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STRUCTURE OF THE GROUND AND EXCITED STATES OF ODD-MASS DEFORMED NUCLEI IN THE REGION $153 \leq A \leq 187$

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1. Introduction

The structure of the ground and excited states of odd-mass deformed nuclei is very simple in the independent quasi-particle model[1]. The ground state and some excited states have a single-quasi-particle structure. Further three-quasi-particle states of the two types follow: 1. (2n, p), (2 p, n) when there are two neutron quasi-particles and one proton quasi-particle or there are two proton quasi-particles and one neutron particle. 2. (3p), (3n) when the three quasi-particles are proton or neutron ones. Then, there must be five-quasi-particle states and so on.

As is known, the interactions of quasi-particles in the nucleus play an important role. In even nuclei the interactions between quasi-particles cause formation of collective non-rotational states which are described as one-phonon excitations. In the odd-mass nucleus there is one quasi-particle in addition to phonons and to quasi-particles of the even nucleus. The interaction of quasi-particles with the even nucleus phonons in odd-A nuclei was considered in refs.[2,3].

Secular equations were obtained the roots of which are the energies of the ground and excited states of odd-mass nuclei. The wave functions were found and it was shown that the interaction between quasi-particles and phonons leads to the appearance of admixtures to the one-quasi-particle states as well as to the formation of collective non-rotational states and complex structure states. A secular equation was studied and some general conclusions about the peculiarities of the collective non-rotational states in odd-mass deformed nuclei were drawn.

In the present paper a further investigation of the secular equations is performed. The energies of the non-rotational states of odd-mass deformed nuclei in the range $153 \leq A \leq 186$ are calculated. The structure of these states is studied. The reduced electromagnetic transition probabilities $B(E\lambda)$, the decoupling parameters and the spectroscopic factors in direct nuclear reactions are calculated.

2. Secular Equation Investigation

The structure of the collective non-rotational states of even deformed nuclei is rather well investigated[4-9]. As is known, the wave function of the collective states is a superposition of the wave functions of two-quasi particle and two-quasi-hole states. In this case, the more strongly is collectivized the state, the larger is the number of two-quasi-particle states which gives a noticeable contribution to the wave function of this state, and the more strongly is lowered the energy of this state with respect to the first pole corresponding to the energy of the two-quasi-particle states. In the ranges $150 \lesssim A \lesssim 190$ and $226 \lesssim A \lesssim 256$ the first $K\pi = 0+$, $2+$, $0-$ states of deformed nuclei are collective in the overwhelming majority of cases. The first $K\pi = 1-$, $2-$ states are weakly collectivized since the admixture of other states to the two-quasi-particle one corresponding to the first pole is about (2-20) percent. The energy and the structure of $K\pi = 0+$, $2+$, $0-$, $1-$ and $2-$ states for the first and second roots of the secular equation are given in ref./9/.

In the odd-mass deformed nucleus there is one quasi-particle in addition to the quasi-particles and the phonons of the even nucleus. Let us take into account the interaction between quasi-particles and phonons describing the even-nucleus collective states without going beyond the framework of the method of approximate second quantization.

The Hamiltonian of the system is written in the form:

$$H = \sum_s \epsilon(s)\, B(ss) + \sum_\nu \epsilon(\nu) B(\nu\nu) - \sum_{\lambda\mu i}{}' \mathfrak{L}_i^{\lambda\mu}\, Q_i(\lambda\mu)^+ Q_i(\lambda\mu) -$$

$$-\frac{1}{4} \sum_{\lambda\mu i} \frac{1}{\sqrt{Y_i(\lambda\mu)}} \Big\{ \sum_{ss'} \big(f^{\lambda\mu}(ss')\, B(ss') + \bar{f}^{\lambda\mu}(ss')\, \bar{B}(ss')\big) V_{ss'} +$$

$$+ \big(f^{\lambda\mu}(\nu\nu')\big)\, B(\nu\nu') + \bar{f}^{\lambda\mu}(\nu\nu')\, \bar{B}(\nu\nu')\big) V_{\nu\nu'} \Big\} \big(Q_i(\lambda\mu)^+ + Q_i(\lambda\mu) \big) + \text{h.c.}, \qquad (1)$$

