ОБЪЕДИНЕННЫЙ
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Дубна

E-2765

ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

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1966

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Submitted to Jad. Phys.

*) Moscow State University.

The problem of simultaneous phase-shift analysis of $n-p$ and $p-p$ data is uniquely solved in the region below pion production threshold /23.1-310 MeV/. The lack of $n-p$ - scattering data has so far made a unique solution above the pion production threshold to be impossible.

Above the threshold a considerable amount of nucleon-nucleon scattering data is available for two energies: 400 and 630 MeV.

Two equally probable phase-shifts sets[1,2] exist at both energies. The angular dependences of the experimental quantities calculated using there two sets [2] differ most for the isotopical spin $T=0$.

This paper is devoted to the planning of an optimal experiment, making it possible to discriminate between the existing sets of phase-shifts at 400 and 630 MeV. The simplest corresponding experiments are the measurements of the Wolfenstein parameters[3] of the determination of the spin-correlation tensor or asymmetry tensor in $n-p$ - scattering[4]. The Wolfenstein parameters can be determined without using a polarized proton target (P.P.T). It is practically impossible to determine the spin correlation coefficients C_{ik} for $n-p$ - scattering without a P.P.T. A P.P.T. is of course necessary to determine an asymmetry tensor.

The dependence of the quantities D_{pn}, R_{pn}, A_{pn}, C^{pn}_{nn}, A^{pn}_{ss} on the θ c.m.s. scattering angle are shown in figs. 1 and 9 for 400 and 630 MeV, respectively.

The planning of experiments is based on the results of refs.[5,6]. The relative time necessary to discriminate between two sets of phase-shifts at a given scattering angle is determined by the relation:

$$t(\Theta) = \frac{\delta^2(\lambda_i+\lambda_k)-(s_i^2-s_k^2)(\lambda_i-\lambda_k)+2\sqrt{(\lambda_k-\lambda_i)(s_k^2\lambda_k-s_i^2\lambda_i)+\delta^4\lambda_i\lambda_k}}{\lambda_i\lambda_k[\delta^4+(s_i^2-s_k^2)-2\delta^2(S_i^2+S_k^2)]} \quad . \quad (1)$$

The formula holds for $\delta^2 > (s_i + s_k)^2$, where

$$\delta^2 = \left(\frac{y_i(\Theta) - y_k(\Theta)}{u_{1-\alpha}} \right)^2 ; \qquad s_i^2 = \sigma_i^2(\Theta) + \sigma_{i0}^2(\Theta) ;$$

$y_i(\Theta)$ – is the experimental quantity D, R, A, C_{nn}, A_{ss}, calculated according to the i-th phase-shift set ;

$\sigma_i(\Theta)$ – is the corridor of errors of the quantity $y_i(\Theta)$;

$\sigma_{i0}(\Theta)$ – is the systematical error of $y_i(\Theta)$;

$u_{1-\alpha}(\Theta)$ – is the normal distribution level ;

Θ; θ – are the l.s. and c.m.s. scattering angles, respectively.

The efficiency $\lambda(\Theta)$ figuring in (1) is determined as

$$\lambda(\Theta_2) = K \int\limits_{\{\Theta_3\}} \int\limits_{\{\Phi_3\}} \frac{I_{02}^2(\Theta_2)\, I_{03}^2(\Theta_3)\, P_{03}^2(\Theta_3)\, f(\Phi_3)}{I_2(\Theta_2)\, I_3(\Theta_3, \Phi_3)}\, d\Omega , \qquad (2)$$

for the parameters D, R and A, where

Θ_j – is the angle of j-th scattering in l.s. ;

Φ_3 – is angle between the normals to the second and third scattering planes;

I_{0j} – is the cross-section on the j-th unpolarized target (for an unpolarized beam);

I_j – is the cross-section on the j-th target;

$P_{03}(\Theta_3)$ – the analysing power of the third target $f(\Phi_3) = \sin^2 \Phi_3$ for R and A, $f(\Phi_3) = \cos^2 \Phi_3$ for the parameter D.

The integration is over the whole detected region in the last scattering. It was assumed in the calculations that parameters D, R, A will be measured using spark chambers for $0° \le \Phi_3 \le 2\pi$ and $4° \le \Theta_3 \le 30°$.

The absolute value of the coefficient K is unknown and can be determined after the first measurement of an arbitrary quantity (D, R or A) /6/. The coefficient K is approximately equal to 1000 measurement hours on the synchrocyclotron of the Laboratory of Nuclear Problems, JINR /8,9/. For the measurement of the parameter R the efficiency λ can be written as

$$\lambda_1(\Theta_2) = K\, I_{02}(\Theta_2) \int\limits_{\{\Theta_3\}} \int\limits_{\{\Phi_3\}} \frac{I_{03}(\Theta_3)\, P_{03}^2(\Theta_3) \sin^2\Phi_3 \sin\Theta_3\, d\Theta_3\, d\Phi_3}{1 + P_{03}(\Theta_3)[P_2 \cos\Phi_3 - P_1 R_1 \sin\Phi_3]} , \qquad (3)$$

where $P_j(\Theta_j)$ – is the polarization of nucleons after j-th scattering. For the parameter A the efficiency λ is given

$$\lambda_1(\Theta_2) = K\, I_{02}(\Theta_3) \int\limits_{\{\Theta_3\}} \int\limits_{\{\Phi_3\}} \frac{I_{03}(\Theta_3)\, P_{03}^2(\Theta_3) \sin^2\Phi_3 \sin\Theta_3\, d\Theta_3\, d\Phi_3}{1 + P_{03}(\Theta_3)[P_2 \cos\Phi_3 - P_1 A_1 \sin\Phi_3]} \qquad (4)$$

and for D is

$$\lambda_1(\Theta_2) = K \frac{I_{02}(\Theta_2)}{1 + P_1 P_2} \int\limits_{\{\Theta_3\}} \int\limits_{\{\Phi_3\}} \frac{I_{03}(\Theta_3)\, P_{03}^2(\Theta_3) \cos^2\Phi_3 \sin\Theta_3\, d\Theta_3\, d\Phi_3}{1 + \frac{P_{03}(\Theta_3)}{1 + P_1 P_2}(P_2 + D_1 P_1)\cos\Phi_3} . \qquad (5)$$

Formula (3) is proved in appendix 1.(We assume that P_2 is the same for all sets.)

