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SYMMETRY BREAKING QUASIPARTICLE METHOD FOR BARIONS AND FOR OUARKS

1. Bound State Problem

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Summary

The quasiparticle method - an essentially nonperturbational approximation of quantum field theory - is applied to an SU_3 -symmetric four fermion interaction between octet-barions N and antibarions $\bar{N}$ (or between quarks and antiquarks). The method is developed in such a way, that the physical particles and resonances themselves appear as quasiparticles in a bound state approximation. The main weight is attached to the understanding of the principles of the method.

The individual quasiparticle operators obtained are of the form $\Omega = uN + v\bar{N}^*$. Their eigenvalue spectrum is compared with the observed barion resonances. A spontaneous dynamical breaking of SU_3 – symmetry appears and leads primarily to a GMO-formula for the gap constants, independently of the coupling strength. (Similar results come out for quarks). The GMO-formula for the barion masses results in the strong coupling limit. (Besides mass formulas for quarks are derived). Further it is shown in detail, that the origin of the symmetry breaking is the fact, that the quasiparticle method is an averaging approximation.

Quasiparticles of higher order are considered in an orientating manner. Especially the meson spectra are treated in the lowest approximation.

It is shown, that the first two of the three typical difficulties of the quasi-particle approximation (1. inequivalent representations and degenerate vacua, 2. parity mixed vacuum, 3. spurious states ≡ Goldstone mesons) are avoided by our bound state approximation. The method is applicable to all compact symmetry groups.

I. Introduction

The large number of particles we are confronted with in particle physics sets the vital question for quantum field theory how to derive the necessary large number of fields from a small number of fundamental fields. Moreover we need a field theory which is consequently nonperturbational. The main aim of this paper is to show that there exists a method which meets both demands - the quasi-particle method. It provides us with a complete dynamical approximation scheme, treating the dense cloud of virtual particles in the interior of the physical particles (core region) as a many particle system.

There exist different directions of application of this method in high-energy physics. Nambu[x/] was the first to use the quasiparticle method, mainly because of its nonperturbational character. Many authors used its capability to produce spontaneous symmetry breaking in a dynamical way, often in connection with the Goldstone model[xx/]. Finally the fact, that the transition from particles to quasi-particles has the purpose to transform away the strongest part of the interaction by respecting it in the structure of the (quasi) particles themselves, was the starting point of Weinberg[xxx/]. (The resulting weakening of the interaction between the quasiparticles is likely to manifest itself in the success of a simple first order breaking of SU_3 and of the one-particle-exchange model in high-energy scattering experiments[xxxx/]).

In the present paper we set a high value especially on (1) the consequent nonperturbational character of the method, (2) its ability to construct families of particles (resonances) or fields out of a small number of fundamental fields, and (3) the spontaneous dynamical breaking of SU_3 symmetry. Finally, (4) a specific point of our formulation starts from the following consideration: Quasi-particles are always an approximate description of metastable systems of states. Now in particle physics the scattering resonances are metastable systems, while in the energy region between the resonances there exist no metastable systems. So the application of the quasi-particle method should be limited to the resonances, and the resonances themselves should appear as quasiparticles. Consequently, the field operator of the resonance will be constructed out of the fundamental field(s). So we are led quite naturally to a nonperturbational and constructive field theory of compound systems.

Probably the most important feature of such a quasi-particle scheme is the change in the qualitative picture. For strong interactions the extension of the particles and the range of the forces are comparable. Therefore two scattering particles - energetically within a resonance - appear now as penetrating clouds losing their individuality by uniting themselves to a common "medium". The virtual or bare particles are moving within that medium and transform therefore into quasi-particles. Formally the expression for this change of the picture is that there is no Bethe-Salpeter equation or something similar. Instead the structure of the quasi-particles is given by certain structure functions u, v or μ, ν as

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x Nambu, Jona Lasinio, Phys.Rev. 122, 345 (1961).

xx/ Goldstone, N.C. 19, 154 (1961)

xxx/ Weinberg, Phys.Rev. 130, 776 (1963).

xxxx/ Moravcik, Ann. of Phys. 30, 10 (1964).

we shall see. We get in this way a very simple dynamical model incorporating the breaking of internal symmetry from the beginning. Its symmetry properties are those of a jj-coupling variant of "kinematical supermultiplets".

As an example of detailed presentation of the method we choose a 4-fermion interaction between octet barions and antibarions (where easily possible, besides we consider the analogous case of triplet barions, ie. quarks and antiquarks). In chapter II after presenting in outline the standard procedure of the quasiparticle method, the specific properties are studied, which are necessary for SU_8 symmetry and for the description of closed systems. Chapter III gives the main application, the determination of the individual quasi-particles as an approximation for the physical barions (quarks). In detail we consider the concrete application to the baryon spectra. After that the spontaneous dynamical breaking of SU_8 - symmetry is studied and shown, that the method leads to this breaking essentially because it is an averaging method. Different mass formula are derived for the barions (and quarks). Expecially the Gell-Mann-Okubo formula comes out in the strong coupling limit. Chapter IV finally contains a preliminary orientation about higher order quasi-particles, especially about the meson spectra.

II. Foundations

The standard procedure of the quasiparticle method can be characterized by the following steps[x/]:

1. Choice of a fundamental field (or some of them).

2. Choice of an orthogonal set of states of independent particles of that field. Definition of creation and destruction operators of the particles in these states.

3. Choice of the Hamiltonian H and derivation of the equations of motion.

4. Division of H into the reduced and rest parts: $H = H_{red} + H_{Rest}$.

5. Quasiparticle approximation for H_{red}, leading to individual quasiparticles in the "stable" approximation.

6. Quasiparticle approximation for $H_{red} + H_{Rest}$, leading to quasiparticles of higher order ("excitons", collective states") in the stable approximation.

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x/ e.g. Bogolubov, Shirkov, Tolmatchov, New Method in the Theory of Superconductivity, Moskwa 1958 and Fortsch.Phys. 6, 605 (1958), for the continuous case; Lane, Nuclear theory, New York, 1964, for the discrete case.

7. Decay of the quasiparticles.

We choose one fundamental field of bare mass m, an octet barion-antibarion field $N - \bar{N}$ with usual anticommutator relations:

$$\{N_\nu \, , N^*_{\nu'}\} = \delta_{\nu\nu'} \qquad \{\bar{N}_\nu \, , \bar{N}^*_{\nu'}\} = \delta_{\nu\nu'} \quad \text{etc} \ldots \tag{1}$$

We use these "bare" barions as building-blocks for the construction of all particles. (By the way, we shall make some preliminary remarks about the most interesting building-blocks, the quarks).

Point 2 of our list requires some more discussion: Should we put our barions into the states of a continuous set (e.g. superconductor: plane waves), or of a discrete set (e.g. nucleus: shell model states)? At first sight it seems to be no doubt, that a continuous set is to be chosen according to the continuous character of the scattering states. But quasiparticles are always a description of metastable states. In scattering of particles we have metastable states, the resonances, which form a discrete set within the continuum of scattering states. Therefore we define our model in such a way, that we approach things "from the bound state side": First we forget the continuum of scattering states between the resonances and idealize the latter to discrete stable (bound) states. Obviously this proceeding is absolutely necessary for a theory, which intends to construct (approximately) the field operator of a resonance. Later the model is to be imbedded again into the continuum, in order to get the decay - the width - of the resonances, point 7.

