Nucl. Phys., 1967, v.B1, N4, p.199-206

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E - 2738

ЛАБОРАТОРИЯ ЯДЕРНЫХ ПРОБЛЕМ

F. Lehar, V.Petrzilka and M.Suk

THE MOMENTUM SPECTRA OF SECONDARY PARTICLES IN (π^-, N) INTERACTIONS AT 7 GeV

1966

E - 2738

F. Lehar, V. Petrzilka* and M. Suk**

THE MOMENTUM SPECTRA OF SECONDARY PARTICLES IN (π^-, N) INTERACTIONS AT 7 GeV

* *The Faculty of Technical and Nuclear Physics, Technical University, Prague.*

** *The Institute of Theoretical Physics of the Copenhagen University.*

I. Introduction

In the last time there were published several papers in which the authors investigated the differential and integral momentum spectra in elastic[1] or inelastic[2-7] nucleon-nucleon and meson-nucleon interactions. For fitting the theoretical or semi-empirical formulae to the histograms of experimental data, they applied several different distributions. The experimental results used for this purpose were taken from investigations either on cosmic ray interactions or on elastic or inelastic collisions produced by artificially accelerated particles. In most cases it was not possible to measure the momenta of secondary particles produced in these interactions by one of the known methods and it was therefore necessary to determine them in an indirect way.

The aim of this paper is to compare the mentioned theoretical and semi-empirical formulas with distributions of measured momenta of secondary π-mesons and recoil protons from the (π^-,N) interactions at 7 GeV. These events were found by the conventional scanning in several stacks of nuclear emulsions NIKFI-R which were irradiated in the external π^--beam of the synchrophasotron of the Joint Institute for Nuclear Research at Dubna. The energy of primary pions was 6.8 GeV, 7.3 GeV and 7.5 GeV. The identification of particles and the measurements of their momenta were done by measurements of ionization and of multiple scattering on their tracks in the collaborating laboratories in Berlin[8], Budapest[9], Dubna[10] and Prague. General as well as some specific results were presented at the Conference on High Energy Physics at Rochester[4] and in Geneva[12]. According to known criteria[13,14] 951 pion-nucleon interactions were selected from which the recoil protons were identified in 208 cases. The measurements of the momenta obtained on these events are used in the present paper.

II. The Momentum Spectra in the Laboratory

The investigations of cosmic ray jets have shown that Heisenberg's formula $N(p)dp \approx p^{-(1+\alpha)}\, dp$[15] can be considered as a good representation only in the region of the high-momentum tail of the spectra, but that even then the constant α is larger than unity[16], or $\alpha > 2$[17], or $\alpha = 1.67$[18]. Besides this, the shape of the experimental distribution of the momentum spectra does not fit Heisenberg's formula in the region of low-momentum values. Therefore, M.F.Kaplon et al.[5] looked for a special formula and found that the differential momentum spectrum in the laboratory system can be well represented by

$$N(p)dp = \frac{N_o}{p^{1+\alpha}} \left[1 - \frac{1}{(10p)^{c_2}} \right] \exp\left(- \frac{c_1}{p^{c_2}} \right) dp, \qquad (1)$$

where α, c_1 and c_2 are constants, N_o is a normalization factor.

We used formula (1) to approximate our experimental differential and integral momentum spectra of secondary π^- mesons and recoil protons from the (π^-, N) interactions at 7 GeV, by the least squares method[19]. Figs. 1 and 2 with the momentum spectra of secondary π-mesons show a better fit than Figs. 3 and 4 with the momentum spectra of recoil protons because of the larger total number of secondary π-mesons (2480), than of recoil protons (379). The corresponding values of N_0, a, c_1 and c_2 and the values of $\chi^2/\overline{\chi^2}$ are given in Table I.

