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CALCULATIONS OF PROPERTIES OF COLLECTIVE STATES IN DEFORMED NUCLEI IN THE REGION $176 \leq A \leq 190$

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Collective states in deformed nuclei are nowadays intensively studied by experimenters and theoreticians. Theoretical investigations of such states are usually based on a microscopic model of atomic nuclei, in which nucleons move in a deformed average field and interact via pairing and multipole-multipole forces. Calculations in which this interaction was taken into account by the method of approximate second quantization in the framework of the superfluid nuclear model have shown, that it is in principle possible to interpret the known experimental data correctly. However, it was also shown that results of the calculations are sensitive to the value of the deformation of the average field.

In papers /1-4/ quadrupole and in /5/ octupole states of even-even deformed nuclei in rare earth and actinide regions were studied. In ref. /1,5/ the single particle energies and Nilsson wave functions were taken with the deformation $\delta = 0.3$ for all rare-earth nuclei. However, for nuclei with $176 \leq A \leq 190$ such an approximation should be rather bad and therefore we recalculated the energies and wave functions of collective states of even-even nuclei in range $176 \leq A \leq 190$ using a modified single-particle level scheme (see table 1) and Nilsson wave functions for $\eta = 4$ ($\delta = 0.2$). For each nucleus we first solved the equations of the superfluid nuclear model and obtained the correlation function C and chemical potential λ . The values of the pairing interaction constants

$$G_N = 0.017 h = \hbar\mathring{\omega}_0 , \quad G_Z = 0.021\, h\, \mathring{\omega}_0$$

were found from a comparison of the theoretical and experimental pairing energies. Further the secular equation

$$1 = 2\kappa^{(\lambda)} \sum_{ss'} \frac{\left(f^{\lambda\mu^2}(ss') + \bar{f}^{\lambda\mu^2}(ss') \right) U^2_{ss'} (\epsilon(s) + \epsilon(s'))}{(\epsilon(s) + \epsilon(s'))^2 - (\omega_i^{\lambda\mu})^2} \qquad (1)$$

was solved (see ref. /1/) and the frequencies $\omega_i^{\lambda\mu}$ of the collective states were found . Here $f^{\lambda\mu}(ss')$ is the matrix element of the operator $r^{\lambda} Y_{\lambda\mu}$, $U_{ss'} = u_s v_{s'} + v_s u_{s'}$, $\kappa^{(\lambda)}$ is the multipole-multipole interaction constant, $\epsilon(s) = \sqrt{(E(s)-\lambda)^2 + C^2}$

The equation (1) was modified according to ref. /1,5/ i.e. the value of the chemical potential λ was taken, which gives the right average number of particles in the excited state. The quantities $\epsilon(s) + \epsilon(s')$ were replaced by the energy of a two quasi-particle state, calculated with the blocking effect. The multipole-

multipole interaction constants were found as usual from a comparison of the experimental and theoretical energies of collective states.

The majority of the known collective states in the region $176 \leq A \leq 190$ are γ-vibrations (states with $K\pi = 2+$). Figure 1 shows the experimental and calculated energies of such states. It is seen, that the agreement between the experimental and theoretical values is quite good, in particular the tendency of the $K\pi = 2+$ states energies to decrease with the increase of the number of protons and neutrons, is reflected correctly.

The same quadrupole-quadrupole interaction constant was taken for protons and neutrons ($\kappa^{(2)}_n = \kappa^{(2)}_p = \kappa^{(2)}_{np} \equiv \kappa^{(2)}$). Let $\kappa^{(2)} = \frac{k}{A^{4/3}} \hbar \mathring{\omega}_0$, then for the parameter k we obtained $k = 4.8$ taking the blocking effect into account and $k = 5.35$ without the blocking. These values are about 30 percent lower than $\kappa^{(2)}$ in ref./1/ but they are quite close to the corresponding $\kappa^{(2)}$ in the paper by Bes et al./3/. Thus we make the conclusion, that it is impossible to describe the energies of γ-vibrational states in the whole rare-earth region with only one value of the quadrupole-quadrupole interaction constant (see also/3/). This effect can be explained by the possible instability of the W and Os isotopes with respect to γ-deformations, by the necessity to use a non-linear theory for vibrations in this region, or by the inadequancy of matrix elements calculated using oscillator single-particle wave functions.