So we consider not only the nucleon, pion etc., but also each resonance as a stable closed system. It occupies therefore only a small finite volume of order 10^{-13} to 10^{-14} cm. For the start of our method we need - as in each quantum field theory - an orthonormalized set of states of independent particles as building-blocks. But independent particles within a small finite volume have according to the rules of quantum mechanics always a discrete spectrum. So we are compelled to start from a discrete set of states, and we use the energy differences of this set as fundamental parameters of the theory. For the formulation it is not necessary to ask for the detailed origin of the set.

Nontheless some words are to be said about this origin. The underlying picture is analogous to the shell model of nuclear physics: We assume, that the complicated forces between the original free barions and antibarions (or quarks) average out within the small volume of our idealized bound system to a simple average potential $V(r)$ (in the c.m.s.). Our independent particles then move on the discrete levels of this central potential. Speaking more generally: The essential key is that in each small, strongly interacting system there exist necessarily strong "average" forces, which keep the system together. If this is right, one always will be able to find a potential, which describes that part of the interaction in a sufficient way.

In order to be concrete, we shall use the simple level system of Fig. 1, which will be completely sufficient to get all families of resonances. (Cut harmonical oscillator $\alpha = 3/2$, $\beta = 1$, square well $\alpha = 0{,}96$, $\beta = 0{,}66$).

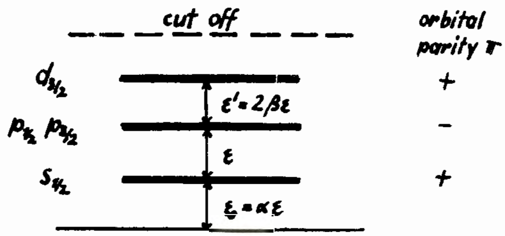

Fig. 1

The operators of (1) are the creation and destruction operators of the particles on their levels. The index ν means therefore the quantum numbers

$$\nu = E_{\bar{\nu}}\, j_\nu\, m_\nu\,,\quad 8_\nu\, i_\nu\, i_{3\nu}\, y_\nu\,,\, \pi_\nu \qquad (2a)$$

($E_{\bar{\nu}}$ = level energy, j_ν = total angular momentum, m_ν = 3 component, 8_ν = "octet" quantum number, i_ν = isospin, $i_{3\nu}$ = 3 component, y_ν = hypercharge, π_ν = orbital parity). Besides we use often the symbols

$$\begin{aligned} -\nu &= E_{-\bar{\nu}}\, j_\nu - m_\nu\,,\ 8_\nu\, i_\nu - i_{3\nu}\, - y_\nu\,,\, -\pi_\nu \\ \bar{\nu} &= E_{\bar{\nu}}\, j_\nu \qquad 8_\nu \qquad \pi_\nu \\ -\bar{\nu} &= E_{\bar{\nu}}\, j_\nu \qquad 8_\nu \qquad -\pi_\nu\,. \end{aligned} \qquad (2b)$$

The necessity to include the orbital parity π_ν will soon come out. For the non-relativistic situation it is clearly $\pi = (-1)^{\ell}$. In our relativistic situation (ϵ = 500 MeV) we define it by $(-1)^{\ell_{up}}$, where ℓ_{up} is the orbital angular momentum of the upper components of the Dirac spinor of the central potential solutions. This definition is rotational invariant.

The use of the discrete terms of a potential means physically, that we have a closed system within a finite volume. So our model has a characteristic length given by the extension of the potential (harm. osc. $R = (\hbar^2 / m\epsilon)^{1/4}$). By this we avoid all peculiarities of the quasiparticle method arising from the inequivalent representations in the case of infinite volume. The cut off of our discrete spectrum (fig. 1) limits our problem to a finite Hilbert space. Its sense is, to limit the discrete starting termsystem, corresponding to the upper end of the resonance region.

Finally we note, that the replacement of barions by quarks means up to now only the replacement of 8- barions by 3-barions; $8_\nu \rightarrow 3_\nu$ in (2).

For the Hamiltonian $H = H_0 + H_{int}$ we take H_0 as the energy of the independent particles on their levels:

$$H_0 = \sum_\nu E_{\bar{\nu}} (N^*_\nu N_\nu + \bar{N}^*_\nu \bar{N}_\nu) . \tag{3}$$

Remark, that the level energies $E_{\bar{\nu}}$ do not depend on the third components, compare (2b), but depend on π_ν ($E_{\bar{\nu}} \neq E_{-\bar{\nu}}$), and are equal for barions and antibarions. For H_{int} we take a barion-antibarion - scattering

$$H_{int} = \sum_{k\lambda\mu\nu} G_{k\lambda\mu\nu} N^*_k \bar{N}^*_\lambda \bar{N}_\mu N_\nu \quad ; \tag{4}$$

later we add similarly barion-barion scattering etc.:

$$H'_{int} = \sum_{k\lambda\mu\nu} F_{k\lambda\mu\nu} N^*_k N^*_\lambda N_\mu N_\nu + \sum_{k\lambda\mu\nu} \bar{F}_{k\lambda\mu\nu} \bar{N}^*_k \bar{N}^*_\lambda \bar{N}_\mu \bar{N}_\nu . \tag{5}$$

The coefficients G (similarly F, $\bar{F}$) have the following properties. From hermiticity

$$G^*_{k\lambda\mu\nu} = G_{\nu\mu\lambda k} . \tag{6}$$

For our closed system we have to require total angular momentum conservation instead of the usual momentum conservation. This and SU_3 - invariance are fulfilled by

$$G_{k\lambda\mu\nu} = \sum_{K\rho} G^{K\rho}_{\bar{k}\bar{\lambda}\bar{\mu}\bar{\nu}} C^{K\rho}_{k\lambda} C^{K\rho}_{\mu\nu} , \quad K = JM8II_3Y , \tag{7a}$$

where the C are Clebsh-Gordan-coefficients:

$$C^{K\rho}_{k\lambda} = C \begin{pmatrix} j_k & J & j_\lambda \\ m_k & -M & -m_\lambda \end{pmatrix} \cdot C \begin{pmatrix} 8_k & 8K\rho & 8_\lambda \\ \phi_k & -\phi_k & -\phi_\lambda \end{pmatrix} \qquad \phi = II_3Y . \tag{7b}$$

$\rho = 1,2$ distinguishes both sets of SU_3 – CG – coefficients for the octet representations 8 and $8'$ out of 8×8. $C^{\rho}(8_k\, 8_K\, 8_\lambda)$ couples both octet particles κ, λ again to an octet, so that we have written down an "octet-interaction". $C(8_k\, 1_K\, 8_\lambda)$ on the other hand would have given a "singlet-interaction". Formulas analogous to (6), (7a) and (7b) have to be written down for the F– and $\bar{F}$–interactions. (Up to now, the formulation is the same for quarks, only $C^{\rho}(8_\mu\, 8_K\, 8_\nu) \to C(3_\mu\, 8_K\, 3^*_\nu)$).

III. Barion Quasiparticles

a) General considerations

According to point 4 we separate off from H_{int} the pairing interaction

$$H_p = \sum_{\nu\mu} G^{K_0}_{\nu-\nu\mu-\mu}\, N^*_\nu \bar{N}^*_{-\nu} \bar{N}_\mu\, N_{-\mu}\ , \quad K_0 = 008000 . \tag{8}$$

Comparing this with (7) one reads off from the CG – coefficients, that H_p transfers no angular momentum from the destructed to the created $N\bar{N}$ – pair, because the upper index K_0 in (8) means, that each pair separately is coupled to angular momentum $J = 0$ $(I = 0)$. On the other hand, H_p transfers SU_3 – spin, because according to (7) or (8) both octet particles (or triplet particles) of each pair are coupled together to "8" by out "octet-interaction". (Only a singlet-interaction makes no transfer). This difference in the transfer-properties of angular momentum and SU_3-spin determines de-decisively the type of symmetry breaking, as we shall see.