Table I

Value of	*Secondary π-mesons*	*Recoil protons*
N_0	$3.844 \cdot 10^8$	$7.344 \cdot 10^7$
$a \pm \Delta a$	3.94 ± 0.13	4.72 ± 0.52
$c_1 \pm \Delta c_1$	13.59 ± 0.06	13.67 ± 0.18
$c_2 \pm \Delta c_2$	0.250 ± 0.008	0.316 ± 0.022
$\chi^2/\overline{\chi^2}$	0.53	1.92

The fact thet the values given in Table I represent the best fit of the curve calculated from formula (1) to the experimental histograms of Figs. 1 and 2 was further proved in the following way: the χ^2 was calculated for $a = 1,2,3,4$ and 5 and its corresponding values are 697.9, 366.9, 83.8, 34.3, 204.0, respectively. They show that χ^2 has its minimum for $a \approx 4$, which characterises the best mathematical fit. It is clear that $a > 1$.

The values of the constants a, c_1 and c_2 are also larger than those given in ref.[6], but this is probably caused by the lower energy of the primary π^--meson (only $\approx$ 7 GeV), by the fact that the momenta were directly measured and by the separation of secondary π-mesons from recoil protons.

We expected the integral momentum spectra and therefore the constant a, c_1 and c_2 to depend on the number of charged particles in the investigated stars. For this reason we can calculate the distribution of differential momentum spectra of secondary π-mesons for four groups of stars: for (I + II + III) -prong stars and separately for the remaining (IV + V + VI + VII + VIII + IX) - prong stars and also for (I + II + III + IV) - prong stars and separately for the remaining (V + VI + VII + VIII + IX) - prong stars. The results are given in Table II.

Table II

Group of stars	a	c_1	c_2
I + II + III *prong stars*	2.14 ± 0.40	5.40 ± 0.25	0.374 ± 0.029
IV + V + VI + VII + VIII + IX *prong stars*	6.06 ± 0.23	15.13 ± 0.09	0.356 ± 0.011
I + II + III + IV - *prong stars*	3.92 ± 0.14	13.23 ± 0.07	0.265 ± 0.008
V + VI + VII + VIII + IX - *prong stars*	6.52 ± 0.42	15.96 ± 0.14	0.357 ± 0.019

With the constans a, c_1 and c_2 for (I + II + III + IV) - prong stars and (V + VI + VII + VIII + IX) prong stars using formula (1) the curve for differential momentum spectra was calculated and is shown in Fig. 5, together with the experimental histograms. The corresponding integral momentum spectra are shown in Fig. 6.

The constants in table II and the curves in Figs.5 and 6 prove clearly the dependence on the number of prongs in the prove clearly the dependence on the number of prongs in the stars. Unfortunately our number of investigated events was not large enough to allow the construction of the momentum spectra for separate groups of stars with equal number of prongs. The applied emulsion method did not make it possible to estimate the π^0-meson momenta.

III. The Distribution of the Transverse Momenta

The distribution of the transverse momenta for the secondary charged particles from the meson-nucleon and nucleon-nucleon interactions has been compared either with the Maxwell-Boltzmann distribution (MBD)/3,4,6/ or with the linear-exponential distribution (LED) or with Planck's law distribution (PD)/7/. In the present paper the experimental results from (π^-, N) - interactions at 7 GeV were used/8-12/, to investigate this distribution again. The values

$$p_t = p \sin \Theta \tag{2}$$

were calculated using the measured values of the momenta p and the corresponding emission angle Θ of the secondary π - meson or the recoil proton. The distribution of the p_t- values of secondary π -meson (2480) is given in Fig. 7 All the three mentioned theoretical distributions are fitted to the experimental results with the value $\chi^2/\overline{\chi^2} = 1,40$ for the MBD/3-6/.

$$N(p_t)\, dp_t = N_o \frac{2 p_t}{A^2} \exp\left(-\frac{p_t^2}{p_o^2}\right) dp_t \tag{3}$$

with the value $\chi^2/\overline{\chi^2} = 1,26$ for the (LED)/7/

$$N(p_t)\, dp_t = N_o \frac{p_t}{p_o^2} \exp\left(-\frac{p_t}{p_o^2}\right) dp_t \tag{4}$$

and with the value $\chi^2/\overline{\chi^2} = 1,40$ for the (PD)/5,7/.

$$N(p_t)\, dp_t = N_o \frac{1}{F(a)} \left(\frac{\sqrt{u^2c^2 + p_t^2c^2}}{kT}\right) \left(\frac{p_t}{kT}\right)_{n=1} \left(K_1\left(\frac{n\sqrt{u^2c^2+p_t^2c^2}}{kT}\right) \frac{dp_t}{kT}\right), \tag{5}$$

In the case of (π^-, N) - interactions at 7 GeV, it is not possible from the mentioned values of χ^2/χ^2 to make a conclusion, which of these distributions gives the better fit.

