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HYPOTHESIS OF CONSERVED TENSOR CURRENTS AND SU(6) GROUP

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The conventional intrinsic groups (isotopic, SU(3)) are connected with conserved vector currents. Fourth components of there currents give secondary quantized generators. Being 3-scalars they can not mix different spin states of fields. We wish to stress that only tensor currents, but not vector ones can generate the transformations of the popular now group SU(6)[1], for which the spin mixing is characteristic. Actually, SU(6) may be connected with the conserved symmetric tensor singlet and octet currents of the form

$$\mathcal{J}_{\mu\nu}(x) = \frac{1}{4}\partial_\rho \left\{ -i\bar{\psi}\, \sigma_{\mu\rho}\overleftrightarrow{\partial}_\nu \psi + b_\rho \overleftrightarrow{\partial}_\mu b_\nu + b^a_\rho \overleftrightarrow{\partial}_\mu b^a_\nu + (\mu\nu)\right\} \tag{1}$$

$$\mathcal{J}^b_{\mu\nu}(x) = \frac{1}{4}\partial_\rho \Big\{ -i\bar{\psi}\lambda_b \sigma_{\mu\rho}\overleftrightarrow{\partial}_\nu\psi + d_{abc} b^a_\rho \overleftrightarrow{\partial}_\mu b^c_\nu + \sqrt{\tfrac{2}{3}}\left(b^b_\rho \overleftrightarrow{\partial}_\mu b_\nu + b_\rho \overleftrightarrow{\partial}_\mu b^b_\nu\right) + \frac{i}{2\mu} f_{abc}\varepsilon_{\mu\rho\sigma\tau}\varphi^a \overleftrightarrow{\partial}_\nu \overleftrightarrow{\partial}_\sigma b^c_\tau + (\mu\nu)\Big\} \tag{2}$$

together with the vector current

$$j^b_\mu(x) = i\bar{\psi}\gamma_\mu\lambda_b\psi + \frac{1}{2} f_{abc}\left(\varphi^a \overleftrightarrow{\partial}_\mu \varphi^c + b^a_\nu \overleftrightarrow{\partial}_\mu b^c_\nu\right) \tag{3}$$

which generates the SU(3) transformations. In (1)-(3) $A\overleftrightarrow{\partial}_\nu B = A\partial_\nu B - \partial_\nu A\cdot B$ and symbol $(\mu\nu)$ denotes the symmetrization. For brevity we have written in (1) -(3) only the contribution of two multiplets: quark one, represented by $\psi(x)$, and 35-plet, consisting of $b^a_\mu(x)$ (the 1^- octet), $b_\mu(x)$ (the 1^- singlet) and of $\varphi^a(x)$ (the 0^- octet). At this stage the fields are supposed to be free, and all these currents are striktly conserved

$$\partial_\mu \mathcal{J}_{\mu\nu} = 0 , \quad \partial_\mu \mathcal{J}^b_{\mu\nu} = 0 , \quad \partial_\mu j^b_\mu = 0 \tag{4}$$

due to usual free equations of motion

$$(\gamma\partial + M)\psi = 0 , \quad (\Box - \mu^2)\varphi^a = 0 , \quad \left[(\Box-\mu^2)\delta_{\mu\nu} - \partial_\mu\partial_\nu\right]\begin{pmatrix} b^a_\nu \\ b_\nu \end{pmatrix} = 0 \tag{5}$$

Note that the moment

$$\mathcal{J}_{\mu,\nu\lambda} = x_\nu \mathcal{J}_{\lambda\mu} - x_\lambda \mathcal{J}_{\nu\mu} \tag{6}$$

of the conserved symmetric current $\mathcal{J}_{\mu\nu}$ is also conserved

$$\partial_\mu \mathcal{J}_{\mu,\nu\lambda} = 0 \tag{6}$$

Consequently, in a theory with the conserved symmetric tensor currents there exist two types of generators

$$V = -i\int d\vec{x}\, \mathcal{J}_{4\nu}\,\omega_\nu \quad (a), \qquad V = -i\int d\vec{x}\, \mathcal{J}_{4,\nu\lambda}\,\omega_{\nu\lambda} \quad (b) \tag{7}$$

which are able to change the spin by 1 (and analogously for $\mathcal{J}^b_{\mu\nu}$) Hence there exist two types of transformations provided that both generators fo not vanish. One example of current $\mathcal{J}_{\mu\nu}$ is well-known: the symmetric energy-momentum tensor.