The lowest order quasiparticle approximation leading to "individual" quasiparticles (point 5) may be called an <u>approximate diagonalization</u> of $H_0 + H_p$ <u>by partly averaging</u> of H_p in the following way: H_p is replaced by

$$\bar{H}_p = \tfrac{1}{2}\sum_{\mu\nu} G^{K_0}_{\nu-\nu\mu-\mu}\, N^*_\nu \bar{N}^*_{-\nu} (\bar{N}_\mu\, N_{-\mu})_0 + \tfrac{1}{2}\sum_{\mu\nu} G^{K_0}_{\mu-\mu\nu-\nu}\, (N^*_\mu \bar{N}^*_{-\mu})_0\, \bar{N}_\nu\, N_{-\nu} \equiv$$

$$\equiv \sum_\nu \Delta_\nu N^*_\nu \bar{N}^*_{-\nu} + \sum_\nu \Delta^*_\nu \bar{N}_{-\nu} N_\nu\ , \tag{9}$$

where the brackets $(\ )_0$ mean the expectation value with a vacuum state 10) which is to be determined. By this averaging $H_0 + \bar{H}_p$ gets obviously a quadratic form in the operators N_ν, N^*_ν, $\bar{N}_{-\nu}$, $\bar{N}^*_{-\nu}$ which can be transformed to principal axis by a Bogolubov-transformation which leads to new operators

$$\Omega_\nu = u_\nu N_\nu + v_\nu \bar{N}^*_{-\nu} \qquad \bar{\Omega}_\nu = u_\nu \bar{N}_\nu - v_\nu N^*_{-\nu} \tag{10a}$$

$$[H_{red}, \Omega_\nu] = [\bar{H}_{red}, \Omega_\nu] = -\omega_\nu \Omega_\nu \tag{10b}$$

$$H_{red} = H_0 + H_p = \bar{H}_{red} = H_0 + \bar{H}_p = \sum_\nu \omega_\nu (\Omega^*_\nu \Omega_\nu + \bar{\Omega}^*_\nu \bar{\Omega}_\nu). \tag{10c}$$

The new (approximate) destruction operators Ω_ν define a new vacuum state, the vacuum of free quasiparticles, by

$$\begin{aligned} \Omega_\nu |0) &= \bar{\Omega}_\nu |0) = 0 \\ |0) &= \prod_\nu (u_\nu - v_\nu N^*_\nu \bar{N}^*_{-\nu}) |0\rangle \end{aligned} \tag{11}$$

($|0\rangle$ = bare vacuum, $N_\nu |0\rangle = \bar{N}_\nu |0\rangle = 0$).

It contains correlated bare barion-antibarion pairs ν $-\nu$ ($n\bar{n}$, $p\bar{p}$, $\Lambda\bar{\Lambda}$ etc.) analogously e.g. to the corresponding electron pairs $p\sigma$, $-p-\sigma$ in the superconductor. The quasiparticle states themselves are then given by

$$|1_\nu) = \Omega^*_\nu |0), \qquad |\bar{1}_\nu) = \bar{\Omega}^*_\nu |0). \tag{12}$$

Requiring

$$u^2_\nu + v^2_\nu = 1, \quad u_\nu = u_{-\nu} \quad v_\nu = v_{-\nu}, \quad u_\nu, v_\nu \text{ real.} \tag{13}$$

the transformation (10a) gets canonical

$$\{\Omega_\nu, \Omega^*_{\nu'}\} = \delta_{\nu\nu'}, \quad \{\Omega_\nu, \Omega_{\nu'}\} = 0 \quad \text{etc.} \tag{14}$$

and unitary:

$$\begin{aligned} \Omega_\nu &= U^{-1} N_\nu U \qquad |0) = U^{-1} |0\rangle, \\ U &= \exp[\sum_\nu \theta_\nu (N_\nu \bar{N}_{-\nu} - N^*_\nu \bar{N}^*_{-\nu})], \qquad u_\nu = \cos\theta_\nu, \quad v_\nu = \sin\theta_\nu. \end{aligned} \tag{15}$$

It conserves barion number, because it mixes destruction of a barion with creation of an antibarion. So here the mixing of destruction and creation maintains a conservation law, quite opposite to the superconductor and nucleus case, where it destroys the conservation of particle number, because no antiparticles come into play. From this we get an interesting remark concerning the applicability of the quasiparticle method in the relativistic domain. The method is usually applied to problems with conservation of particle number (superconductor, nucleus), though it violates this conservation law. In our case the situation is reversed: There is no such violation, and instead we have a conservation law which is absolutely needed.

Quasiparticle operators of the type nucleon+(antinucleon)* were first considered extensively by Bacry and Mandelbrojt[x], though these authors run into parity difficulties Namely fermion and antifermion always have opposite relative (intrinsic) parity.

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[x] Bacry, Mandelbrojt, N.C. 23, 564 (1962). See also Nambu l.c.

Correlated pairs with opposite momenta, $N^*_{\bar p} N^*_{-\bar p} |0\rangle$, are in an s-state and have therefore total parity -1. So their presence in the vacuum state analogous to (11) leads to a parity mixed vacuum. Our formulation in terms of angular momenta is able to avoid this by placing both particles of a pair into orbits with opposite orbital parity: ν, $-\nu$, compare the definition of $-\nu$ in (2b).

From our requirement that $H_0 + \bar H_p$ takes the form (10c), one gets the eigenvalues ω_ν of the states $|1_\nu\rangle$ and the coefficients

$$\left.\begin{matrix} u_\nu^2 \\ v_\nu^2 \end{matrix}\right\} = \tfrac{1}{2}\left(1 \pm \frac{E_{-\bar\nu} + E_{+\bar\nu}}{2\omega_\nu + \Delta E_{\bar\nu}}\right) \qquad \Delta E_{\bar\nu} = E_{-\bar\nu} - E_{\bar\nu} \tag{16a}$$

$$u_\nu v_\nu = \frac{\Delta_\nu}{2\omega_\nu + \Delta E_{\bar\nu}} \tag{16b}$$

$$\omega_\nu = -\frac{\Delta E_{\bar\nu}}{2} \pm \sqrt{\tfrac{1}{4}(E_{\bar\nu} + E_{-\bar\nu})^2 + \Delta_\nu^2} \equiv -\frac{\Delta E_{\bar\nu}}{2} \pm R_\nu \tag{16c}$$

$$E_0 = \sum_\nu \left[(E_{\bar\nu} + E_{-\bar\nu}) v_\nu^2 - 2\Delta_\nu u_\nu v_\nu\right] = \sum_\nu (E_{\bar\nu} - \omega_\nu) \tag{16d}$$

$$\Delta_\nu = \sum_{\nu'} G_{\nu -\nu \nu' -\nu'} \frac{\Delta_{\nu'}}{\Delta E_{\bar\nu'} + 2\omega_{\nu'}} . \tag{16e}$$

Exactly the same results come out, if one uses the equation of motion method: One requires (10b)

$$[H_{red}, \Omega_\nu] = -\omega_\nu \Omega_\nu .$$

inserts (10a) and uses the equations of motion for N_ν and $\bar N^*_{-\nu}$, in which exactly as in (9) pairs of particles are replaced by their vacuum expectation values. So one gets the linear homogeneous equations for u_ν, v_ν:

$$\begin{aligned} (E_{\bar\nu} - \omega_\nu) u_\nu - \Delta^*_\nu v_\nu &= 0 \\ (E_{-\bar\nu} + \omega_\nu) v_\nu + \Delta_\nu u_\nu &= 0 , \end{aligned} \tag{17}$$

which again lead to (16).