A beam of polarized neutrons and a P.P.T./7/ were assumed in the measurement of the spin correlation coefficient C_{nn}. Using scintillation counters with a fixed solid angle $d\Omega$ /in a large region of scattering angles Θ /. In this case the efficiency λ is:

$$\lambda_1(\Theta) = K \frac{I_0(\Theta)\, P_1^2\, P_2^2}{1 + P_0(\vec{P}_1 \vec{n}_1) + P_0(\vec{P}_2 \vec{n}_2) + C_{nn}^1 (\vec{P}_1 \vec{n})(\vec{P}_2 \vec{n})} \qquad (6)$$

If the component of the asymmetry tensor A_{ss} is determined under the same conditions as C_{nn}, the efficiency λ is

$$\lambda_1(\Theta) = K \frac{I_0(\Theta)\, P_1^2\, P_2^2}{1 + A_{ss}^1 (\vec{P}_1 \vec{s})(\vec{P}_2 \vec{s})} . \qquad (7)$$

In eqs. (6) and (7)

P_1 – is the polarization of the incident particle /in our case $P_1 = 0.37$/;

P_2 – is the polarization of P.P. T. /we assume $P_2 = 0.30$/ ;

P_0 – is the polarization in the scattering of unpolarized particles on an unpolarized target $\vec{n}_1 = \vec{k}_0 \times \vec{k}_1$, $\vec{s} = \vec{n}_1 \times \vec{k}_0$, where $\vec{k}_0$, $\vec{k}_1$ – are unit in the directions of the incident and scattered particle momenta, respectively.

The calculations have shown that the efficiencies λ_i / $i = 1,2$/ in all the experiments D_{pn}, R_{pn}, A_{pn}, C_{nn}^{pn}, A_{ss}^{pn} coincide within 0.01 for both phase-shifts at 400 and 630 MeV.

The dependence $\lambda(\theta)$ for all mentioned quantities at 400 MeV are given in figs. 2, 3. The level $u_{1-\alpha}$ /remember that α is the probability of excluding a correct hypothesis/ is always assumed to be equal to 1. The time necessary to discriminate between two sets of phase-shifts at 400 MeV is given as a function of the scattering angle for various experiments in figs. 4-8. The optimal angles for each experiment and corresponding necessary time in arbitrary units are shown in table 1.

The planning of experiments discriminating between two sets of phase-shifts is performed for the same group of experiments in n-p -scattering at 630 MeV all data on the n-p -scattering parameter and on the phase-shifts analysis at 630 MeV are taken from ref./8/. The efficiencies λ for R, D, A, C_{nn}, A_{ss} at 630 MeV are shown in figs. 10, 11. The necessary time is given in figs. 12-16 as a functions of the measuring angle. The optimal angles for each experiment and the necessary measuring time in arbitrary units are given in table 2. The time necessary to discriminate between two sets of phase-shifts at 400 and 630 MeV, is expressed in the same units only inside the following groups of parameter (R , D , A) and (C_{nn} , A_{ss}).

The results of the planning of experiments show that measurements of the parameters D and A are the most effective way to discriminate between two existing sets of phase-shifts at both energies . It should be noted, that planning was performed, assuming the sets of phase-shifts to be stable with respect to the addition of new experimental data. Actually new data can sometimes noticeably change the phase-shifts analysis solutions. This makes a new planning of experiments necessary and results can differ significantly from those in tables 1 and 2. Such an affect is improbable at 630 MeV, since much experimental data on p-p -scattering exists as well as the cross-section and polarization in n-p-scattering and same data on triple--scattering n-p-parameters/8,9/. At 400 MeV only the cross-section and polarization are determined for n-p - scattering so that the predicted angular dependences of the experimental quantities can change significantly if new data with errors of the order 0,1 are added.

In conclusion the authors wish to express their deep gratitude to Yu.M. Kazarinov, N.P. Klepikov and P. Winternitz for valuable discussions; to E.Dudova, T.D. Timofeyeva, N.V. Volczkova for help in the work.

Table 1.

Parameter	T /time in arbitrary units/	$\vartheta^{optim.}_{c.m.s.}$
D_{pn}	0.068	60°
A_{pn}	0.100	55°
R_{pn}	3.2	115°
Parameter	T'/time in arbitrary units/	$\vartheta^{optim.}_{c.m.s.}$
C^{pn}_{nn}	65,/200/	60°,/115°/
A^{pn}_{ss}	85,/100/	110°,/55°/

Table 2.

Parameter	T /time in arbitrary units/	$\vartheta^{optim.}_{c.m.s.}$
D_{pn}	0.035	115°
A_{pn}	0.190	130°
R_{pn}	0.690	115°
Parameter	T'/time in arbitrary units/	$\vartheta^{optim.}_{c.m.s.}$
C^{pn}_{nn}	75,/500/	150°,/70°/
A^{pn}_{ss}	11, 24, 30	25°,65°,155°

T and T' are measurement times in arbitrary units. Measurements on the parameters D_{nn}, R_{pn}, A_{pn} have been planned under the assumption that in measuring spark chamber should be used. In the experiment on the determination of C^{pn}_{nn} and A^{pn}_{ss} the polarized proton target and the polarized neutron beam are to be used. The scales of T and T' are different.

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Received by Publishing Department
on May 30, 1966.

Appendix 1

Derivation of the efficiency λ. for an R experiment

The number of particles , scattered in a time unit by the analysing target in the direction Θ_3 and Φ_3 in the solid angle element $d\Omega$ is proportional to:

$$N(\Theta_3, \Phi_3, \Theta_2) \approx I_2(\Theta_2)\; I_3(\Theta_3, \Phi_3)\, d\Omega \;. \qquad (A1.\ 1)$$

Let the polarization $\vec{P}_1$ of the beam incident on the second target be parallel to $\vec{s}$ (fig. 17) . Then $I_2(\Theta_2) = I_{02}(\Theta_2)$ and the polarization $\vec{P}_2$ after the second scattering will have the following components/10/.

$$\begin{aligned} P_{n_2} &= P_{02} \;, \\ P_{s'} &= D_{s's}\, P_1 = R\, P_1 \;, \\ P_{k_2} &= D_{k's}\, P_1 = R'\, P_1 \;. \end{aligned} \qquad (A1.\ 2)$$

The diffential cross-section on the analysing target is equal to

$$\begin{aligned} I_3(\Theta_3, \Phi_3) &= I_{03}(\Theta_3)[\,1 + P_{03}(\Theta_3)\,(\vec{P}_2\, \vec{n}_3)] = \\ &= I_{03}(\Theta_3)\,[\,1 - P_{03}(\Theta_3) R\, P_1 \sin\Phi_3 + P_{03}(\Theta_3)\, P_{02}(\Theta_2) \cos\Phi_3\,]. \end{aligned} \qquad (A1.\ 3)$$

From (A1. 1) and (A1. 3) we have

$$\begin{aligned} N(\Theta_3, \Phi_3, \Theta_2) \approx I_{02}(\Theta_2)\, I_{03}(\Theta_3)\,[\,1 - P_{03}(\Theta_3)\, R(\Theta_2)\, P_1 \sin\Phi_3 + \\ + P_{03}(\Theta_3) P_{02}(\Theta_2) \cos\Phi_3\,] \cdot d\Omega \;. \end{aligned} \qquad (A1.4)$$

