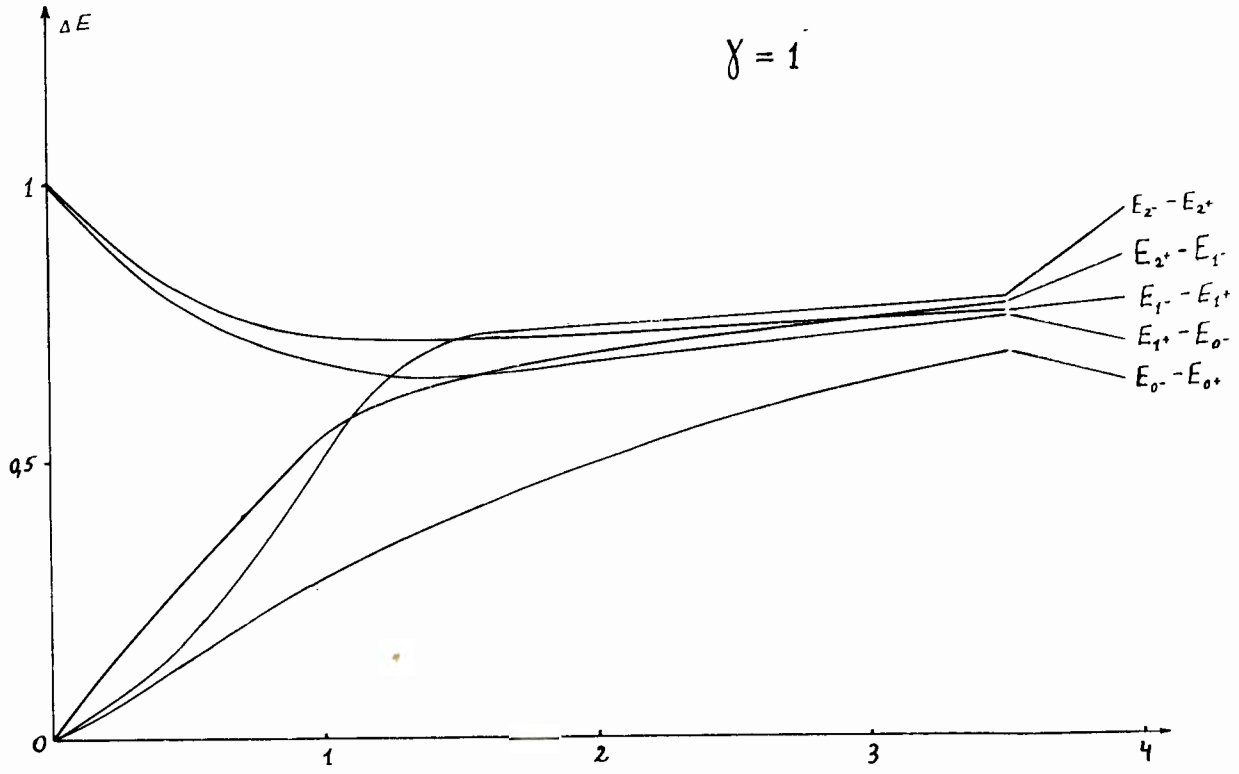

Fig. 10. Differences of the energies of states when $\gamma = 1.0$.