There are only qualitative reasons/3,6,7/, which can decide what distribution should be preferred for approximation. From the (LED) fitted by means of (3) to the experimental histogramm of p_t, for secondary π-meson we obtained the value $\langle p_t \rangle = 2\, p_o = 0.305 \pm 0.011$ GeV/c agreeing within error with the experimental average value of p_t, found to be

$$\langle p_t \rangle = (0.299 \pm 0.010)\ \text{GeV/c}. \tag{7}$$

From $A = 0.304$ in the (MBD), which is related to $\langle p_t \rangle$ by $\langle p_t \rangle = \sqrt{\pi}\, A/2$ according to/7/, we obtained from the fit to the experimental histogramm of p_t the value $\langle p_t \rangle = 0.269$ GeV/c, which is 10% smaller than the experimental value $\langle p_t \rangle$.

If we compare equation (3) with equation (1) from refs./3/,/4/, we obtain the relation $A^2 = \langle p_t^2 \rangle$. The value $\langle p_t^2 \rangle = 0.133$ obtained from the experimental results on secondary π -mesons and the value

$A^2 = (0.304)^2 = 0.0924$ differ by 30%. Nevertheless the relation $(\langle p_t^2 \rangle)^{1/2} = (4/\pi)^{1/2} \langle p_t \rangle$ [3] is fulfilled with the error only of 7.4%.

Using the relations for **kT** from [7] we obtained from the **(LED)** fit for **kT** the value **kT = 0.153** GeV.

The distribution of the p_t - values for recoil protons from the same (π^-, N) - interactions has recently been published in refs. [3,4] and is therefore omitted here.

The angular distribution of the p_t - values for secondary π - mesons divided by the average value [7] of p_t is shown in Fig. 8. It illustrates very clearly that it is possible for many purposes to consider the value of p_t as constant in the region of angles from 20^0 to 160^0. However in the neighbourhood of 0^0 and 180^0 it may be very dangerous to apply this assumption. This is the case of cosmic ray jets produced by primary particles of very high energy, in which the secondary π -mesons are collimated within a very narrow cone in the laboratory system.

Consider this point of view we shall make an attempt to use the value $\langle p_t \rangle$ to evaluate the momentum spectra of the secondary π - mesons.

4. Evaluation of Momentum Spectra Using the $\langle p_t \rangle$ - Value

Using the mean value of $\langle p_t \rangle \pi$ for secondary π-mesons, the values of **p** for all secondary particles (secondary π -mesons and recoil protons) emitted at the angle Θ were calculated from equation (2). Taking into account that, according to the results shown in Fig. 8, the values of p_t for secondary particles differ significantly from the value of $\bar{p}_t$ in the angle region of **0** and **180** ,we made the cut off for **p** at 7 GeV/c. From the 2840 calculated momenta **p**, 2751 values are in the interval from 0 to 7 GeV/c, whereas only 89 cases, i.e. 3.1%, have the values of **p** higher than 7 GeV/c.

The curves calculated from formula (1) fitted to these experimental spectra gave the following values for the constans $a = 4.27 \pm 0.12$, $c_1 = 13.88 \pm 0.06$, $c_2 = 0.278 \pm 0.007$ and the value $\chi^2/\bar{\chi}^2 = 1.97$. The fit can be considered to be good*. The shape of these curves is practically the same as in Figs. 1, 2 and 3.

This first procedure prooved applicable for the interactions, for which it was possible to measure the momenta **p** , of secondary particles can be used to in such cases - for example in cosmic ray physics - where it is difficult to determine **p** directly.