Using the known expression /11/ for the dispersion

$$2\, D(a + bx) = b^2 D(x)\,, \qquad (A1.5)$$

where

a and b are constants and x is a random quantity. Assuming all parameters in (A1. 4) except R to be known obtain the dispersion of the parameter

$$D(R) \approx \frac{D[N(\Theta_3, \Phi_3, \Theta_2)]}{I_{02}^2(\Theta_2)\; I_{03}^2(\Theta_3)\; P_{03}^2(\Theta_3) \sin^2\Phi_3\,(d\Omega)^2} \;. \qquad (A1.6)$$

It is shown in chapter 1 of /12/, that the dispersion obtained in a unit of time is proportional to

$$D[N(\Theta_3,\Phi_3,\Theta_2)] \approx I_2(\Theta_2)\, I_3(\Theta_3,\Phi_3)\, d\Phi \quad . \qquad \text{(A1.7)}$$

It follows, that the weight obtained in a unit of time is equal to

$$W(\Theta_3,\Phi_3,\Theta_2) = \frac{1}{D(R)} \approx \frac{I_{02}^2(\Theta_2)\, I_{03}^2(\Theta_3)\, P_{03}^2 \sin^2\Phi_3\, d\Omega}{I_2(\Theta_2)\, I_3(\Theta_3,\Phi_3)} . \qquad \text{(A1.8)}$$

If the detector after the third scattering covers the region $\{\Theta_3\},\{\Phi_3\}$, then

$$W(\Theta_2) \approx \int_{\{\Theta_3\}}\int_{\{\Phi_3\}} \frac{I_{02}^2(\Theta_2)\, I_{03}^2(\Theta_3)\, P_{03}^2(\Theta_3)\, \sin^2\Phi_3 \sin\Theta_3\, d\Theta_3\, d\Phi_3}{I_2(\Theta_2)\, I_3(\Theta_3,\Phi_3)} \qquad \text{(A1.9)}$$

and, the definition of the efficiency /12/ gives

$$\lambda(\Theta_2) = K \int_{\{\Theta_3\}}\int_{\{\Phi_3\}} \frac{I_{02}^2(\Theta_2)\, I_{03}^2(\Theta_3)\, P_{03}^2(\Theta_3)\, \sin^2\Phi_3\, \sin\Theta_3\, d\Theta_3\, d\Phi_3}{I_2(\Theta_2)\, I_3(\Theta_3,\Phi_3)} . \qquad \text{(A.10)}$$

It is easy to show, using similar consideration that the efficiency of experiments measuring D and A . will be determined by an analogous formula.

The unknown coefficient K is present due to difficulties connected with the calculations of number of constants, depending on the intensity of the initial beam, the target density, geometry of the experiment e.t.c.

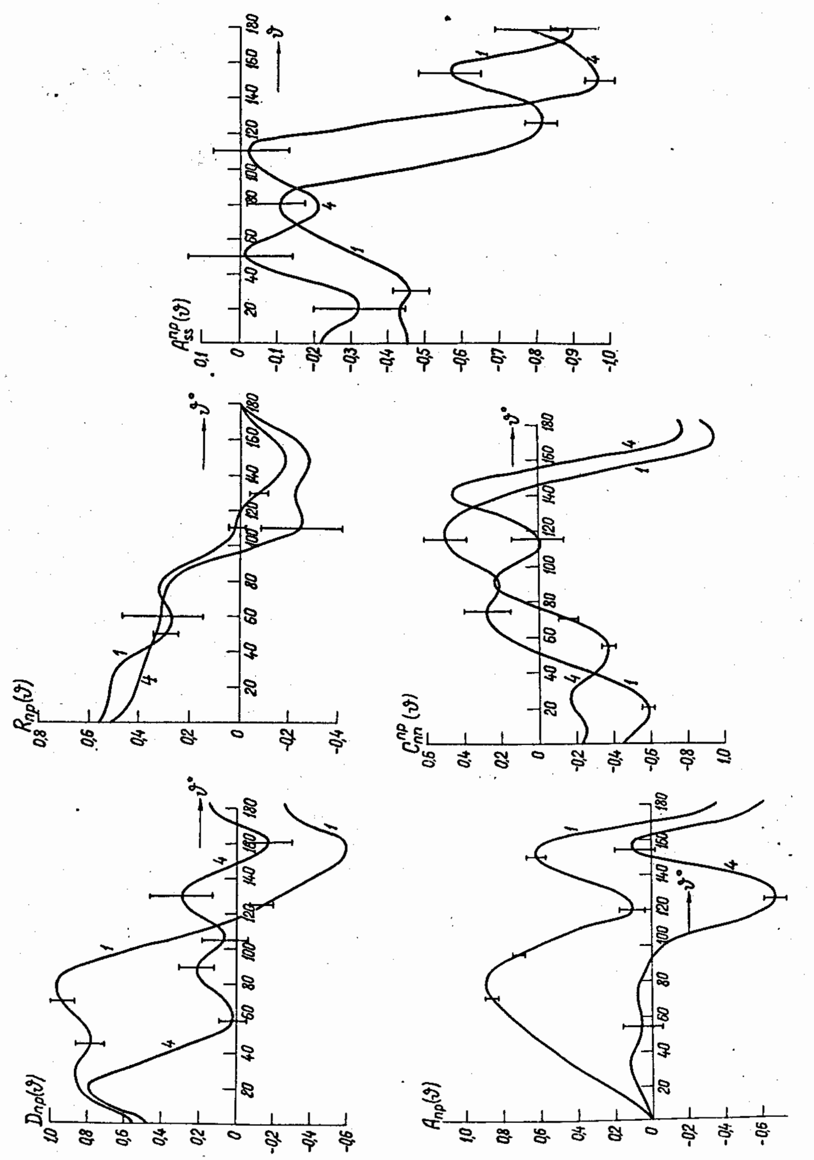

Fig. 1.

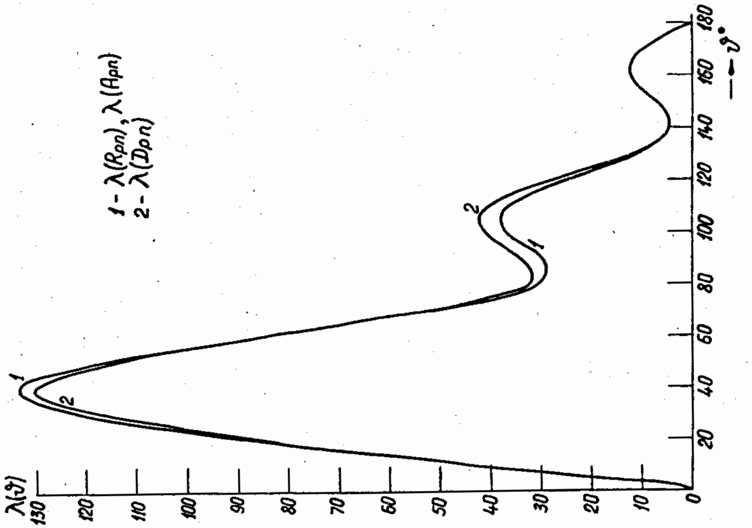

Fig. 2. The dependence of efficiency λ on the scattering angle θ (c.m.s.) on np-scattering for the parameters R_{pn}, D_{pn}, A_{pn} at 400 MeV.