where $Q_i(\lambda\mu)$ is the phonon operator of multipolarity λ with the projection μ, $\mathfrak{L}_i^{\lambda\mu}$ is the part of the multipole-multipole interaction taken into account in the method of approximate second quantization in the investigation of the collective states of even nuclei, an explicit form of $\mathfrak{L}_i^{\lambda\mu}$ is given in ref./9/. The quantity $\mathfrak{L}_i^{\lambda\mu}$ contains the $f^{\lambda\mu}(ss')$, $\bar{f}^{\lambda\mu}(ss')$ matrix elements of the multipole momentum operator $(\lambda\mu)$. $\omega_i^{\lambda\mu}$ is the energy of the collective non-rotational state of the even nucleus with A-1 nucleons. Here the sum is taken over the one-particle levels of the average field of the neutron (proton) system, $\epsilon(s) = \sqrt{C_n^2 + \{E(s) - \lambda_n\}^2}$, C_n is the correlation function, λ_n is the chemical potential in the neutron system:

$$U_{ss'} = u_s v_{s'} + u_{s'} v_s, \qquad V_{ss'} = u_s u_{s'} - v_s v_{s'},$$

$$u_s^2 = \frac{1}{2}\Big(1 - \frac{E(s) - \lambda_n}{\epsilon(s)}\Big), \qquad v_s^2 = 1 - u_s^2$$

$$B(ss') = \sum_\sigma a_{s\sigma}^+ a_{s'\sigma}, \qquad \bar{B}(ss') = \sum_\sigma \sigma\, a_{s\sigma}^+ a_{s'\sigma}$$

$\sigma=\pm 1$, $a_{s\sigma}$ is the quasi-particle operator.

$$Y^{I}(\lambda\mu) = \sum_{ss'} \frac{(f^{\lambda\mu}(ss')^2 + \bar{f}^{\lambda\mu}(ss')^2)\, U_{ss'}\, \omega_i^{\lambda\mu}(\epsilon(s)+\epsilon(s'))}{((\epsilon(s)+\epsilon(s'))^2 - (\omega_i^{\lambda\mu})^2)^2} + \\ + \sum_{\nu\nu'} \frac{(f^{\lambda\mu}(\nu\nu')^2 + \bar{f}^{\lambda\mu}(\nu\nu')^2)\, U_{ss'}\, \omega_i^{\lambda\mu}(\epsilon(\nu)+\epsilon(\nu'))}{((\epsilon(\nu)+\epsilon(\nu'))^2 - (\omega_i^{\lambda\mu})^2)^2}, \quad (2)$$

The quantity $Y^{I}(\lambda\mu)$ is comparatively small for the collective states and $Y^{I}(\lambda\mu)\to\infty$ when $\omega_i^{\lambda\mu}\to\epsilon(\nu)+\epsilon(\nu')$ i.e. it is very large for states close to the two-quasi-particle ones.

It should be noted that writting the Hamiltonian (1) we have used the secular equations determining $\omega_i^{\lambda\mu}$ the energies of collective state in even nucleus.

For $\kappa_n^{\lambda}=\kappa_p^{\lambda}=\kappa_{np}^{\lambda}\equiv\kappa^{(\lambda)}$ this equation is of the form

$$1 = 2\kappa^{(\lambda)}\Big\{ \sum_{ss'} \frac{(f^{\lambda\mu}(ss')^2 + \bar{f}^{\lambda\mu}(ss')^2)\, U^2_{ss'}}{\epsilon(s)+\epsilon(s') - \frac{(\omega_i^{\lambda\mu})^2}{\epsilon(s)+\epsilon(s')}} + \\ \sum_{\nu\nu'} \frac{(f^{\lambda\mu}(\nu\nu')^2 + \bar{f}^{\lambda\mu}(\nu\nu')^2)\, U^2_{\nu\nu'}}{\epsilon(\nu)+\epsilon(\nu') - \frac{(\omega_i^{\lambda\mu})^2}{\epsilon(\nu)+\epsilon(\nu')}} \quad (3)$$

The summation in eq. (1) over i is the summation over the roots of (3). Owing to (3) the last terms of eq.(1) do not contain the multipole-multipole interaction constant $\kappa^{(\lambda)}$.

