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S-MATRIX, NON-COMPACT SYMMETRY GROUPS AND THEIR COLLINEAR SUBGROUPS

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In the present paper we study the structure of the S - matrix in the symmetry with non-compact group

$$G = \mathcal{P} \cdot \mathcal{S} \quad , \tag{1}$$

where $\mathcal{P}$ is the Poincare group, $\mathcal{S}$ is a non-compact semi-simple group of internal symmetry, and the point between $\mathcal{P}$ and $\mathcal{S}$ denotes the semidirect product. We consider also connection between the collinear subgroup of $\mathcal{S}$ and the $SU(6)_W$ symmetry of Lipkin and Meshkov /1/ and Dashen and Gell-Mann /2/. The symmetry group (1) was suggested by Budini and Fronsdal /3/ and studied in a series of papers by Fronsdal /4/, Delbourgo, Salam and Strathdee /5,6/ and Ruhl /7/. A detailed version will be published in /8/.

We introduce some notations. The generators of the symmetry group $\mathcal{S}$ and the homogeneous Lorentz group $\mathcal{L}$ will be denoted by S_ξ and $I_{\mu\nu}$, respectively. $\mathcal{S}$ contains some subgroup isomorphic to $\mathcal{L}$. We denote this subgroup by $\mathcal{S}_{\mathcal{L}}$ and its generators - by $s_{\mu\nu}$. For the corresponding infinitesimal operators of the representations we use the same notations, and we put

$$I'_{\mu\nu} = I_{\mu\nu} - s_{\mu\nu} \ .$$

The operators $I'_{\mu\nu}$ form a Lie algebra of some group $\mathcal{L}'$ isomorphic to $\mathcal{L}$ and commuting with $\mathcal{S}$. $\mathcal{L}'$ together with the translation group forms some group $\mathcal{P}'$ isomorphic to $\mathcal{P}$ and G is the direct product of $\mathcal{P}'$ and $\mathcal{S}$

$$G = \mathcal{P}' \times \mathcal{S} \ .$$

Let $\mathcal{S}_0$ is the maximal compact subgroup of $\mathcal{S}$ which contains the $SU(2)$ group with generators s_{ij} , $i, j = 1, 2, 3$. Let an irreducible unitary representation of $\mathcal{S}$ be characterized by a set of parameters a. These representations split into direct sums of the irreducible representations of the subgroups $\mathcal{S}_0$ each of which is characterized by a set of parameters j the basis vectors with a given j being characterized by a set of parameters ν. This basis will be called a canonical one, and the basis vectors will be denoted by

$|\alpha\, j\, \nu\rangle$. The representations of the group $\mathcal{P}'$ are characterized by two numbers: p^2 and s' We shall consider the case $s' = 0$ and the basis of the representations of $\mathcal{P}'$ will be denoted by $|p\rangle$.

Consider the rest particles and denote by $\hat{p}_\mu$ their momenta: $\hat{p}_i = 0$, $i = 1, 2, 3$. State vectors are of the form

$$|\hat{p}\; \alpha\; j\; \nu\rangle = |\hat{p}\rangle \times |\alpha\; j\; \nu\rangle\ . \tag{3}$$

In a pure Lorentz transformation $\Lambda_{p \leftarrow \hat{p}}$ they transform in the following manner

$$|\hat{p}\; \alpha\; j\; \nu\rangle \xrightarrow{\Lambda_{p \leftarrow \hat{p}}} e^{iI_{\mu\nu}\omega_{\mu\nu}}\, |\hat{p}\; \alpha\; j\; \nu\rangle = e^{iL_{\mu\nu}\omega_{\mu\nu}}\, |\hat{p}\rangle \times e^{is_{\mu\nu}\omega_{\mu\nu}}\, |\alpha\, j\, \nu\rangle\ , \tag{4}$$

where $\omega_{\mu\nu}$ are the parameters of the transformation $\Lambda_{p \leftarrow \hat{p}}$. The first factor in the right-hand side of Eq. (4) is equal to $|p\rangle$ by definition. We denote the second factor by $|\alpha\, j_p\, \nu_p\rangle$. Instead of (4) we have

$$|\hat{p}\; \alpha\; j\; \nu\rangle \xrightarrow{\Lambda_{p \leftarrow \hat{p}}} |p\, \alpha\, j_p\, \nu_p\rangle \tag{5}$$

The vector $|p\, \alpha\, j_p\, \nu_p\rangle$ is the state vector of a particle with momentum p. We put

$$S_\xi\,(p) = e^{is_{\mu\nu}\omega_{\mu\nu}}\; S_\xi\, e^{-is_{\mu\nu}\omega_{\mu\nu}}\ . \tag{6}$$

These operators satisfy the same commutation relations as the generators S_ξ. In the basis $|\alpha\, j_p\, \nu_p\rangle$ they have exactly the same matrix elements as S_ξ have in the basis $|\alpha\, j\, \nu\rangle$. In other words $|\alpha\, j_p\, \nu_p\rangle$ is the canonical basis of the group with generators $S_\xi(p)$ depending on p.