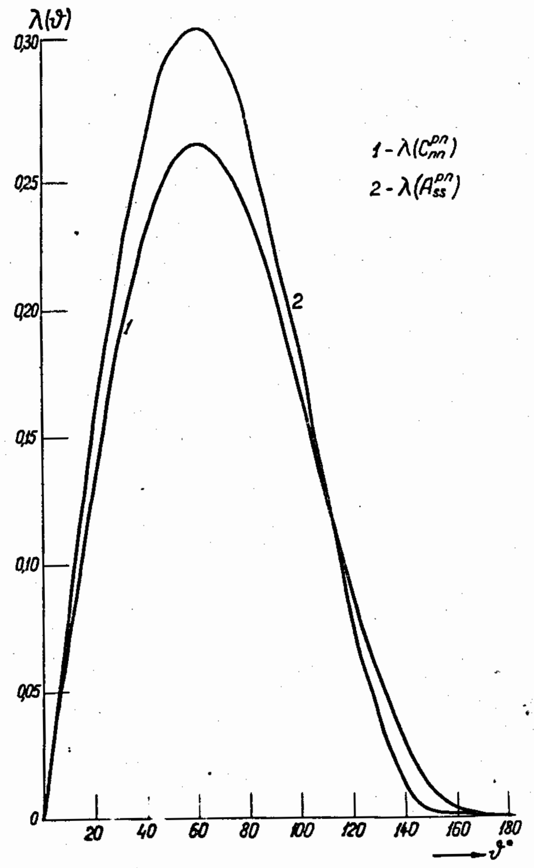

Fig. 3. The dependence of efficiency λ on the scattering angle θ (c.m.s.) of pn-scattering for the parameters C_{nn}^{pn}, A_{ss}^{pn} at 400 MeV.

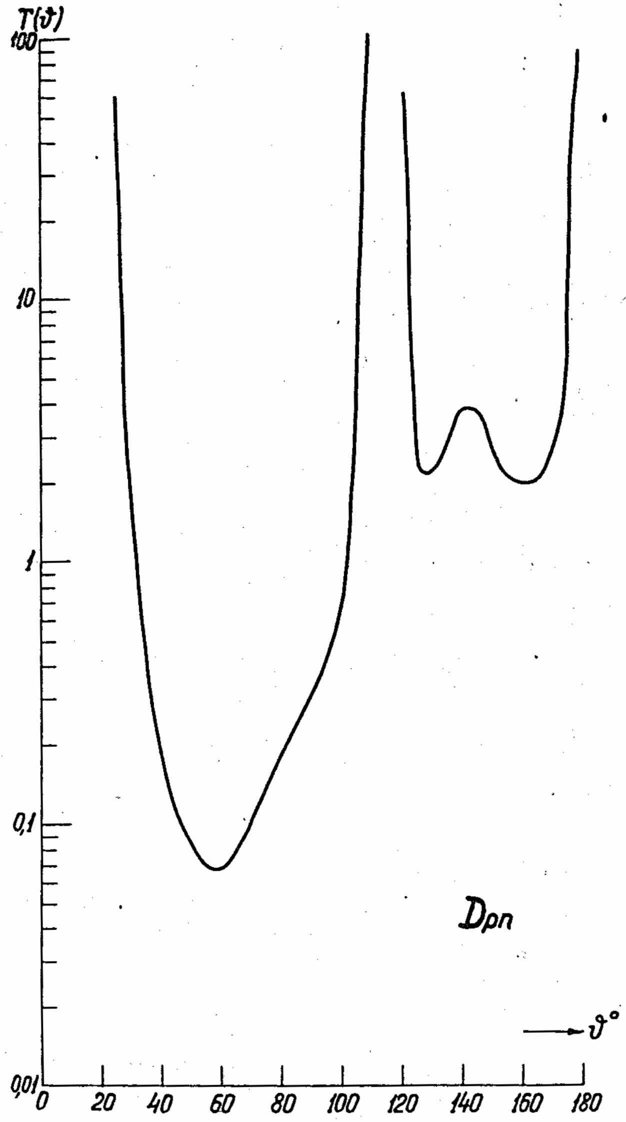

Fig. 4. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter D_{pn} at 400 MeV.

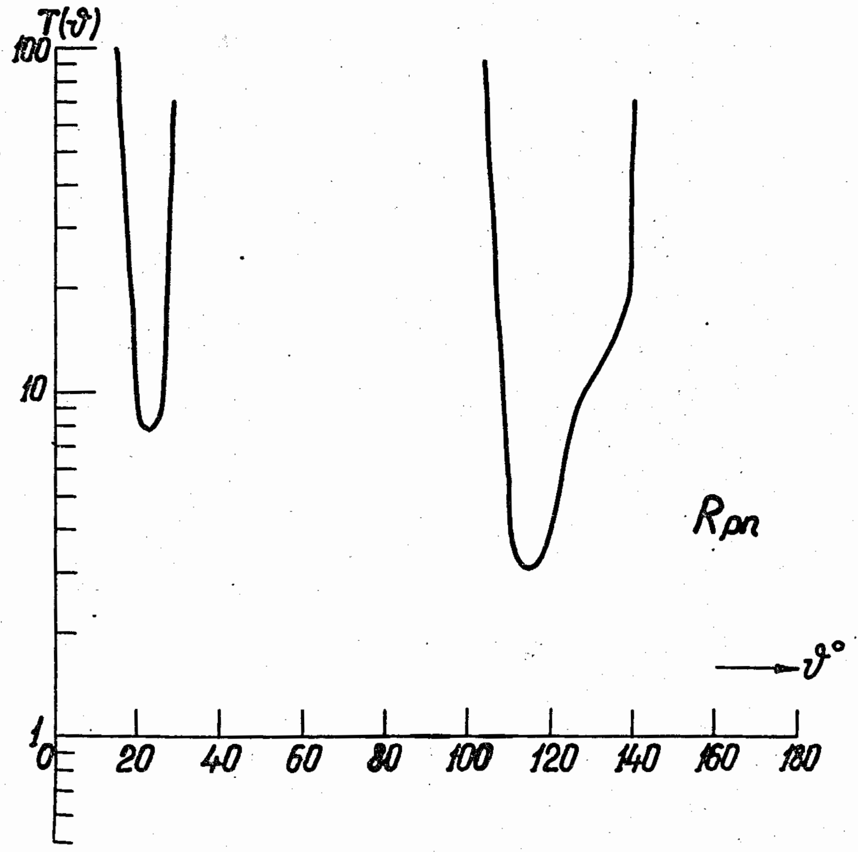

Fig. 5. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter R_{pn} at 400 MeV.