All formulas in (16) are very similar in form to the analogous expressions in the superconductor or nucleus. Indeed, they would be identical with them, if we had not $E_{\bar\nu} \neq E_{-\bar\nu}$ according to our termsystem Fig. 1.

The energy spectrum (16c) gives for each ν a dublet, whose splitting grows with growing $E_{\bar\nu}$: (Fig. 2). So the lowest.

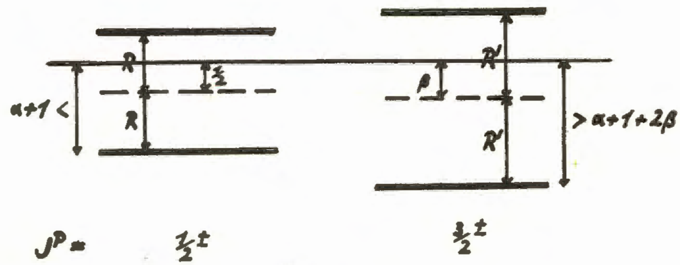

Fig. 2.

state is completely cutoff dependent ($\beta < 0$ quantum mechanically forbidden): Addition e.g. of a fourth higher level in Fig. 1 would produce a new lowest state. This is unphysical, but fortunately can be brought in order by a hole theory in the well known Dirac manner. Namely the second commutation relation (14) means $\Omega_\nu^2 = 0$, so that each of our states ω_ν can be occupied only once. Defining the state in which all levels ω_ν below $-\frac{\Delta E_{\bar\nu}}{2}$ are occupied, above empty, as the new vacuum state

$$\overline{|0)} = \Omega^*_\nu \ldots\ldots \Omega^*_n |0) \qquad \omega_\nu < -\frac{\Delta E_n}{2}, \ldots\ldots, \omega_n < -\frac{\Delta E_n}{2},$$

changes the eigenvalue spectrum into

$$\overset{\pm}{\omega}_\nu = \pm\frac{\Delta E_{\bar\nu}}{2} + \sqrt{\tfrac{1}{4}(E_{\bar\nu} + E_{-\bar\nu})^2 + \Delta_\nu^2} \equiv \pm\frac{\Delta E_{\bar\nu}}{2} + R_\nu . \tag{18}$$

So Ω-holes appear now as $\bar\Omega$ - and $\bar\Omega$-holes as Ω-states. The necessity of a hole theory is clearly characteristic of a theory treating barions and antibarions (or quarks and antiquarks).

It is important to note, that all steps done up to now can be repeated for the continuous case too, i.e. for free spherical waves instead of bound ones. The formulas are not changed by this.

b) Eigenvalues

We begin the concrete application to the barion spectra with some general remarks. The form of $\Omega = uN + v\bar N^*$ shows, that Ω automatically has the same quantum numbers as N, the bare 8-barion. Therefore we get as individual quasiparticles Ω only octet barions, $\bar\Omega$ as octet antibarions[x]. (Anologously with N = triplet barion = bare quark, the Ω are quarks). No other individual quasiparticles are possible.

Secondly, what means the vacuum state $\overline{|0)}$ physically? Having barion number 0, he could be a meson in principle. But we consider our individual

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x/ This statement is to be corrected a little, as we shall see later.

quasiparticles as an approximation for the physical barion and barion-resonance states. Therefore $\overline{|0)}$, the state with no quasiparticle present, is necessarily our approximation for the physical vacuum (within our small volume). Therefore its energy $(\overline{0|}H\overline{|0}) = \bar{E}_0$ is to be taken as the zero point of the energy scale, instead of $\langle 0|H|0\rangle$, as up to now. The energy relations are then that of Fig. 3:

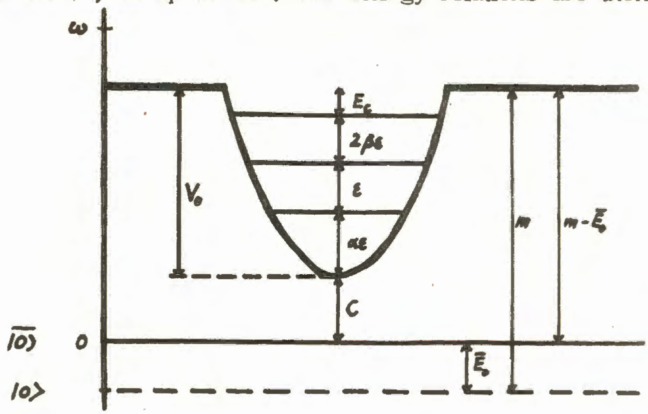

Fig. 3.

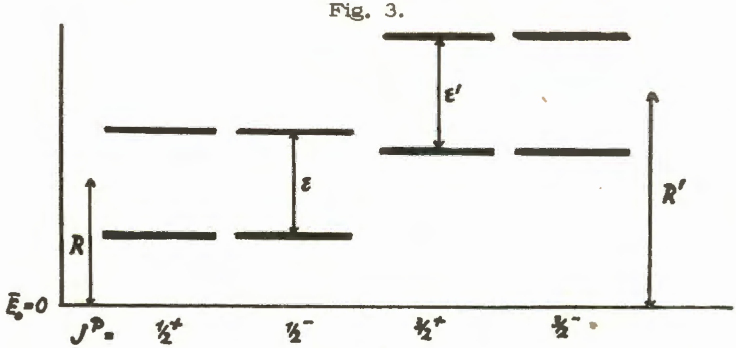

Fig. 4.

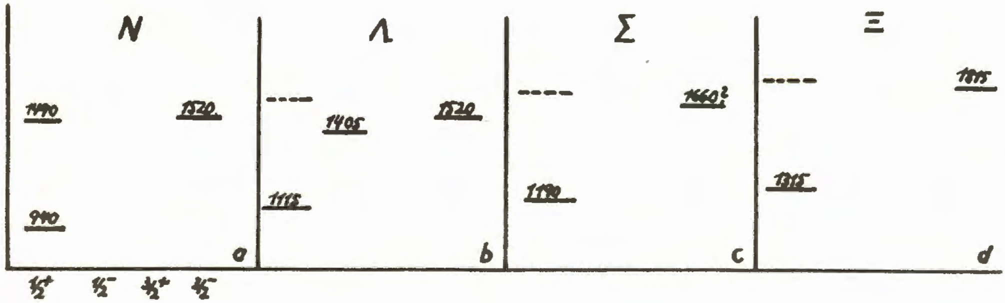

Fig. 5.

Fig. 4 shows the eigenvalues predicted by (18) for the term system of Fig. 1. Let us confirm this first with the observed nucleon resonances of Fig. 5a[x]. The most remarkable fact is the absence of ½− , 3/2+ states. The usual interpretation[xx] is, that states ½−, 3/2+, 5/2 − , ... do not appear as a consequence of a definite exchange character of the interaction. Indeed let us assume for H_{int} the exchange character

$$G_{k\lambda\mu\nu} = \pm G_{-\lambda-k\mu\nu} \qquad \begin{matrix} \frac{1}{2}+ & \frac{3}{2}- \\ \frac{1}{2}- & \frac{3}{2}+ \end{matrix} \qquad \text{for } k, \lambda . \tag{19}$$

Then it is easily seen, that H_p (8) does not contain contributions from $\frac{1}{2}-$, $\frac{3}{2}+$ at all.