The reduced probabilities of electromagnetic transitions for the Coulomb excitation of the $K\pi = 2+$ states were calculated, too. As in ref./6/ the influence of distant proton states was eliminated by means of an effective charge, the same for protons and neutrons, i.e. $e_p = e + e_{eff}$, $e_n = e_{eff}$. In the electronic computer calculations an error was made in/6/ which did not affect the values of B(E2) but led to a wrong determination of e_{eff}. On Fig. 2 the quantities B(E2) are plotted in Weisskopf units with blocking ($e_{eff} = 0.3\, e$) and without blocking ($e_{eff} = 0.2\, e$). These values of e_{eff} agree well with the corrected values of e_{eff} in ref./6/. Let us point out, that e_{eff} was determined from the comparison between the theoretical and experimental values of B(E2) and that it depends on the number of single-particle levels taken into account in eq. (1). From this point of view $e_{eff} \approx 0{,}2 - 0{,}3e$ is too small and our calculations confirm the conclusion made earth in ref./3/, that the theoretical values of B(E2) are overestimated.

In the region of nuclei $176 \leq A \leq 190$ only few experimental data on collective states with $K\pi = 0+$ (β-vibrations) and on octupole states with $K\pi = 0-, 1-, 2-$ are available. Therefore the comparison between theory and experiment could only be very rough. All known states of such a type are comparatively high-lying (more than 1 MeV) and their collective nature is not so clearly pronounced as in the case of γ -vibrations. In studying $K\pi = 0+$ states one has to take into account not only quadrupole-quadrupole interaction, but also the residual pairing interaction in order to exclude the spurious state connected with the nonconservation of the number of particles. The results of the calculations confirm the conclusion made in ref.[1,4] that states with $K\pi = 0+$ have energies in the interval 1.0 - -1.5 MeV close to 2C, and that their structure is essentially determined by the pairing interaction. Only in the end of the deformation region, for ^{188}Os and ^{190}Os the $K\pi = 0+$ states are lowered and their character is closer to the quadrupole β - -vibrations.

Octupole states with $K\pi = 0-$ are very high-lying ($\approx$ 1.7 MeV) and their energies do not change very rapidly from nucleus to nucleus. However, they have a collective nature, i.e. many two-quasi-particle states contribute considerably to their wave functions. Calculations made in this study confirm also, that states with $K\pi = 1-$, $2-$ are close to two-quasi-particle ones. Therefore their energy is very sensitive to the details of the single-particle level scheme, i.e. to quantities which are known with the largest uncertainty. The calculations have shown that for nuclei in the given region there are no low-lying (below 1 MeV) negative parity collective states.

Odd-mass nuclei.

There is one quasi-particle above the even-even core in odd-mass nuclei. Therefore according to the most primitive model the excited states of odd-mass nuclei should be one-quasi-particle ones, further one-quasi-particle+phonon states and so on. The multipole-multipole interaction contains a part, which corresponds to the scattering of a quasi-particle on a phonon and is responsible for the mixing of one quasi-particle states with one-quasi-particle + phonon states. Let us take this interaction into account by a method proposed in ref.[7]. We choose the trial wave function for the variational procedure in the form

$$\Psi(K\pi) = \frac{1}{\sqrt{2}} C_\rho \Big(\sum_\sigma \alpha^+_{\rho\sigma} + \sum_{\substack{\lambda\mu i\\ \nu\sigma}} D^{\lambda\mu i}_{\rho\nu\sigma}\, \alpha^+_{\nu\sigma}\, Q^i(\lambda\mu)^+\Big)\Psi_0 \qquad (2)$$