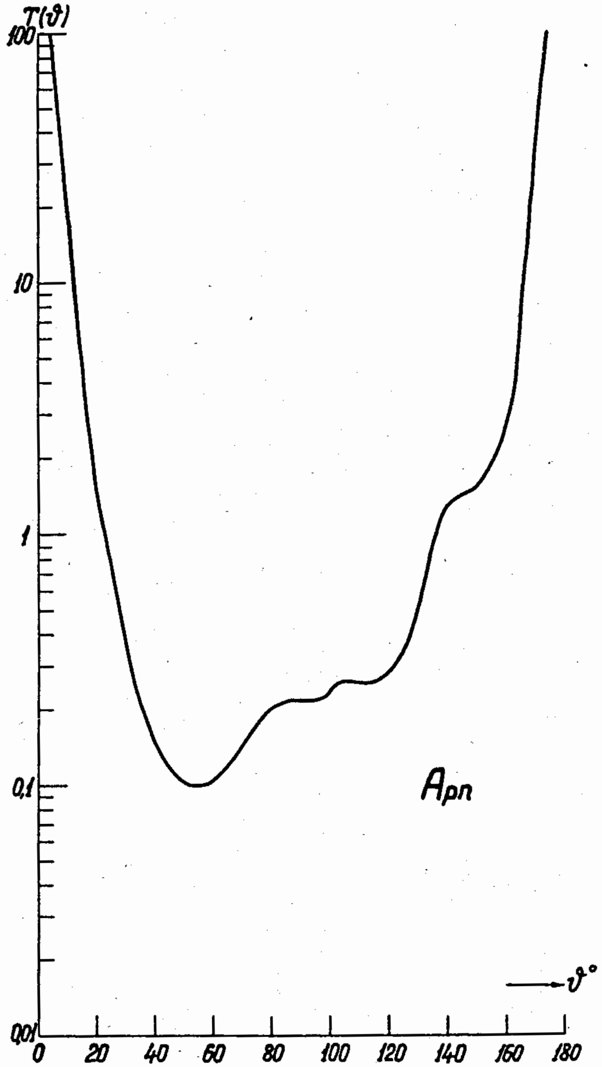

Fig. 6. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter A_{pn} at 400 MeV.

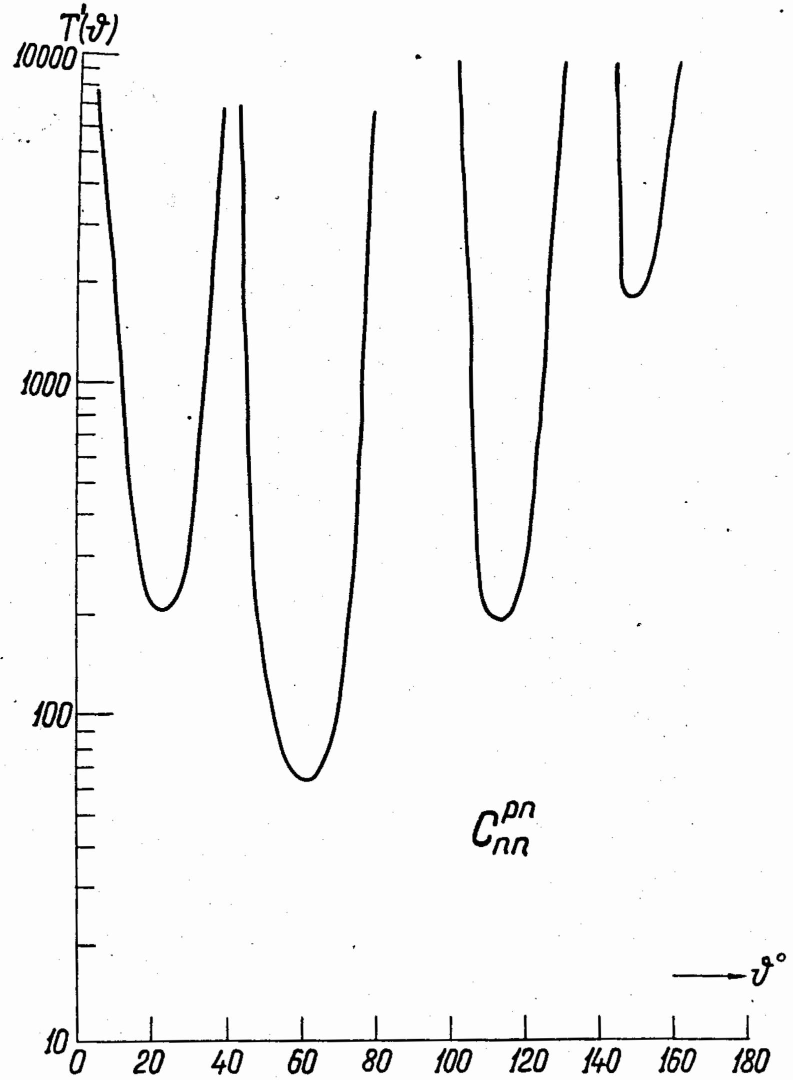

Fig. 7. The dependence of measurement time T' on the scattering angle θ (c.m.s.) for the parameter C_{nn}^{pn} at 400 MeV.

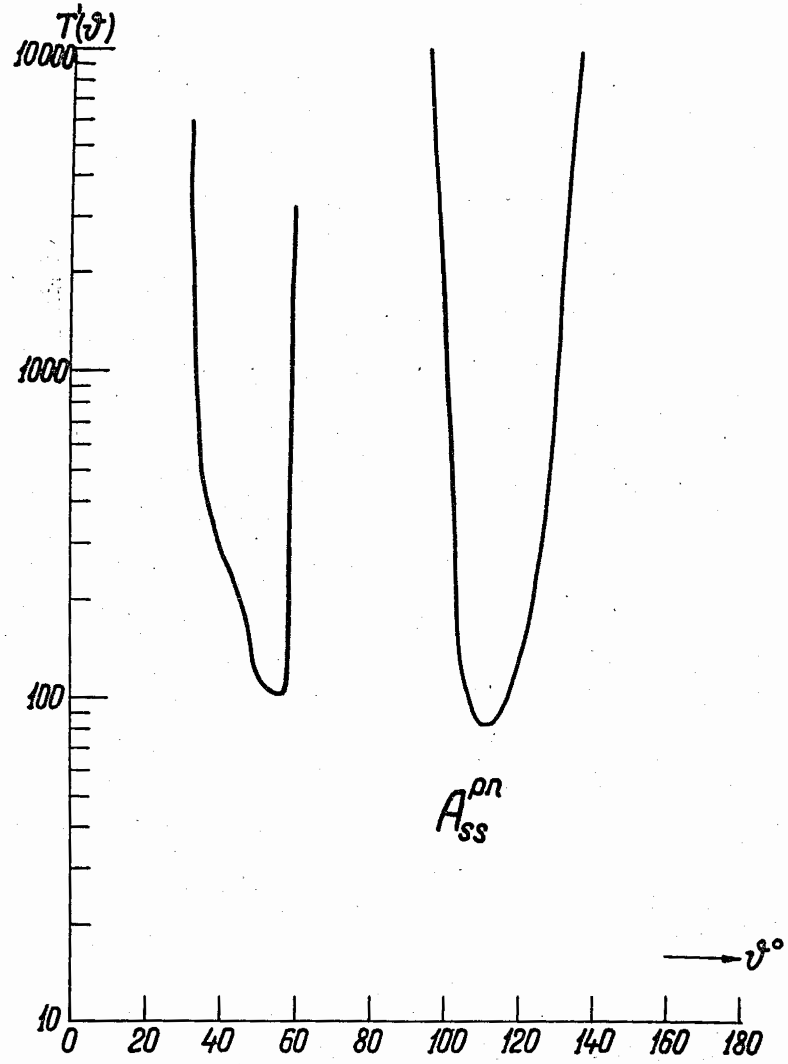

Fig. 8. The dependence of measurement time T' on the scattering angle θ (c.m.s.) for the parameter A^{pn}_{ss} at 400 MeV.