Next we read off directly that

$$\epsilon = 1490 - 940 = 550 \text{ MeV} .$$

This corresponds to an extension of our system of

$$R = \left(\frac{h^2}{m\epsilon}\right)^{½} = 2{,}5 \cdot 10^{-14} \text{cm} \qquad (m = m_{bare} \approx m_N) .$$

The system we describe is therefore essentially the dense "core" of the nucleon, while the meson cloud is not included. On the other hand ϵ' cannot be determined, because the higher partner of the $\frac{3}{2}-$ dublet lacks. Either this means, that it exists, but was not observed up to now. This is well possible, for with $\epsilon' \approx \epsilon$ we predict a $\frac{3}{2}-$ state around 1520 + 550 ≈ 2100 MeV, which could be masked by the $\frac{7}{2}+$ -resonance at 2190 MeV. Or the dublet partner lacks, because the splitting vanishes: $\epsilon' = 0$. Therefore we discuss always two cases:

case A: $\epsilon' = \epsilon$ ($\beta = ½$) , case B: $\epsilon' = 0$ ($\beta = 0$) .

Besides ϵ, ϵ' our model contains the energy parameter a, the bare mass m , and the eight interaction parameter[xxxx]

$$G^{K_0\rho}_{\bar\nu\, -\bar\nu \bar\mu -\bar\mu} \qquad \bar\nu, \bar\mu = \tfrac{1}{2}+ , \tfrac{3}{2}- \;;\; \rho = 1,2 \;;\; K_0 = 008000 .$$

It is useful to take instead of the $G's$ the eight parameters

$$\Delta_\nu \qquad \nu = \tfrac{1}{2} + N\Lambda\Sigma\Xi \;,\; \tfrac{3}{2} - N\Lambda\Sigma\Xi .$$

Namely as in other applications of the quasiparticle method the gap constants[xxx] $\Delta_\nu = \delta_\nu \cdot \epsilon$ are the dynamically decisive parameters determining the energy

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[x] Fig. 5 was drawn according to Rosenfeld et al. UCRL 8030, Aug. 65. Only states with $J \leq 3/2$ are given. Notes of interrogation mean non-established quantum numbers.

[xx] E.g. Frautschi, Regge Poles and S -Matrix Theory, New, York, 1963, p.148.

[xxx] We call Δ gap constant, though in our theory it has lost this sense by our transition to hole theory .

[xxxx] This high number of parameters can be reduced to four (coupling constant, 2 potential parameters, m) using the local special case of our (4). This requires computer work (for the G's).

spectrum. Equating our eigenvalues ω_ν (18) with the observed masses $E_{exp} = e \cdot \epsilon$, we get the relations:

$$J = \tfrac{1}{2}+ : \quad \delta^2 = (e_B + \tfrac{1}{2})^2 - (\alpha + \tfrac{1}{2} + c(\alpha\beta\mu_o))^2 \tag{20a}$$

$$J = \tfrac{3}{2}- \; ; \quad \delta'^2 = (e_{B*} + \tfrac{1}{2})^2 - (\alpha + \beta + 1 + c)^2 \tag{20b}$$

$$e_B = 1{,}71(N), \quad 2{,}02(\Lambda); \quad 2{,}16(\Sigma); \quad 2{,}42(\Xi)$$

$$e_{B*} = 2{,}76(N^*_{1520}); \; 2{,}76(\Lambda^*_{1520}); \; 3{,}02(\Sigma^*_{1660}); \; 3{,}30(\Xi^*_{1815})$$

$$c = c_0 + c_1\alpha + c_2\beta \quad c_0 = 321 - 191\mu_o \quad c_1 = -1 \quad c_2 = 382 \; ^{xx/}\,,$$

compare Fig. 5. Remarkably the requirement $\delta^2 \geq 0$ determines the parameter $\mu_o = \mu - e_o$ $(m = \mu \cdot \epsilon, \; E_o = e_o \cdot \epsilon)$ within very narrow limits:

A: $2{,}67 \leq \mu_o \leq 2{,}70$ B: $1{,}67 \leq \mu_o \leq 1{,}70$.

Now $e_o \lesssim 1$ according to Fig. 3. So the bare mass will be around

A: $m \approx 3\epsilon \approx 1500$ MeV B: $m \approx 2\epsilon \approx 1000$ MeV .

Table 1 gives the numerical δ -values for $\mu_o = 2{,}68$ (A), 1,69 (B).

Table 1: $\Delta = \delta \cdot \epsilon$

	CASE A: $\beta = \tfrac{1}{2}$		CASE B: $\beta = 0$	
	$\delta(\tfrac{1}{2}+)$	$\delta'(\tfrac{3}{2}-)$	$\delta(\tfrac{1}{2}+)$	$\delta'(\tfrac{3}{2}-)$
N	2,15	2,89	1,64	3,1
Λ	2,46	2,89	2,02	3,1
Σ	2,61	3,19	2,20	3,4
Ξ	2,88	3,40	2,50	3,6

If necessary, the corresponding G value can be determined with the help of equation (16e), which is an inhomogeneous linear system of equations for the unknown G's . Inspection of table 1 shows that the δ's are always of the order of ϵ . So certainly we are not in a weak coupling situation.

Returning to the comparison of Fig. 4 and 5, we find in the Λ family an $\tfrac{1}{2}-$ state at 1405 MeV. Lacking its lower dublet partner near the Λ -mass, he does *not fit into our* scheme and must have another dynamical origin$^{xx/}$.

x/ $c = c \cdot \epsilon = m - \bar{E}_0(\beta\mu_o) - V_0(\alpha\beta)$, compare Fig. 3. α drops out in δ^2, δ'^2. Therefore in table 1: case A = harm. Oscillator $(\alpha = \tfrac{3}{2})$.

xx/ Usual interpretation: singlet barion, i.e.quasiparticle of 3. order. At the Oxford conference there appeared a new resonance $N^*_{\frac{1}{2}}(\tfrac{1}{2}-)$ at 1700 MeV. If it exists, one has to apply an analogous argumentation to it.

This interpretation is confirmed by the lack of any $\frac{1}{2}-$ in Σ – and Ξ – family. Finally $\Sigma^*(1385)$ and Ξ^* (1530) are members of the decuplet, so that now there is no term against our assumption of exchange character (19). The $\frac{1}{2}+$ dublet splitting of $\epsilon = 550$ MeV clearly has to appear in all members of the dublet (for A and B). So $N^*(1480)$ gets predicted octet partners

$$\tfrac{1}{2}+: \quad \Lambda^*(1665), \qquad \Sigma^*(1740), \qquad \Xi^*(1865) \qquad \text{(predicted)},$$

appearing in Fig. 5 as broken lines. An analogous prediction of a higher $\frac{3}{2}-$ octet results in case A.

c) Dynamical Breaking of SU_8 – Symmetry

We turn now to some questions of principle, giving insight into the problem, how the quasiparticle approximation breaks symmetry, and from this, what this approximation essentially means.