5. Discussion and Conclusions

Remembering that in the (π^-, N) - interactions at 7GeV it was possible to measure on the tracks of secondary particles in the emulsions the momenta as well as the transverse momenta, we can make the following conclusions from the obtained experimental measurements and from their comparison with the fitted curves of different distributions:

1) We can confirm the results of ref. [6] that the shape of the secondary particle momenta, in high energy interactions can be well represented by semi-empirical formula (1).

2) The constant a has larger values for stars with larger number of prongs. In all cases the value a is larger than 1.

3) The **(MBD)**, **(LED)** and **(PD)** distributions can be fitted with practically the same $\chi^2/\bar{\chi}^2$ to the experimental distribution of $p_t \cdot \tau$ There are different qualitative reasons for using one of these distributions. The

---

* The application of formula (1) should be in this case more correct if we use the value 8.3 p instead of 10 p in formula (1).

best agreement with the average value of p_t was found with the value $<p_t>$ obtained from the fit of (LED).

4) Taking into account the angular distribution of p_t (Fig. 8), it is possible to use the relation $p = <p_t> \sin\Theta$ for the construction of momentum spectra in cases when it is not possible to measure the values of p directly.

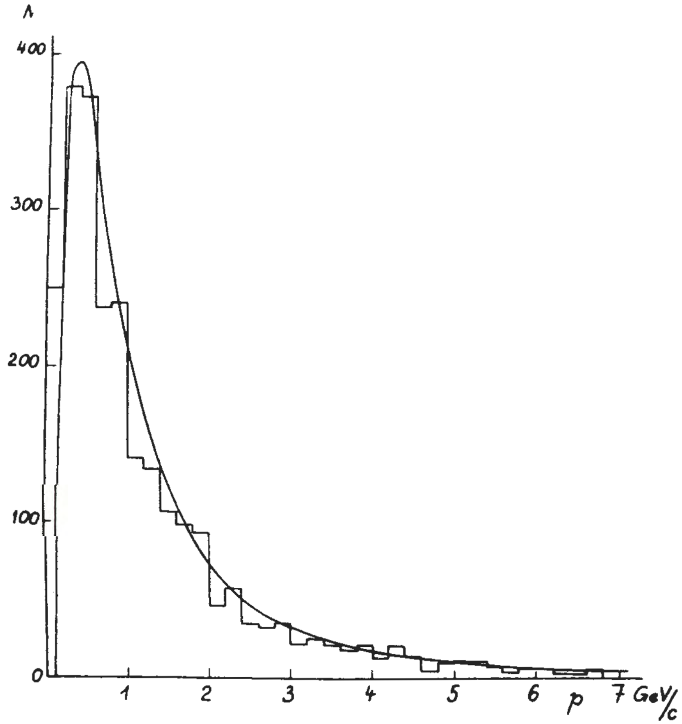
* * *

The authors express their sincere thanks to E.Fenyves, K.Lanius and K.D.Tolstov for permission to use also their experimental data, to J.Marek for help in the work and to J.Vanicek for the fit of distributions to the p_t - histogramms.