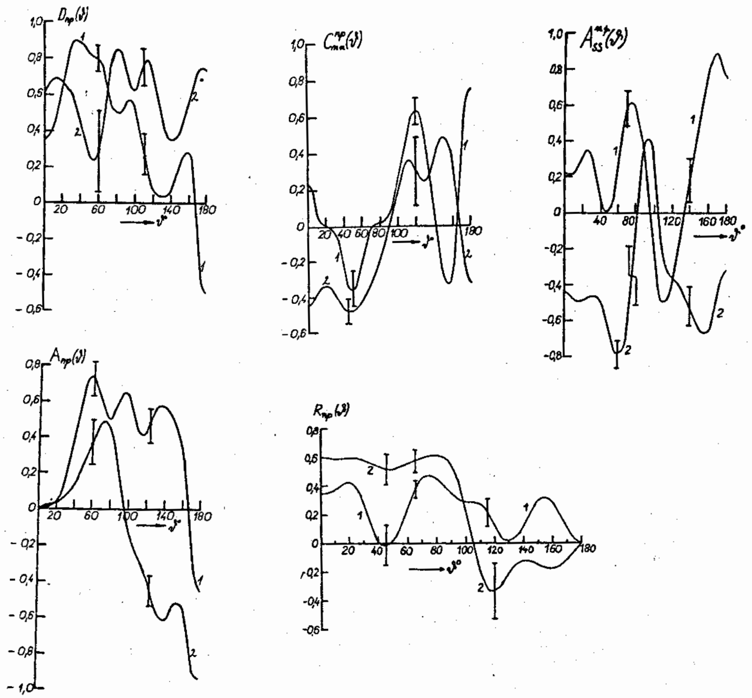

Fig. 9.

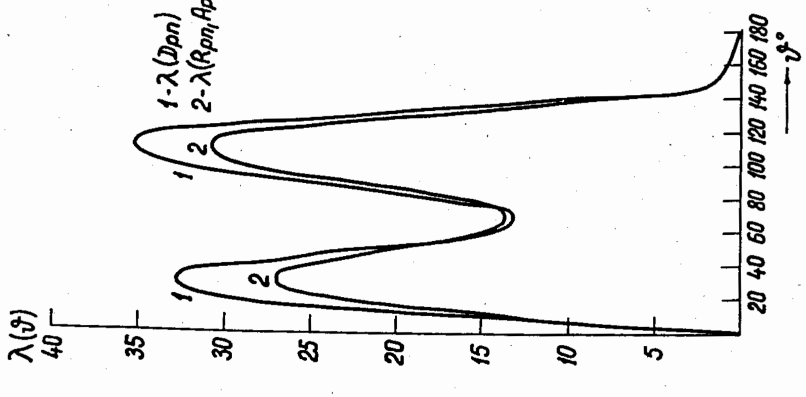

Fig. 10. The dependence of efficiency λ on the scattering angle θ (c.m.s.) of pn-scattering for the parameters R_{pn}, D_{pn}, A_{pn} at 630 MeV.

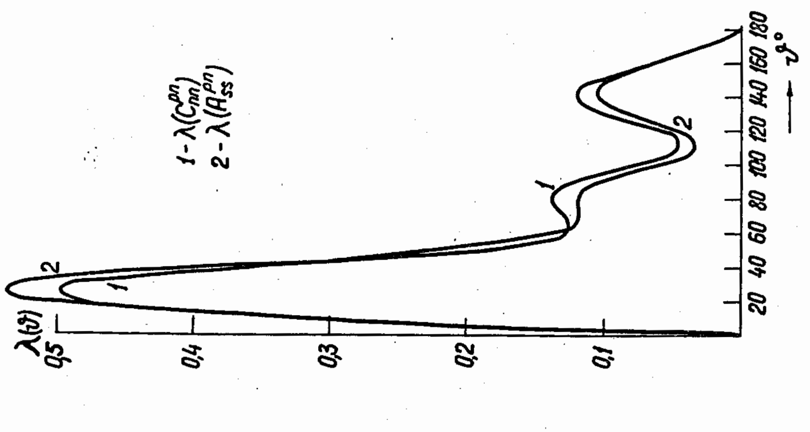

Fig. 11. The dependence of efficiency λ on the scattering angle θ (c.m.s.) of pn-scattering for the parameters C^{pn}_{nn}, A^{pn}_{ss} at 630 MeV.

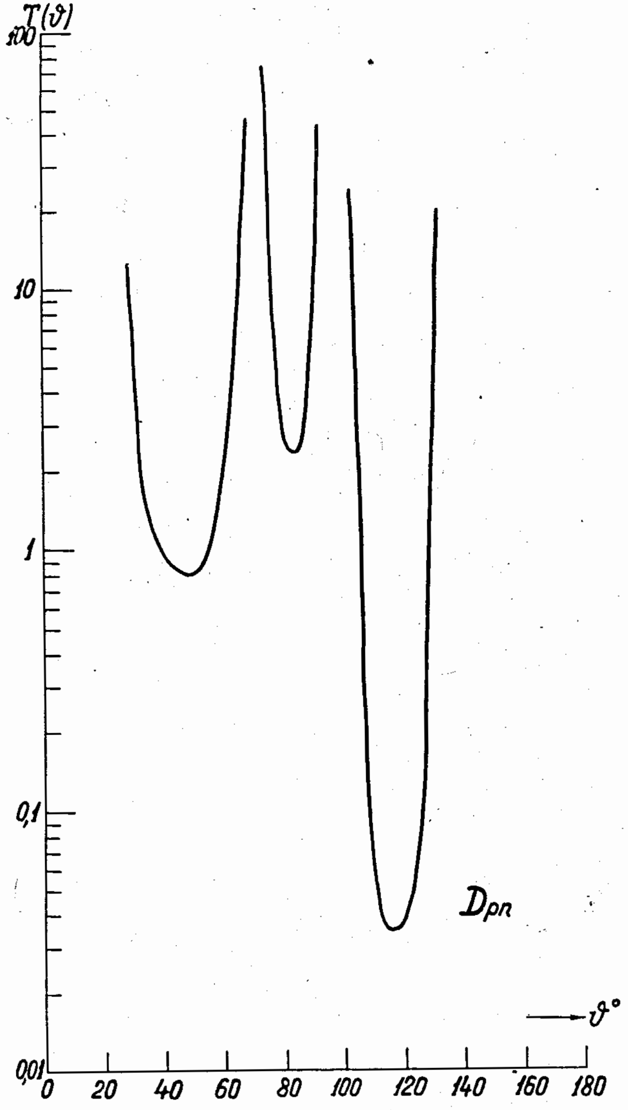

Fig. 12. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter D_{pn} at 630 MeV.