Already the mass formula (18) showed the presence of symmetry breaking, because ω_ν contains

$$\Lambda_\nu = \Lambda_{j_\nu\, m_\nu\, 8_\nu\, i_\nu\, i_{8\nu}\, y_\nu}$$

and so depends on the "third components" m_ν and $\phi_\nu = i_\nu i_{8\nu} y_\nu$. Writing Λ_ν explicitly

$$\Delta_\nu = \tfrac{1}{2}\sum_{\mu\rho} G^{\kappa_0\rho}_{\bar\nu-\bar\nu\bar\mu-\bar\mu}\; C\binom{j_\nu\;0\;j_\nu}{m_\nu 0\, m_\nu}\, C\binom{j_\mu\;0\;j_\mu}{m_\mu 0\, m_\mu} C^\rho\binom{8_\nu\,8\,8_\nu}{\phi_\nu 0 \phi_\nu} C^\rho\binom{8_\mu 8 8_\mu}{\phi_\mu 0 \phi_\mu}\cdot(\bar N_\mu\, N_{-\mu})_0\;, \qquad (21)$$

we find, that the dependency on the third ν – components is contained exclusively in the CG – coefficients. Moreover because of

$$C\binom{j\;0\;j}{m\;0\;m} = 1$$

there is really no dependency on m_ν, so that rotational symmetry or angular momentum conservation is automatically <u>not</u> broken, as it must be for a closed system. On the other hand

$$C^\rho\binom{8_\nu\;8\;8_\nu}{\phi_\nu\;0\;\phi_\nu} \neq 1$$

depends really on $\phi_\nu = i_\nu i_{8\nu} y_\nu$. The resulting SU_3 breaking is therefore completely expressed by writing

$$\Delta_\nu = \Delta_{j_\nu 8_\nu i_\nu i_{8\nu} y_\nu} = \sum_\rho C^\rho\binom{8_\nu\;8\;\;8_\nu}{\phi_\nu\;0\;\;\phi_\nu} C^\rho_{\bar\nu} \qquad (22a)$$

But (22a) is exactly the (Wigner-Eckart) form of a quantity, which obeys [illegible] Gell-Mann- Okubo formula, namely of a tensor component $T^{(8)}_{000}$. So we have *)

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*) Comp. e.g. Kadyshevsky, Muradyan, Dubna preprint P- [illegible]24, formula (6[illegible]

a GMO-formula for the gap constants:

$$\Delta_N + \Delta_\Xi = \tfrac{3}{2}\Delta_\Lambda + \tfrac{1}{2}\Delta_\Sigma \qquad \text{for } \tfrac{1}{2}+,\ \tfrac{3}{2}- \qquad (22b)$$

Analogously for quarks we have $8_\nu \to 3_\nu$:

$$\Delta_\nu = C\begin{pmatrix} 3_\nu & 8 & 3_{\bar\nu}^* \\ \phi_\nu & 0 & \phi_\nu \end{pmatrix} \mathcal{C}_{\bar\nu} \ ,$$

so that

$$\frac{\Delta_1}{C_1} = \frac{\Delta_2}{C_2} = \frac{\Delta_8}{C_8} = \mathcal{C}_{\bar\nu} \ , \quad C_1 = C_2 = \frac{1}{\sqrt{6}} \quad C_8 = -\frac{2}{\sqrt{6}} \qquad (23a)$$

$$\Delta_1 = \Delta_2 = -\tfrac{1}{2}\Delta_8 \ . \qquad (23b)$$

It is important to note, that both relations (22) and (23) are completely independent of the coupling strength. Let us see, whether our "experimental" δ_ν, δ'_ν of table 1 fulfil the relation (22B). One finds:

Table 2

		left-hand side	right-hand side of (22b)	error
Case A:	$\delta(\frac{1}{2}+)$	5,03	4,99	1%
	$\delta'(\frac{3}{2}-)$	6,29	5,94	5%
Case B:	$\delta(\frac{1}{2}+)$	4,14	4,13	0,3%
	$\delta'(\frac{3}{2}-)$	6,7	6,35	6%

Having a GMO-formula for the gap constants Δ, what about the masses? According to (18), Δ appears squared under a root in the mass. So obviously only in the strong coupling limit $\Delta_\nu \gg E_{\pm\bar\nu}$ we get the right GMO-formula for the barion masses:

$$\omega_\nu^{\pm} = \pm\frac{\Delta E_{\bar\nu}}{2} + \Delta_\nu\left(1 + \frac{(E_{\bar\nu} + E_{-\bar\nu})^2}{8\Delta_\nu^2}\right), \quad \text{strong coupling}$$

because now the main dependency on the third components is $\sim\Delta_\nu$. (The small term $\sim\frac{1}{\Delta_\nu}$ leads to small deviations from the GMO-formula. An additional singlet interaction would not contribute to the breaking: $\Delta_\nu = \Delta_{\bar\nu}^{(1)} + \Delta_\nu^{(8)}$). The relative small errors in table 2 are now easily understandable: The δ of table 1 were calculated from the experimental masses e_B (e_{B^*}), which fulfil the GMO-formula within 1% (8%), , and in this calculation we assumed values of μ_0 very near the strong coupling value $\mu_{0\,str} = 2{,}683$ (A) ; 1,683 (B) , defined by $\delta^2 = \max$ or $c(\alpha\beta\mu_0) = -\alpha - \frac{1}{2}$, compare (20a).

In the weak coupling limit, on the other hand, the breaking term is proportional to Δ_ν^2, giving a "rooted" GMO-formula (a = const):

$$(m_N - a)^{\frac{1}{2}} + (m_\Xi - a)^{1/2} = \frac{3}{2}(m_\Lambda - a)^{\frac{1}{2}} + \frac{1}{2}(m_\Sigma - a)^{\frac{1}{2}} \qquad \text{weak coupl.}$$

$$m_\Lambda = m_\Sigma \,.$$

So we can note the important fact, that the appearance of a GMO-formula for the masses is not necessarily a proof for a weaker first order breaking interaction, as is usually believed, the standard method of derivation using a first order breaking. Further we note, that our results concerning symmetry breaking are independent of the choice of the (discrete) states of the bare particles. For the form (21) of Δ_ν and the mass formula (18) do not depend on this choice. So these results are characteristic of the quasiparticle approximation with bare barions in general.

For quarks the strong coupling limit gives according to (23b):

$$\frac{m_1 - b}{C_1} = \frac{m_2 - b}{C_2} = \frac{m_3 - b}{C_3} \qquad \text{strong coupl.,}$$

$$m_1 = m_2 = \frac{1}{2}(3b - m_3)^{xx/}$$

the weak coupling the same with C_1 replaced by C_1^2. Here we have no compelling reason to prefer the strong coupling, because nothing is known about the masses of the quarks $\Omega^* \overline{10})$.