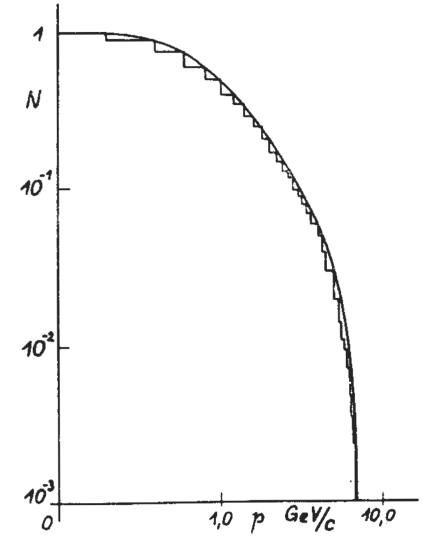
References

1. G.Cocconi. Nuovo Cimento, 33, 643, 1964.
2. V.Petrzilka and M.Suk. Preprints JINR, Dubna, P-964, p. 28, 1962, P-2269, p.3, 1965.
3. К.Д. Толстов. Препринт ОИЯИ Р-1469, Дубна 1963; Ядерная физика 1, 832 (1965).
4. V.V.Glagolev and K.D.Tolstov. Nuclear Physics, 68, 657 (1965).
5. K.Imaeda. Nuovo Cimento, 28 908 (1963).
6. H.H.Aly, M.F.Kaplon, and M.L.Shen. Nuovo Cimento, 32, 905 (1964).
7. K.Imaeda and J.Avidan. Nuovo Cimento, 32, 1497 (1964).
8. C.Grote et al. Nucl.Phys., 34, 648, 659, 676 (1962).
9. E.Bozoki et al. Communication at the Session of the Emulsion Committee at Dubna 1961.
10. В.А. Беляков и др. ЖЭТФ 39, 937, 1960; Препринт ОИЯИ Р-530, Дубна 1960.
11. V. Petrzilka. Proc. of the Ann.Conf. on High Energy Physics, Rochester 1969, p. 82.
12. K.Lanius. Proc. of the Ann.Conf. on High Energy Physics, Geneva 1962, p. 617
13. W.D.Walker and J. Crussard. Phys.Rev., 98, 1416 (1955).
14. И.Н. Граменицкий. Материалы сессии эмульсионного комитета. Препринт ОИЯИ Р-553, Дубна 1960.
15. W.Heisenberg. Koschmische Strahlung, Berlin 1953, p. 153.
16. М.А. Добротин. Космические лучи. Москва 1954, стр. 184.
17. J.Pernegr. V. Simak and M.Votruba. Nuovo Cimento, 10, 1 (1960).
18. Yash Pal and B.Peters, Matematisk-Fysiske Meddelelser, Det Kongelige Danske Videnskabernes Selskab, Bind 33, nr. 15., 1964.
19. С.Н. Соколов, И.Н. Силин. Препринт ОИЯИ Д-810, Дубна 1961.

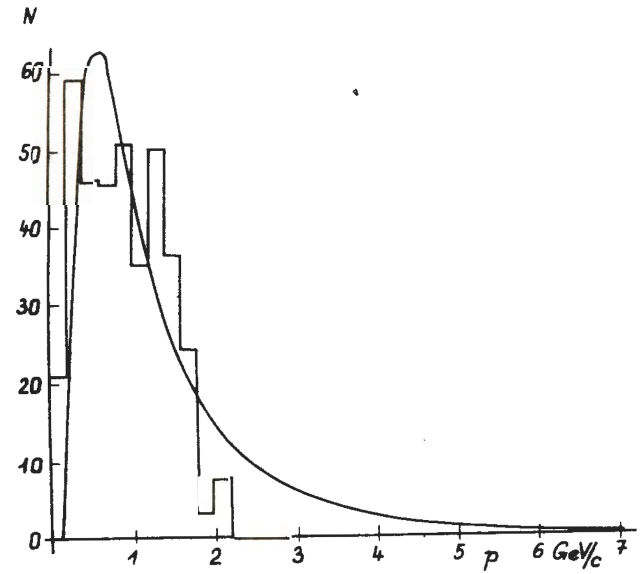
Received by Publishing Department
on May 11, 1966.



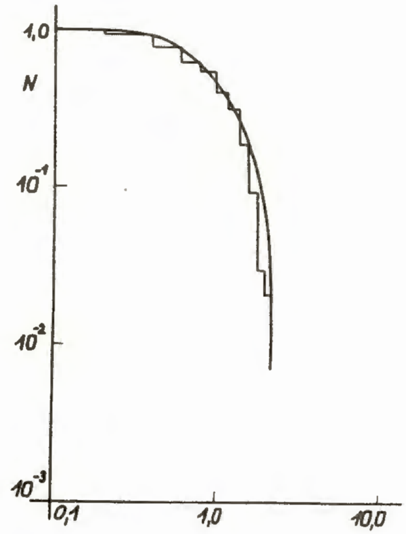
F i g. 1. *Differential momentum spectrum of secondary π-mesons from (π⁻, N)-interactions at 7 GeV.*