In the scattering processes particles have different momenta, and their state are classified according to different canonical basis. Therefore in constructing S -matrix we have to choose a common basis for all particles. We do this in the following manner. From the state vectors $|p\, \alpha\, j\, \nu\rangle$ we construct formerly their linear combination $|p\, \alpha\, j\, \nu\rangle$ in such a manner that in the basis $|p\, \alpha\, j\, \nu\rangle$ the operator S_ξ has the same matrix elements as in the basis $|\hat{p}\, \alpha\, j\, \nu\rangle$ for the rest particles. We have

$$| p\, \alpha\, j_p\, \nu_p > \; = D^{\alpha}_{j_p \nu_p\, j'\nu'} (\lambda_{p \leftarrow \hat{p}})\, | p\, \alpha\, j'\nu' > \, , \tag{7}$$

and the matrix elements of the S -matrix between the states $| p\, \alpha\, j_p\, \nu_p >$ are expressed linearly throughout the matrix elements between $| p\, \alpha\, j\, \nu >$, the general form of which can be found from the invariance considerations. For simplicity we denote the pair $j\nu$ by a and omit the index p . From the argument of the functions D one can see what index is to be added to a . The product of two representations can contain some representation many times. In order to distinct these equivalent representations we introduce the index ξ . We denote by $C^{a\, a\, \xi}_{a_1 a_1 a_2 a_2}$ the Clebsh-Gordan coefficients of the group $\mathcal{S}$. Then the amplitude of a binary scattering process is

$$T\, (q_2\, \alpha_2\, a_2;\; p_2 \beta_2\, b_2 \,|\, q_1 \alpha_1\, a_1;\; p_1\, \beta_1\, b_1) = D^{\alpha_2}_{a_2 a''} (\lambda_{q_2 \leftarrow \hat{q}_2})\, D^{\beta_2}_{b_2 b''} (\lambda_{p_2 \leftarrow \hat{p}_2})$$

$$D^{\alpha_1}_{a_1 a'} (\lambda_{q_1 \leftarrow \hat{q}_1})\, D^{\beta_1}_{b_1 b'} (\lambda_{p_1 \leftarrow \hat{p}_1}) \sum_{\gamma_c \xi_1 \xi_2} \overline{C^{\gamma_c \xi_2}_{\alpha_2 a'' \beta_2 b''}}\; C^{\gamma_c \xi_1}_{\alpha_1 a' \beta_1 b'}\; F^{\gamma}_{\xi_2 \xi_1}(s, t)\, . \tag{8}$$

Putting this expression into the right-hand side of relation

$$\frac{1}{i}\, (T - T^{+}) = T^{+}\, T$$

we see that in the two-particle approximation the antihermitic part have the same structure as the suggested structure of the matrix element (8), and the unitarity condition leads only to the integral equations

$$\mathrm{Im}\, F^{\gamma}_{\xi_2 \xi_1}(s,t) = \frac{1}{8\pi^2} \int \frac{d^3 p'}{2p'_o} \frac{d^3 q'}{2q'_o}\, \delta^4 (p' + q' - p_1 - q_1) \sum_{\xi_3} \overline{F^{\gamma}_{\xi_3 \xi_2}(s, t_2)}\, F^{\gamma}_{\xi_3 \xi_1}(s, t_1)\, . \tag{9}$$

If we construct the amplitudes of other inelastic processes with the particle creation and we put these amplitudes into the right-hand side of the unitarity condition, then we obtain again the structure of the form (8). Thus in the symmetry theory under consideration there exists no contradiction with the unitarity condition.

Let the group $\mathcal{S}$ be U(6,6). In this case there exists a subgroup called $U(6)_W$ whose generators S^{W}_{ξ} satisfy the condition

$$S^{W}_{\xi}(p) = S^{W}_{\xi}\, , \quad \vec{p} \,//\, \vec{z}\, ,$$

i.e. do not change in the transition from the rest system to the system under consideration. This subgroup is common for all particles involved in the given collinear process and the study of the invariance under this subgroup can be carried out using the technics of the $U(6)$ symmetry.

The collinear subgroup $U(6)_w$ under consideration coincides with the $U(6)_w$ group in /1,2/. However, the consequences of the symmetry $U(6)_w$ obtained by the method developed here do not coincide with the predictions obtained by the method of Lipkin and Meshkov /1/.

Indeed, in the theory of Lipkin-Meshkov /1/ the representations of the groups $U(6)_w$ and $U(6)_p$ coincide but in the theory developed in the present paper the vectors in each irreducible representation of the group $U(6)_w$ are, generally speaking, the linear combinations of the vectors from different multiplets of the group $U(6)_p$ with generator obtained from the generators of the group $U(6)$ by means of the transformation (6). This difference leads to the differences of the physical consequences of two theories.

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