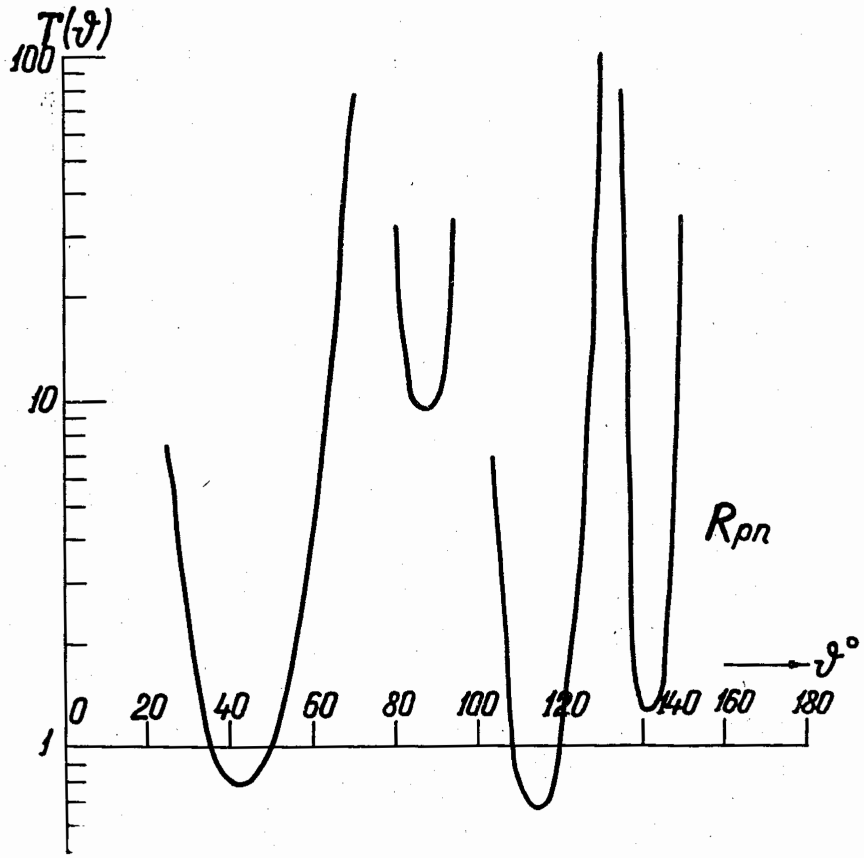

Fig. 13. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter R_{pn} at 630 MeV.

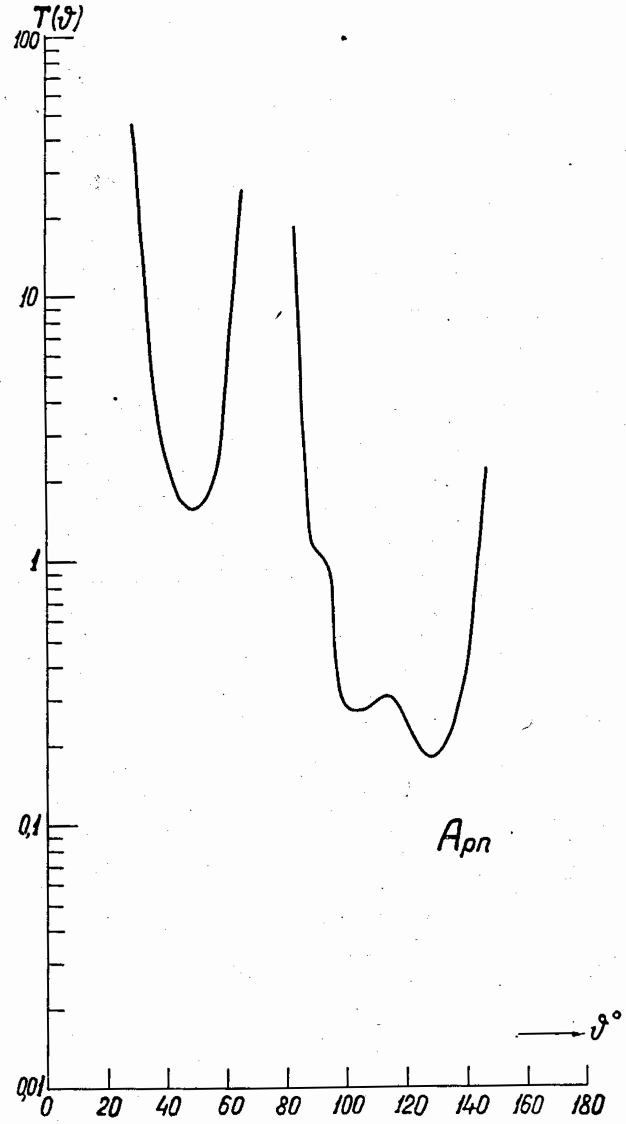

Fig. 14. The dependence of measurement time T on the scattering angle θ (c.m.s.) for the parameter A_{pn} at 630 MeV.