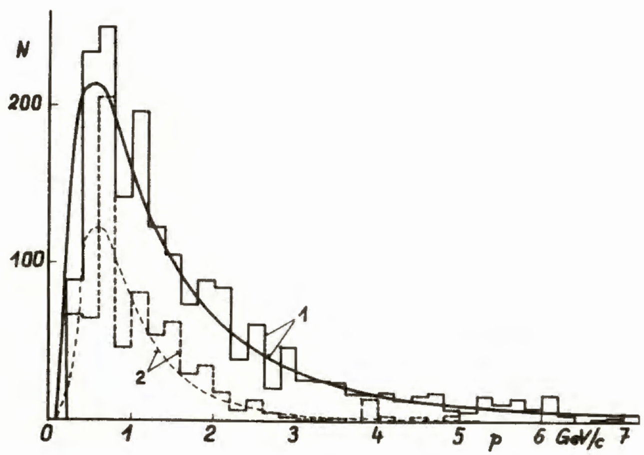
F i g. 2. *Integral momentum spectrum of secondary π-mesons from (π⁻, N)-interactions at 7 GeV.*



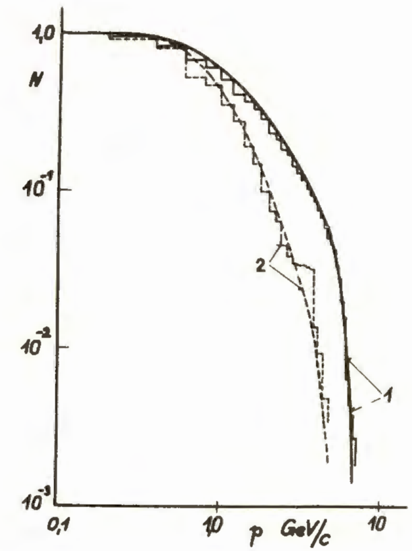
F i g. 3. *Differential momentum spectrum of recoil protons from (π⁻, N) - interactions at 7 GeV.*



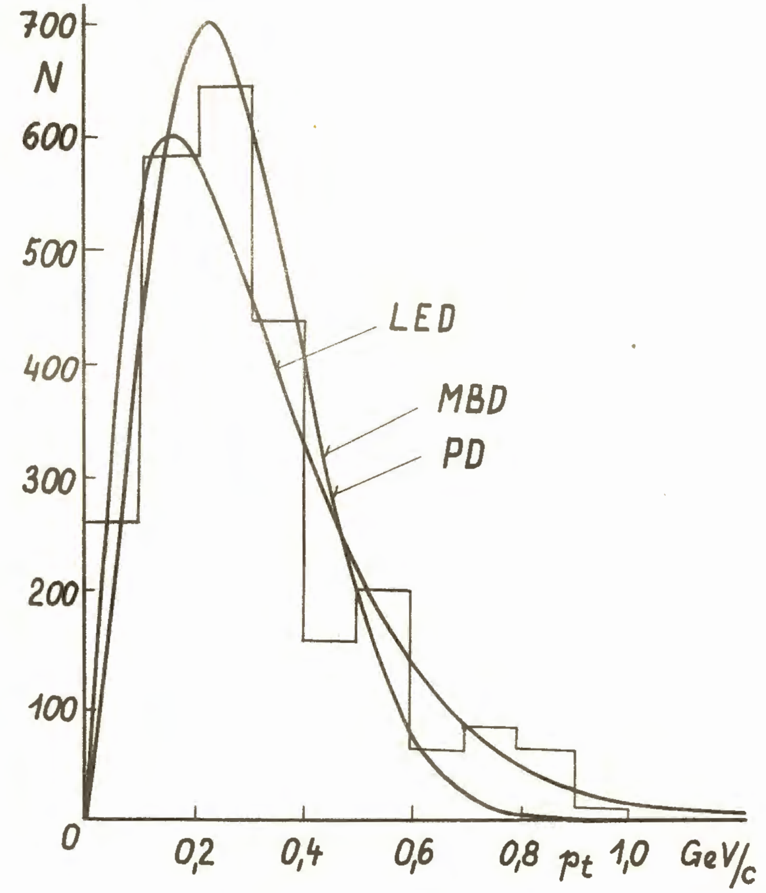
F i g. 4. Integral momentum spectrum of recoil protons from (π^- , N) - interactions at 7 GeV.