Here $\alpha^+_{\rho\sigma}$ is the operator of the quasi-particle ρ with a given $K\pi$, $Q_i(\lambda\mu)^+$ is the phonon operator of multipolarity (λ, μ) which together with the quasi-particle ν creates a state with the same $K\pi$. From the variational principle i.e. from the minimum average value of H we get a secular equation for determining the unknown frequency η_j of the state $\Psi(K\pi)$

$$\epsilon(\rho) - \eta_i - \tfrac{1}{4} \sum_{\lambda\mu i\nu} \frac{V^2_{\rho\nu}}{Y_i(\lambda\mu)} \; \frac{f^{\lambda\mu^2}(\rho\nu) + f^{-\lambda\mu^2}(\rho\nu)}{\epsilon(\nu) + \omega_i^{\lambda\mu} - \eta_i} = 0 \qquad (3)$$

The quantity $V_{\rho\nu} = u_\rho u_\nu - v_\rho v_\nu$ shows that the interaction is preferably of particle-particle or hole-hole type, $\epsilon(\rho)$ is the quasi-particle energy, $\omega_i^{\lambda\mu}$ is the frequency of a phonon in the even-even nucleus, $Y_i(\lambda\mu)$ is the derivative of the right-hand side of eq. 1 with respect to $\omega_i^{\lambda\mu}$. From the normalization condition of $\Psi(K\pi)$ we could obtain the quantities C_ρ and $D_{\rho\nu}^{\lambda\mu i}$. The quantity C_ρ^2 is the probability of finding quasi-particle ρ in the state $\Psi(K\pi)$, $C_\rho^2(D_{\rho\nu}^{\lambda\mu i})^2$ is the probability of finding quasi-particle ν + phonon $\lambda\mu i$.

The interaction of quasi-particles with phonons will strongly affect states, which are usually interpreted as γ-vibrations in odd-mass nuclei. Such states are known in ^{185}Re and ^{187}Re /8/. The comparison of experimental and theoretical data on states with $K = K_0 - 2$ and $K = K_0 + 2$ (K_0 refers to the ground states) in Re isotopes is given in table 2. It follows from the table that states with $K = K + 2$ i.e. $K\pi = 9/2+$ are really γ-vibrations. The calculated energies and B(E2) values agree well with the experiment. The energies and structure of $K\pi = 1/2+$ states are very sensitive to the value of the one-particle energy for the 1/2+400 state, which is underestimated in our scheme. Therefore the contribution of the one-quasi-particle 1/2+400 state is overestimated and the energy and B(E2) value underestimated in our calculation. However, from the calculations and also from the analysis of the experiment, one can make the conclusion that the first $K\pi = 1/2+$ states in Re isotopes contain a large admixture of the single-particle 1/2+400 state. This is also in accordance with the experimental value of the decouling parameter a. The large experimental value of B(E2) should be connected not only with the pure collective effect, but also with other effects (fast single-particle transition coherently added to the collective one, Coriolis interaction etc.). Further, it follows from the calculations that another $K\pi = 1/2+$ state with an energy 1-1.2 MeV and a large B(E2) value should exist.

Since no experimental data on collective states in odd-neutron nuclei exist in the considered region we shall only show the main results of our calculations. Comparing them with future experimental data, one has to have in mind their rather qualitative character (the main uncertainly is connected with the single-particle level scheme).

The 1/2-510 state is the ground one in nuclei with N=109. The first $K\pi = 3/2-$ state i.e. the $K_0 = K_0 - 2$ one is very low lying (of the order of 200 KeV) and contains 90 percent of the one-quasi-particle 3/2-512 state. The next $K\pi = 3/2-$ state has the energy 650 keV and structure close to the one-quasi-particle 3/2-501 The first $K\pi = 5/2-$ state has the energy 800 keV and contains 80-90 percent of the 5/2-512 state. The really collective $K_0 - 2$ and $K_0 + 2$ states lie higher than 1 MeV in these nuclei.