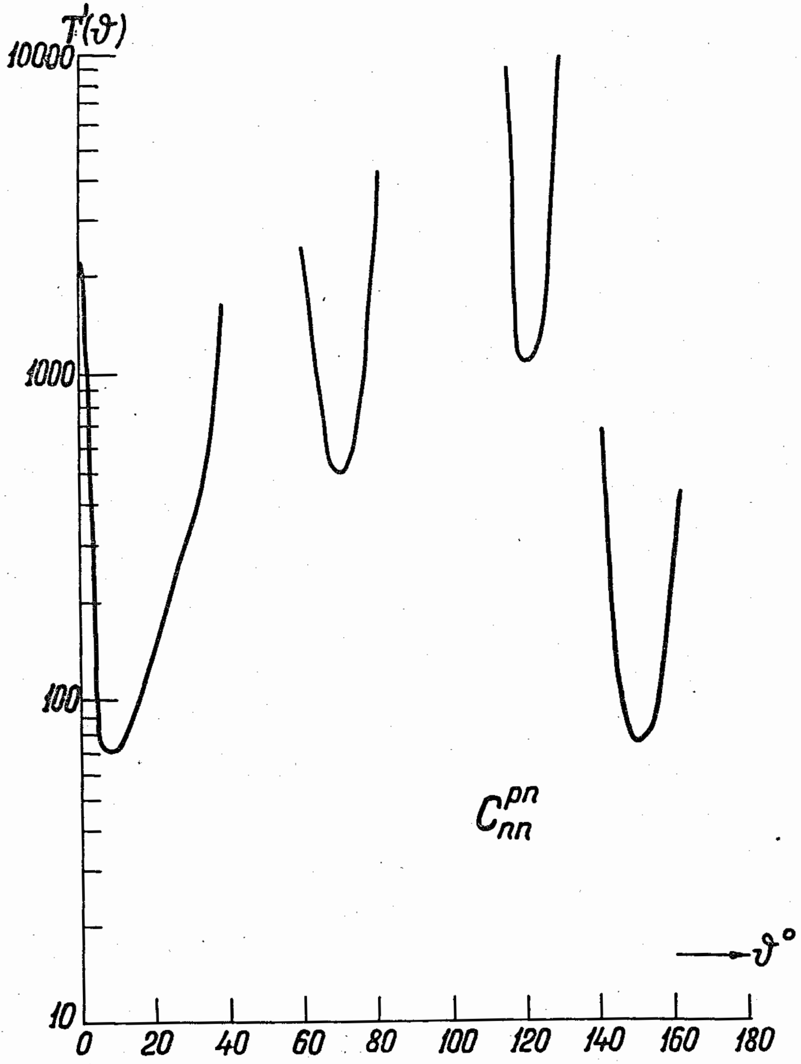

Fig. 15. The dependence of measurement time T_{pn} on the scattering angle θ (c.m.s.) for the parameter C_{nn}^{pn} at 630 MeV.

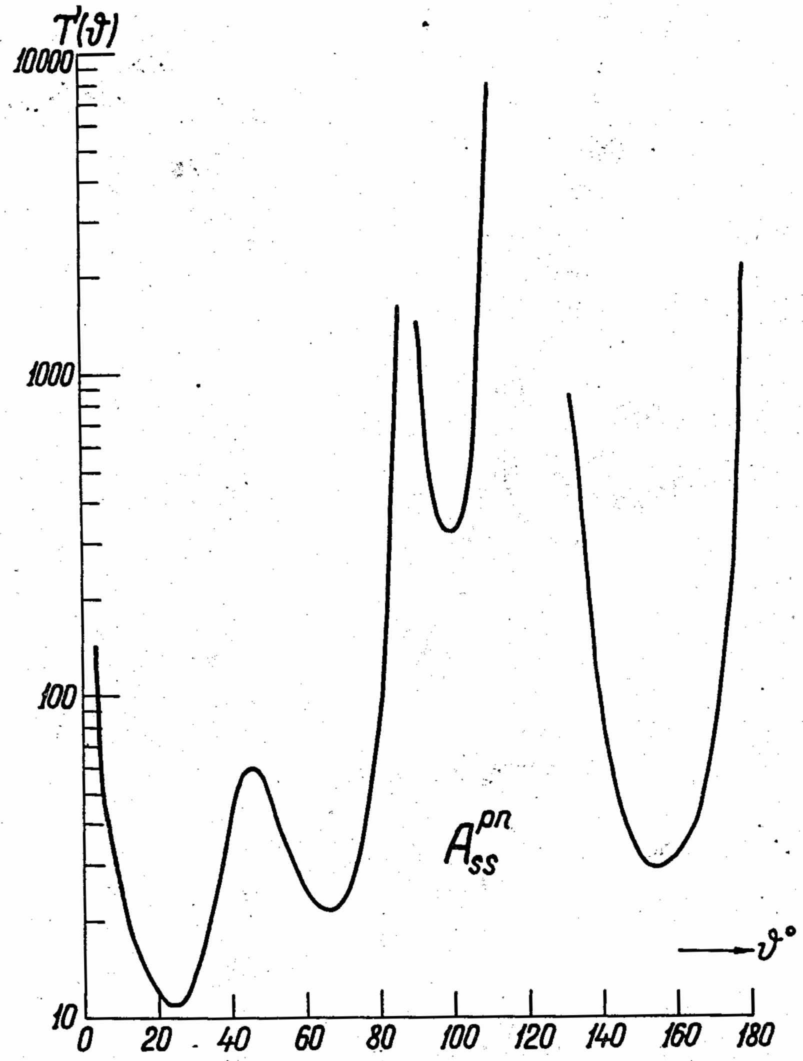

Fig. 16. The dependence of measure ment time T'_{pn} on the scattering angle θ (c.m.s.) for the parameter A_{nn}^{pn} at 630 MeV.

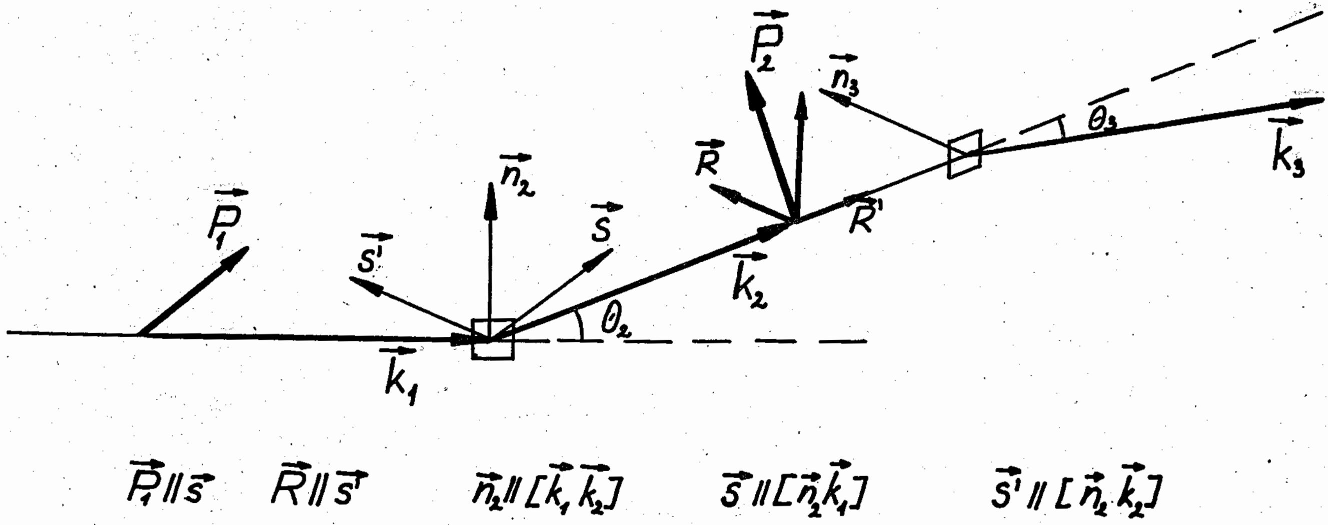

Fig. 17. The scheme of experiment for measurement of triple scattering parameter R.