The currents (1) and (2) are such, that the generators (7a) for them vanish, and the geaerators (7b) are

$$U = \int \frac{d\vec{p}}{2p_0}\,\omega_{\mu\nu}\left\{\frac{1}{4}\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{M}\bar{u}\gamma_\rho\gamma_5 u + CC - i b^+_\mu b_\nu - i b^{a+}_\mu b^a_\nu\right\} \tag{8}$$

$$W = \int \frac{d\vec{p}}{2p_0}\,\omega^b_{\mu\nu}\left\{\frac{1}{4}\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{M}\bar{u}\lambda_b\gamma_\rho\gamma_5 u + CC - i d_{abc} b^{a+}_\mu b^c_\nu + \right. \tag{9}$$

$$\left. -i\sqrt{\frac{2}{3}}\left(b^+_\mu b^b_\nu - b^{b+}_\nu b_\mu\right) - \frac{i}{2} f_{abc}\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{\mu}\left(b^{a+}_\rho \varphi^c + \varphi^{c+} b^a_\rho\right)\right\}$$

[2]

where all the field operators depend on $\vec{p}$: $u(\vec{p}), \bar{u}(\vec{p}), b_\mu(\vec{p})$ etc; CC stands for charge-conjugated terms for antiquarks. The generator, which corresponds to the vector current, is a usually

$$Q = -i\int d\vec{x}\, j_4 = \int \frac{d\vec{p}}{2p_0}\,\omega^b\left\{\frac{1}{2}\bar{u}\lambda_b u + CC + i f_{abc}\left(\varphi^{a+}\varphi^c + b^{a+}_\mu b^c_\mu\right)\right\} \tag{10}$$

The generators Q, U and W produce infinitesimal transformations of the fields f

$$\delta f = -i\left[Q + U + W, f\right]$$

So, for quarks

$$\delta\psi(x)=\frac{i}{4}\left[2\omega^a\lambda_a+\hat{\sigma}_{\mu\nu}(\omega_{\mu\nu}+\omega^a_{\mu\nu}\lambda_a)\right]\psi\ ;\ \hat{\sigma}_{\mu\nu}=\left(\delta_{\mu\mu'}+\frac{p_\mu p_{\mu'}}{M^2}\right)\sigma_{\mu'\nu'}\left(\delta_{\nu'\nu}+\frac{p_{\nu'}p_\nu}{M^2}\right) \quad (11)$$

or, equivalently, in the free case,

$$\delta\psi(x)=\frac{i}{4}\left[2\omega^a\lambda_a+\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{M}\gamma_\rho\gamma_5(\omega_{\mu\nu}+\omega^a_{\mu\nu}\lambda_a)\right]\psi \quad (11')$$

The transformations of the 35-plet are

$$\delta\varphi^a(x)=-f_{abc}\left(\omega^b\varphi^c+\frac{1}{2}\omega^b_{\mu\nu}\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{M}b^c_\rho\right);\ \delta b_\mu=\hat{\omega}_{\mu\nu}b_\nu+\sqrt{\tfrac{2}{3}}\,\hat{\omega}^b_{\mu\nu}b^b_\nu$$

$$\delta b^a_\mu(x)=-\omega^b f_{abc}b^c_\mu+\hat{\omega}_{\mu\nu}b^a_\nu+\hat{\omega}^b_{\mu\nu}d_{abc}b^c_\nu+\sqrt{\tfrac{2}{3}}\,\hat{\omega}^a_{\mu\nu}b_\nu+ \quad (12)$$

$$+\frac{1}{2}f_{abc}\,\omega^b_{\alpha\beta}\,\varepsilon_{\alpha\beta\gamma\mu}\frac{p_\gamma}{M}\varphi^c$$

where $\hat{\omega}_{\mu\nu}=\left(\delta_{\mu\mu'}+\frac{p_\mu p_{\mu'}}{M^2}\right)\omega_{\mu'\nu'}\left(\delta_{\nu'\nu}+\frac{p_{\nu'}p_\nu}{M^2}\right)$ and $p_\mu=\frac{1}{i}\partial_\mu$.

In the rest system $(\vec{p}=0)$ the transformations (11) and (12) coincide with the SU(6) transformations. There is no need in superfluous momenta in such a relativization (in contrast with SL(6), $\tilde{U}_{12}$ and so on), and the problem of non-invariance of equationsof motion do not arise: the equations (5) are striktly invariant under the transformations obtained. For the quarks this relativization coincides with Salam one[3]. The only distinction is that we complement three non-relativistic spin matrices to antisymmetric tensor, instead of 4-vector, in general frame of reference. Our parameters $\omega_{\mu\nu}$ and the Salam ones E_ρ are related by the formula $\omega_{\mu\nu}\varepsilon_{\mu\nu\lambda\rho}\frac{p_\lambda}{M}=E_\rho$. The Salam transformation corresponds also to tensor carrents, but the latter are non-symmetric and conserved only in one index.

When omitting parameters $\omega_{\mu\nu}\varepsilon_{\mu\nu\lambda\rho}p_\lambda$ and $\omega^b_{\mu\nu}\varepsilon_{\mu\nu\lambda\rho}p_\lambda$ in U and W one obtains pseudovector quantities of the form $\int\frac{d\vec{p}}{2p_0}\,j_{\rho 5}(\vec{p})$ e.g. for quarks $\int\frac{d\vec{p}}{2p_0}\,\bar{u}(\vec{p})\gamma_\rho\gamma_5 u(\vec{p})$. Therefore the relation is revealed to the pseudovector currents, in the algebra of currents[4]. Thus, in fact, <u>the "pseudovector currents"</u>

are originated by the fourth components of some conserved tensor currents. It takes off the question about the non-conservation of the pseudovector currents, and explains their connection with SU(6).