The states 1/2-510 and 3/2-512 are very close to each other in nuclei with N=111 and our model cannot decide which of them is the ground state. The excited states with $K\pi$ =3/2- and 1/2- and with the energy 1MeV in ^{185}W and 800 KeV in ^{187}Os are close to collective γ -vibrations. States with $K\pi$ = 5/2 - and 7/2- have a complex structure. $K\pi$ =5/2- states with the energy 800 KeV and 1200 keV in ^{185}W and 650 keV and 1200 keV in ^{187}Os would contain roughly 50 percent of the one-quasi-particle 5/2-512 and the collective 1/2-510+ $Q_1(22)$ state each. The first $K\pi$ =7/2- state is a single-particle 7/2- 503 one, the second and third states have the energies 800 keV and 1100 keV (^{185}W) and 800 keV and 950 keV (^{187}Os). All of them contain the 7/2-514 and collective 3/2+ $Q_1(22)$ state.

The 3/2-512 state is the ground one in nuclei with N=113. The first $K\pi$ =1/2- state with the energy 140 keV is close to the 1/2-510 one. A collective $K\pi$ =1/2- state has the energy 1MeV in ^{187}W and 900 keV in ^{189}Os. The first $K\pi$ =7/2- state is the single-particle 7/2-503 one, the second with the energy 900 keV in ^{187}W and 750 keV in ^{189}Os is a mixture of the single-particle 7/2-514 state and of the γ -vibrational state. Let us point out, that there should be a negative splitting in this case, i.e. the energy of the collective K_0+2 state should be less than the energy of the K_0-2 state.

Thus the calculations have shown, that a model with pairing and multipole-multipole interactions can describe well the experimental data on collective levels in the range $176 \leq A \leq 190$. The accuracy is limited mainly by a bad knowledge of the energies and wave functions of the average field.

In conclusion we express our deep gratitude to Prof. V.G.Soloviev for many stimulating discussions and to A.A.Korneichuk, K.M.Zheleznova and G.Jungklaussen for their help with the numerical calculations.

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Table 1

Single-particle levels of the average field and the characteristics of the ground states (energies in $\hbar\omega_0$)

Neutrons						Protons			
N	$K\,\pi\,Nn_z\Lambda$	E(s)	C_N	λ_N	Z	$K\,\pi\,Nn_z\Lambda$	E(v)	C_p	λ_p
	7/2-523	0.06				1/2+440	0.48		
	1/2+400	0.21				3/2+431	0.56		
	1/2-541	0.22				3/2-3o1	0.60		
	9/2-514	0.23				5/2-3o3	0.67		
	3/2+4o2	0.24				5/2+422	0.68		
	1/2-53o	0.32				1/2-3o1	0.71		
	3/2-532	0.35				7/2+413	0.83		
	11/2-5o5	0.38				1/2+431	0.88		
	1/2+660	0.42				9/2+4o4	1.02		
	3/2-521	0.48				3/2+422	1.o4		
	3/2+651	0.51				1/2+420	1.05		
	5/2-523	0.53				1/2-55o	1.06		
	5/2+642	0.55				3/2-541	1.11		
	7/2+633	0.64				5/2-532	1.21		
	1/2-521	0.65				5/2+413	1.24		
1o4	5/2-512	0.67	o.119	0.720		3/2+411	1.25		
1o6	7/2-514	0.73	0.113	0.769		7/2-523	1.29		
1o8	9/2+624	o.77	o.109	0.829	70	1/2+411	1.34	0.150	1.389
110	1/2-51o	o.88	o.115	o.88o	72	7/2+4o4	1.39	o.142	1.443
112	3/2-512	o.94	o.123	o.926	74	9/2-514	1.46	o.134	1.5o2
114	9/2-5o5	0.96	o.127	o.968	76	5/2+4o2	1.52	o.128	1.565
116	7/2-5o3	o.99	o.129	1.010	78	3/2+4o2	1.61	o.126	1.626
	3/2-5o1	1.02				1/2+4oo	1.65		
	11/2+615	1.o5				11/2-5o5	1.71		
	1/2+651	1.11				1/2-541	1.76		
	13/2+606	1.16				1/2-53o	1.82		
	1/2+64o	1.18				3/2-532	1.83		
	3/2+642	1.21				5/2-523	1.99		
	5/2-5o3	1.22				3/2-521	2.00		
	3/2+631	1.3o				7/2-514	2.18		
	1/2-5o1	1.3o				5/2-512	2.20		
	5/2+633	1.35				9/2-5o5	2.40		
	5/2+622	1.47				7/2-5o3	2.45		
	1/2+631	1.52				1/2+660	2.55		
	7/2+624	1.53				3/2+651	2.60		
	7/2+613	1.67				5/2+642	2.63		