After we have seen the consequences of the fact, that our approximation leads to a GMO symmetry breaking for the gap constants Δ_ν, we wish to understand in detail, how the quasiparticle approximation leads to this breaking. The partly averaging of H_p to $\bar{H}_p$ (9) replaces one of the two $N\bar{N}$ pairs interacting in H_p by a vacuum expectation value, i.e. by a classical function, leading to a "potential" in $\bar{H}_p$ in a generalized sense, Fig. 6. (9) shows, that Δ_ν is just this potential. Between both pairs there was no transfer of angular momentum, as already stated. So the potential now exchanges no angular momentum with the remaining pair, and we have angular momentum conservation. On the other hand the pairs exchanged unitary spin because of the octet character of the chosen interaction. So there is now transfer of unitary spin between the potential and the remaining pair. Consequently there is breaking of unitary spin conservation in an analogous way, as we would have breaking of angular-momentum conservation, if we had a potential exchanging angular momentum x/

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x/ Example Zeeman effect: $V \sim L_z$.

xx/ So $3b \geq m_3 - m_1 \geq -\frac{3}{2}b$, $b > 0$

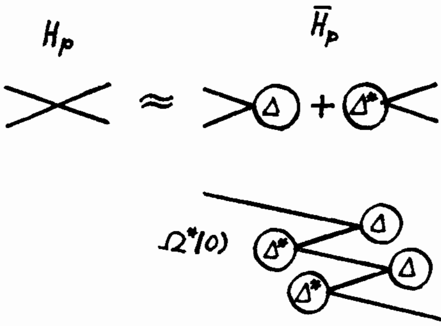

Fig. 6.

with the remaining pair. The replacement of a dynamically interacting pair of particles by the "stiff" vacuum expectation value is the origin of the dynamical breaking.

So we have found a simple method to read off directly from H_p which of its symmetries will be broken by the quasiparticle method: only the symmetries transferred "within" H_p . Reversing the argument, the appearance of a GMO-breaking gives information about the "internal" structure of H_p , about its transfer properties. It may well be, that this statement is more general than our quasiparticle approximation.

Summarizing we can state: If we use (bare) barions as building-blocks, and assume that the averaging of the quasiparticle approximation is allowed, we are led to strong coupling within the dense matter of the core. It is quite natural, that in such a dense packed strongly interacting medium averaging methods are useful. But let us not forget, that we assumed tacitly, that the (suppressed) meson cloud does not contribute considerably to the masses. For quarks as building-blocks on the other hand our considerations give no limit to prefer the strong (or any other) coupling.

Finally we have to remove an important incompleteness of our treatment. Having states with broken symmetries, one awaits that the quasiparticle vacuum $|0\rangle$ itself is not invariant. An invariant pairing of two particles $N\bar{N}$ has necessarily the form:

$$|P_{\bar\nu}\rangle = \sum_{m_\nu}\sum_{\phi_\nu} C\begin{pmatrix} j_\nu & 0 & j_\nu \\ m_\nu & 0 & m_\nu \end{pmatrix} C\begin{pmatrix} 8_\nu & 1 & 8_\nu \\ \phi_\nu & 0 & \phi_\nu \end{pmatrix} N^*_\nu \bar{N}^*_{-\nu} |0\rangle =$$

$$= \sum_{m_\nu} \sum_{\phi_\nu} N^*_\nu \bar{N}^*_{-\nu} |0\rangle \equiv \sum_{m_\nu \phi_\nu} P^*_\nu \; |0\rangle ,$$

in which both CG-coefficients are equal to unity. Instead we have in 10)

$$|0) = \prod_{\nu} u_{\nu} \cdot [1 - \sum_{\lambda} \frac{v_{\lambda}}{u_{\lambda}} P^*_{\lambda} + \sum_{\lambda \neq \lambda'} \sum \frac{v_{\lambda} v_{\lambda'}}{u_{\lambda} u_{\lambda'}} P^*_{\lambda} P^*_{\lambda'} - + \ldots] |0> .$$

So, as far as u_{λ}, v_{λ} depend on the third components m_{ν}, ϕ_{ν} – i.e. breaking really occurs – we have not the invariant expressions $|P_{\bar{\nu}}>$ in $|0)$. The consequences of this noninvariance have to be settled.

First we can state, that this makes no essential change in Δ_{ν}, because according to (21) the ν-dependency depends not on the vacuum expectation value in Δ_{ν} at all. So our considerations about Δ_{ν} and ω_{ν} are unchanged. Secondly the noninvariant vacuum leads in a well known manner to superflous states with energy $= 0$, called "spurious states" (nuclear physics x/) or Goldstone mesons (particle physics xx/).

Third let us study a first order breaking

$$g_{\lambda} \equiv \frac{v_{\lambda}}{u_{\lambda}} = g_{\bar{\lambda}} + \gamma_{\lambda} \qquad \gamma_{\lambda} \ll g_{\bar{\lambda}} .$$

This leads to

$$|0) = |0)_{inv} - \sum \gamma_{\lambda} P^*_{\lambda} |0> ,$$

therefore to

$$|1_{\nu}) = \Omega^*_{\nu}|0) = \Omega^*_{\nu}|0)_{inv} + \gamma_{\nu} v_{\nu} N^*_{\nu} |0> - u_{\nu} \sum_{\lambda} \gamma_{\lambda} N^*_{\nu} N^*_{\lambda} \bar{N}^*_{-\lambda} |0> .$$

The first two terms transform clearly as an octet, but the last term represents a small admixture of type 8x8x8, i.e. of 1+ ... + 64. The breaking therefore shifts not only the eigenvalues, but also mixes the representations. This will have consequences in the calculation of matrix elements in general. All our statements concerning SU_8 multiplets therefore have to be understood in this sense.

d) Supplementary Remarks

If one includes the pure barion-barion and antibarion-antibarion interactions F and $\bar{F}$, one awaits no essential changes. For our quasiparticles, pairing N and $\bar{N}$, depend mainly on the barion-antibarion interaction. Indeed, the mass formula (18) retains its form, only the replacement

$$E_{\bar{\nu}} \rightarrow E_{\bar{\nu}} + \epsilon_{\nu} \qquad \epsilon_{\nu} = 2 \sum_{\mu} F_{\nu\mu\mu\nu} (N^*_{\mu} N_{\mu})_0$$

is to be performed. Restricting analogously to our choice of H_p to the terms F^{K_0}, $\bar{F}^{K_0}$, $K_0 = 008000$, one gets $\Delta\epsilon_{\nu} = \epsilon_{-\nu} - \epsilon_{+\nu} = 0$. The only change in (18) is the appearance of ϵ_{ν}-containing terms under the root. They contribu-

x/ Baranger, l.c.

xx/ Kibble, Proc. Oxford conf. (1965), Goldstone l.c.

te to deviations from GMO in the strong coupling limit, because ϵ_ν contains $C^\rho \binom{8_\nu\ 8\ 8_\nu}{\phi_\nu 0\ \phi_\nu}$ squared. So these deviations give some information about F, $\bar{F}$.

Some preliminary remarks about renormalisation can be made. The mass renormalization we performed automatically in calculating the physical mass ω_ν. So for each level extra we can define a mass renormalization

either $\delta\omega_\nu = \omega_\nu - m \qquad (m = m_{bare})$

or
$$\delta\omega_\nu = \omega_\nu - E_{\bar{\nu}} .$$

The wave function renormalization constant Z_ν of a level is the probability to find a bare particle N_ν in the state $|1_\nu) = \Omega^*_\nu |0)$. It can be read off, if one expresses Ω^*_ν and $|0)$ by bare particles (abbreviated)

$$|1_\nu) = \Omega^*_\nu |0) = (Z_\nu N^*_\nu + Y_\nu N^*_\nu (N^*\bar{N}^*) + X_\nu N^*_\nu (N^*\bar{N}^*)(N^*\bar{N}^*) + \dots) |0> .$$

One gets
$$Z_\nu = \frac{u_1 \dots\dots u_n}{u_\nu} .$$

The full renormalization problem arises only, if we go beyond the bound state approximation and include the scattering states. Because we do not use perturbation theory (according to which our 4-fermion interaction would be non-renormalizable), we are confronted with a completely new situation: Our bound state approximation separates off the mass (≡ self energy) problem from the scattering problem.