A theory with interactions may be conjectured as one, in which tensor currents $J_{\mu\nu}$ and $J^{b}_{\mu\nu}$ serve as sources of 2^+ fields (in analogy with conserved vector currents which are the sources of 1^- fields[5]). Hence it follows, firstly, the fundamental role of the 189 supermultiplet, containing just octet and singlet of the 2^+ states. Its role is the same as of regular representations e.g. the SU(3) octet for 1^- fields. Note, that the experimentally found 2^+ resonances form just octet and singlet: $K^*(1430)$, $T=\frac{1}{2}$, $Y=\pm 1$; $A_2(1320)$, $T=1$, $Y=0$; $f(1250)$, $f'(1525)$, $T=0$, $Y=0$ [6].

In this connection it is important to identify other states of the 189-plet[7]. Secondly, the common current-source implies the universality of the 2^+ field interactions and, therefore, the definite relations between the constants of interactions of all the fields with the 2^+ fields. In different aspect the universality hypothesis has been discussed in[8].

In the free fields transformations obtained the parameters depend on 4-velocity. Actually , if we apply the Lee bracket operation $(\delta_2\delta_1-\delta_1\delta_2)f=\delta_{br}f$ where the variations δ_1, δ_2 are transformations (11) (12) with parameters $\omega_1^{\alpha}, \omega_{1\mu\nu}, \omega^{\alpha}_{1\mu\nu}$ and $\omega_2^{\alpha}, \omega_{2\mu\nu}, \omega^{\alpha}_{2\mu\nu}$, respectively, then we shall find for the parameters of the bracket variation δ_{br}

$$\omega^a_{br} = -f_{abc}\left(\omega_1^b\omega_2^c + \tfrac{1}{2}\omega^b_{\mu\nu}\omega^c_{\mu\nu} + u_\mu u_\nu \omega^b_{1\mu\alpha}\omega^c_{2\nu\alpha}\right)$$

$$\omega_{\mu\nu br} = -\left(\delta_{\alpha\beta}+u_\alpha u_\beta\right)\left(\omega_{1\mu\alpha}\omega_{2\nu\beta} + \tfrac{2}{3}\,\omega^a_{1\mu\alpha}\omega^a_{2\nu\beta} - (\mu\nu)\right)$$

$$\omega^a_{\mu\nu br} = -\left(\delta_{\alpha\beta}+u_\alpha u_\beta\right)\left(\omega^a_{1\mu\alpha}\omega_{2\nu\beta} + \omega_{1\mu\alpha}\omega^a_{2\nu\beta} + d_{abc}\omega^b_{1\mu\alpha}\omega^c_{2\nu\beta} - (\mu\nu)\right)$$
$$- f_{abc}\left(\omega_1^b\omega^c_{2\mu\nu} - \omega_2^b\omega^c_{1\mu\nu}\right)$$

Therefore, to each value of 4-velocity $u_\mu = \dfrac{P_\mu^{quark}}{M} = \dfrac{P_\mu^{35}}{\mu} = \ldots$ there corresponds its own group. The dependence on the 4-velocity is an essential defect[9]: if these transformations were applicable in theory with interaction, they would impose an infinite number of limitations on reaction amplitudes and make the latter to be zero. But they appear to be non-applicable, since an interaction will violate the conservation of the above currents $J_{\mu\nu}$ and $J^b_{\mu\nu}$. Therefore, the transformations and the currents: need to be modified: transformations and currents must be essentially non-linear, if interaction is present. Such a situation is analogous to one in the field theoretical approach to the Einstein gravitation theory[10,11]. There the interaction essentially modifies the tensor current (the energy-momentum tensor) in a non-linear manner[11]. As to the non-linearity of laws of transformations the gravitation theory gives also such an example (up to now unique in the field theory): it is the law of transformations of spinors, which becomes an infinite series in powers gravitation field, when the gravitation interaction is taken into account[12].

For the time being the problem of re-establishment of a true theory is still at the initial stage. However, we can already now draw some conclusions concerning the role of the SU(6) and the reasons for difficulties of its straight-forward linear relativizations. In the true theory S-matrix will be invariant under some new non-linear transformations, but not under the SU(6) transformations. This general group will reduce to SU(6) applied to one-particle states and to static effects only. As to the reaction amplitudes, the new group will, generally speaking, establish relations between processes with different numbers of particles. Therefore, the SU(6) group is dynamical group in Pais language. Note also, that when modifying the current, the neutral tensor current will apparently be complemented to the energy-momentum tensor, so that the true non-linear group will contain the homogeneous Lorentz group and the SU(3) group as its subgroups.

In any case, it is just the conserved tensor currents that generate the group, which reduces to SU(6) in the rest system, and it suggests that they will be the object of am extensive investigations in the nearest future. More detail investigation will be published elsewhere.

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