Table 2

Collective states in Re isotopes

Nucleus			Energy (keV)		B(E2) / B(E2)$_{s.p.}$		Decoupling parameters a		C_s^2	$C_s^2(D_{2\nu}^{\lambda\mu i})^2$
			exp.	calcul.	exp.	calcul.	exp.	calcul.		
^{183}Re	1/2+	400↑		460		0.5		0.34	0.87	0.05
	1/2+	411↓	1103	900		-			0.94	0
	1/2+	400↑		1400		2.5	0		0.02	0.95
	9/2+	404↑		1350		2.5			0.04	0.95
^{185}Re	1/2+	400↑	647	400	3.6	0.5	0.38	0.32	0.79	0.07
	1/2+	400↑		1100		1.8		0	0.04	0.90
	9/2+	404↑	966	1020	2.6	1.9			0.03	0.95
^{187}Re	1/2+	400↑	511	400	3.1	0.6	0.38	0.31	0.78	0.09
	1/2+	411↓	618	850			-1.1		0.90	0
	1/2+	400↑		1150		2.4		0	0.03	0.90
	9/2+	404↑	840	1080	3.8 (2.5)	2.6			0.02	0.95

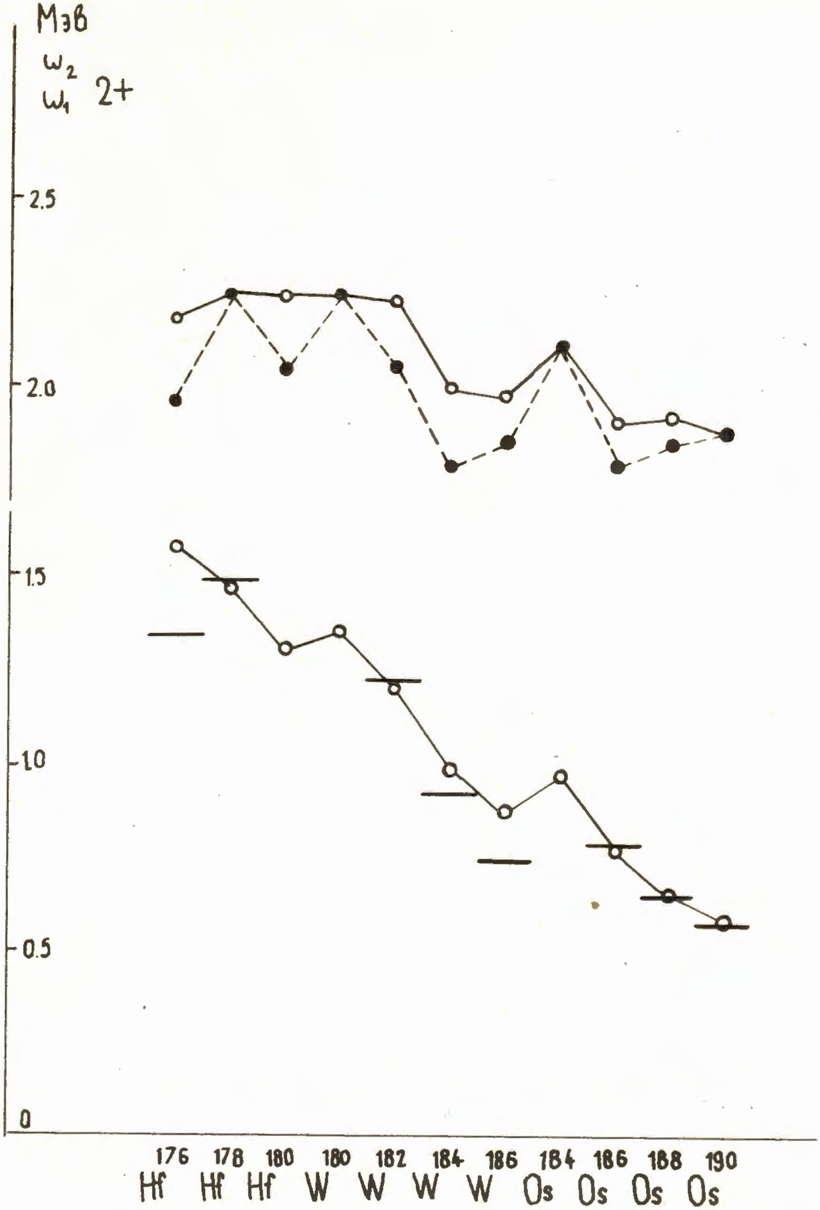

Fig.1. Energies of the first and second $K\pi = 2+$ states (full line) and of the first poles of the secular equation (broken line). The experimental values are denoted by short horizontal lines.

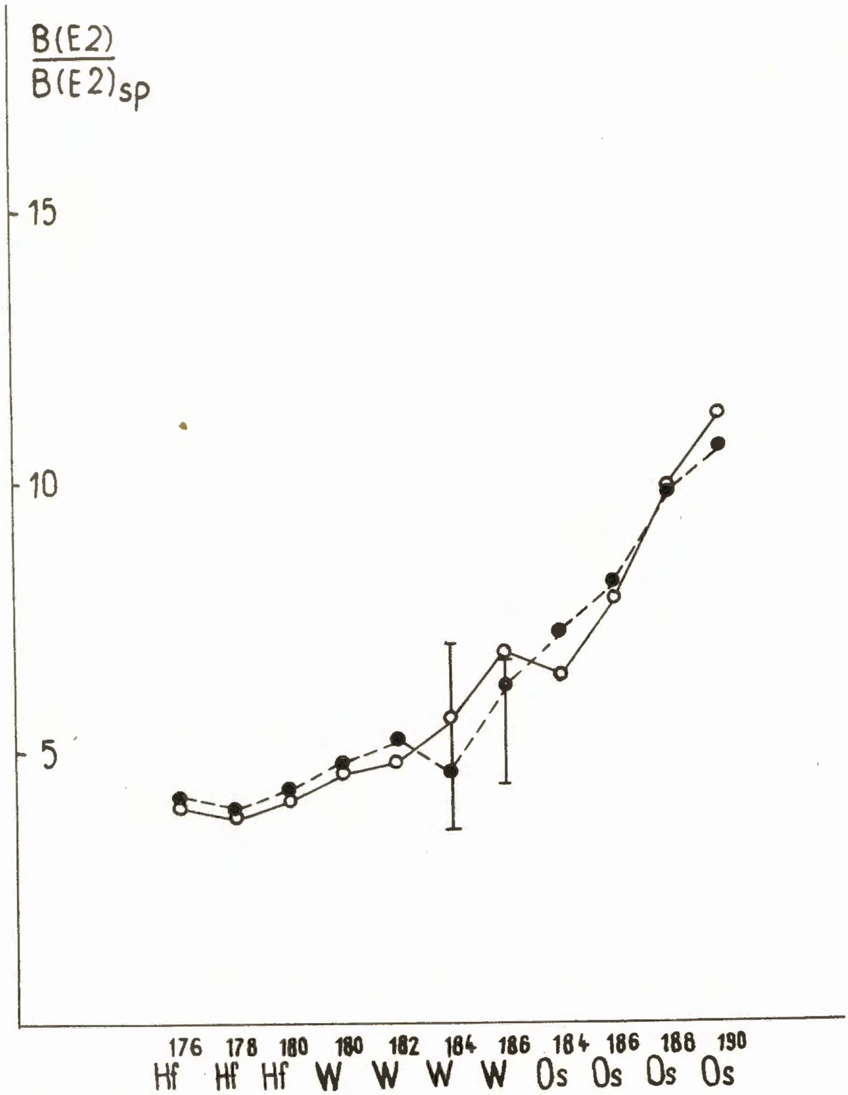

Fig.2. The quantities B(E2) for excitation of the $K\pi = 2+$ states in the single-particle units. The broken line is with blocking, full line without blocking.