It is easy to write down propagators along the standard way, using
$$\Omega_\nu(t) = e^{i\mathcal{H}_0 t}\, \Omega_\nu\, e^{-i\mathcal{H}_0 t} \qquad \mathcal{H}_0 = \sum_\nu \omega_\nu \Omega^*_\nu \Omega_\nu .$$

The Fourier transform $\mathcal{F}_{t-t'}$ of the propagator gets

$$\mathcal{F}\,(0|\,T(\Omega_\nu(t)\Omega_\nu(t'))|0) = \frac{1}{\omega_\nu - E + i\delta} ,$$

the usual form for a particle of mass ω_ν in its rest system. Besides often another "propagator with regard to the bare vacuum" is defined:

$$\mathcal{F} <0|\,T(\Omega_\nu(t)\Omega_\nu(t'))\,|0> = \frac{u_\nu^2}{\omega_\nu + E - i\delta} - \frac{v_\nu^2}{\omega_\nu + E + i\delta} .$$

IY. Meson Quasiparticles

a) General considerations

The next step of proceeding (point 6) is the construction of higher order quasiparticles (superconductor: excitons, nucleus: collective states) taking individual quasiparticles Ω as building-blocks, which are bound by the interactions of H_{Rest}. For instance the destruction operator of a meson for $\Omega =$ barion has the form $(K = JM\underline{n}II_8Y)$

$$M^K = \sum_{\nu\nu'} (\mu^K_{\nu\nu'} \Omega_\nu \bar{\Omega}_{\nu'} + \nu^K_{\nu\nu'} \Omega^*_\nu \bar{\Omega}^*_{\nu'}) . \qquad (24)$$

The coefficients μ, ν contain convenient Clebsh-Gordan coefficients, e.g.

$$C(j_\nu\, j_{\nu'} J) \cdot C^\rho (8_\nu\, 8_{\nu'}\, 8) ,$$

if M is an octet meson. The rest of the coefficients is determited from the requirement, that M behaves as destruction operator with regard to the full Hamiltonian:

$$[H_{red} + H_{Rest}, M^K] = -\omega^K_M M^K .$$

This leads to equations for μ, ν of a similar type, as we had for u, v (17):

$$[(\omega_1 + \omega_2 - G_{12})^2 - \omega^2_M]\mu_{12} = \Sigma \text{ (interaction terms)}. \qquad (25)$$

Remark, that automatically the meson mass appears squared.

The decuplet-barions require clearly quasiparticles of 3. order:

$$D^K = \sum_{\nu\nu'\nu''} (\delta^K_{\nu\nu'\nu''} \Omega_\nu\, \Omega_{\nu'}\, \bar{\Omega}_{\nu''} + \epsilon^K_{\nu\nu'\nu''} \Omega^*_\nu\, \Omega^*_{\nu'}\, \bar{\Omega}^*_{\nu''}).$$

For $\Omega =$ quarks on the other hand the barions themselves are of third order

$$B^K = \sum_{\nu\nu'\nu''} (\beta^K_{\nu\nu'\nu''} \Omega_\nu\, \Omega_{\nu'}\, \Omega_{\nu''} + \gamma^K_{\nu\nu'\nu''} \Omega^*_\nu\, \Omega^*_{\nu'}\, \Omega^*_{\nu''}) ,$$

while the mesons retain the form (24).

b) Preliminary Treatment of Meson Spectra

Let us study in an orientating manner the octet mesons. Clearly equations like (25) can be solved only approximately. Here we restrict ourselves to the simplest possible treatment, which gives the quantum numbers and the succession of levels right, but the energies of the levels bad in general. It approximates ω^8_M by

$$\omega^8_M \approx \omega_1 + \omega_2 - G_{12} , \qquad (26)$$

compare (25). In the simpler case (17) the corresponding approximation is $\omega_\nu \approx E_{\bar{\nu}} =$ energy of the independent particle. That means, that in this case all interaction is neglected (exept that creating the levels). In our present case not all interaction between the individual quasiparticles is neglected, as is shown by the appearance of G_{12}, a matrix element of the rest interaction, in (26), which means a crude "binding energy" between the quasiparticles in M. Knowing nothing about G_{12}, we approximate it by an overall binding energy

$$G_{12} \approx G = \text{const} .$$

Then the mass ω_M is essentially the sum of the masses of both bound individual quasiparticles, shifted down by G. For case A we give the resulting term system in Fig. 7. For case B the upper two lines of terms are absent. The corresponding observed established mesons are indicated.

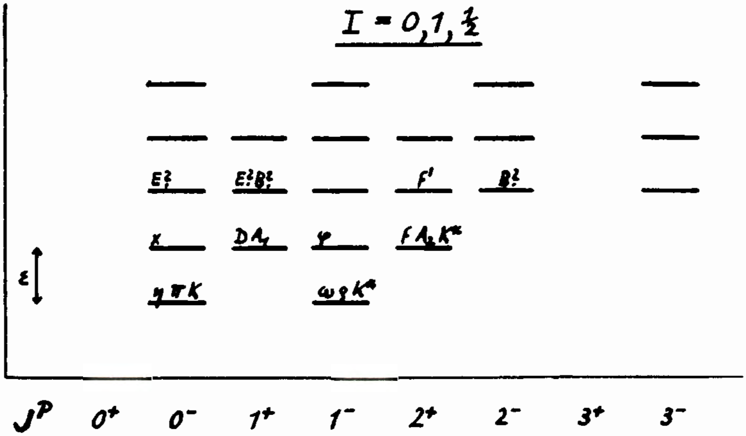

Fig. 7.

Besides this overal orientation we mention two specific predictions. From parity considerations and the exclusion of $\frac{1}{2}-$, $\frac{3}{2}+$ (19) it follows that there should be no 0+ and 3+ mesons, at least as long as only the jj -coupling of Ω and $\bar{\Omega}$ in (24) is admitted. The observation of scalar mesons 0+, like s_0 and K_1K_1, therefore points to the necessity of other coupling types in M, e.g. L – S -coupling. This remark is important, for L – S -coupling is intimately related to SU_6 -symmetry, because it decouples spin from angular momentum.

The second remark proves right for all quasiparticles of order 2 or higher. All field operators M,D,B etc. have no "elementary" commutation relations, for instance

$$[M^K , M^{K'}] = \delta_{KK'} 1 + \underline{\text{operators}}.$$

From this one derives easily, that the states created by them are not strictly orthogonal one to another. This leads to mixture of states within each column in Fig. 7, so between ω and ϕ, or η and $\varkappa$, even if ϕ and $\varkappa$

are assumed to belong to an octet, as we did in Fig. 7.

V. Conclusion

As we have seen, the quasiparticle approximation provides us with a simple complete dynamics, applicable to either barions or quarks, and leading spontaneously to the right symmetry breaking. It is "constructive", calculating the field operators and states of all physical particles in terms of the fundamental field. Further it can be easily used for other symmetries than SU_3 . For all symmetry properties are contained in the CG-coefficients. So the method is applicable to all compact groups. Further it can be generalized to Yukawa interactions instead of four fermion interactions.

We considered the resonances only in the "stable" approximation. The study of the scattering problem is therefore one of the next steps, embedding our discrete solutions into the continuum scattering states. This leads to a reactance-matrix instead of an S -matrix scheme. Further this problem is intimately connected with the question of the deeper sense of our approximation "by partly averaging".

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