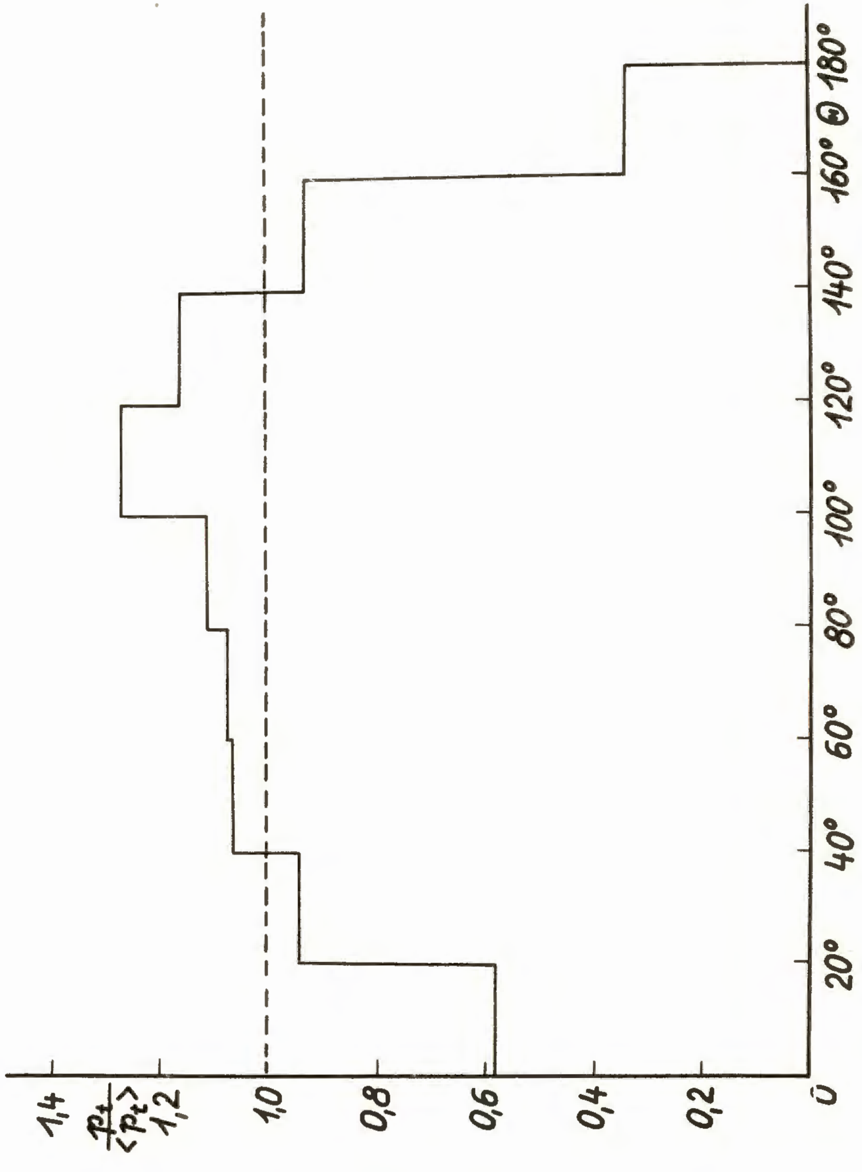
i g. 5. Differential momentum spectrum of secondary π -mesons from (1+2+3+4) - prong stars (curve 1) and from (5+6+7+8+9) - prong stars (curve 2) induced by (π^-, N) interactions at 7 GeV.



F i g. 6. Integral momentum spectrum of secondary π - mesons from (1+2+3+4) - prong stars (curve 1) and from (5+6+7+8+9) -prong stars (curve 2) induced by (π^-,N) interactions at 7 GeV.



F i g. 7. p_t-distribution of secondary π - mesons from (π^-, N) - interactions at 7 GeV.



F i g. 8. Angular distribution of p_t - values for secondary π - mesons from (π^-, N) - interactions at 7 GeV.