To take into account the interaction of quasi-particles with phonons $\lambda=2$, $\mu=0$ it is necessary, in addition, to exclude the spurious state. For this, use is made of the two terms of the total hamiltonian $H'(n)$ and $H'(p)$, where

$$H'(n) = \frac{G_N}{\sqrt{2}} \sum_{ss'} (u_s^2 - v_s^2)\, u_{s'} v_{s'} \sum_i \{(\psi^i_{ss'}\, Q_i(20)^+ + \\ + \phi^i_{ss'}\, Q_i(20))\, B(s's') + B(s's')(\psi^i_{ss'}\, Q_i(20) + \phi^i_{ss'}\, Q_i(20)^+)\}, \quad (4)$$

(for the notations see refs.[8,9]).

Let us take into account the interaction of quasi-particles with phonons having different values of $\lambda\mu i$. The wave function for, e.g., an odd-proton nucleus which describes the states with the projection of the momentum on the nuclear symmetry axis K and the parity π , is written in the form

$$\Psi(K\pi) = \Omega(K\pi)^+\,\Psi_0 \quad (5)$$

where

$$Q_i(\lambda\mu)\Psi_o = 0 \qquad (6)$$

$$\Omega^+(K\pi) = \frac{C_\rho}{\sqrt{2}}\{\sum_\sigma a^+_{\rho\sigma} + \sum_{\lambda\mu i}\sum_{\nu\sigma} D^{\lambda\mu i}_{\rho\nu\sigma}\, a^+_{\nu\sigma} Q^+_i(\lambda\mu)\}, \qquad (7)$$

where ρ (or ρ_q) denotes the average field level with a given value of $K\pi$. The normalization condition (5) is written as follows:

$$C^2_\rho\{1 + \tfrac{1}{2}\sum_{\lambda\mu i}\sum_{\nu\sigma}(D^{\lambda\mu i}_{\rho\nu\sigma})^2\} = 1 \qquad (8)$$

Let us find the mean value of H in the state $\Psi(K\pi)$ and determine C_ρ and $D^{\lambda\mu i}_{\rho\nu\sigma}$ using the variational principles in the form

$$\delta\{<\Omega(K\pi) H \Omega^+(K\pi)> - \eta_i[C^2_\rho(1 + \tfrac{1}{2}\sum_{\lambda\mu i}\sum_{\nu\sigma}(D^{\lambda\mu i}_{\rho\nu\sigma})^2) - 1]\} = 0 \qquad (9)$$

where η_i is the Lagrangian multiplier. After some calculations we get the following secular equation/2/.

$$P(\eta) \equiv \tfrac{1}{4}\sum_{\lambda\mu i}\sum_\nu \frac{V^2_{\rho\nu}}{Y^i(\lambda\mu)}\frac{f^{\lambda\mu}(\rho\nu)^2 + \bar{f}^{\lambda\mu}(\rho\nu)^2}{\epsilon(\nu) + \omega^{\lambda\mu}_i - \eta_i} - (\epsilon(\rho) - \eta_i) = 0 \qquad (10)$$

The poles in (10) correspond to the sum of the quasi-particle energy $\epsilon(\nu)$ and the phonon one $\omega^{\lambda\mu}_i$, the roots of eq. (10) η_i determine the energies of the non-rotational states in odd-mass nuclei, i.e. if (9) is taken into account then $<\Omega(K\pi) H\Omega(K\pi)^+> = \eta_i$. The multiplier $V^2_{\rho\nu}$ points out that the particle-particle and hole-hole-interactions are preferred to the particle-hole ones. It should be noted that the terms in (10) with $\lambda > 3$ and $i > 2$ give a very small contribution since the quantity $Y^i(\lambda\mu)^{-1}$ tends to zero when the corresponding even nucleus state approaches the two-quasi-particle one.

The behaviour of $P(\eta)$ for $K\pi$=3/2-states in ^{171}Yb is given in Fig. 1, where the dot-and-dash line denotes $P(\eta)$ for ρ=521↑, ϵ(521↑)= 0,317$\hbar\mathring{\omega}_o$ x) the dashed line is $P(\eta)$ for ρ=512↓, ϵ(512↓) = 0,405$\hbar\mathring{\omega}_o$. The first pole corresponds to ϵ(521↓)+ω^{22}_1 = = 0.34 $\hbar\mathring{\omega}_o$, the second one to ϵ(521)+ω^{22}_2 = 0.340$\hbar\mathring{\omega}_o$. From Fig. 1 it is seen that $P(\eta)$ has up to the first pole, one root, between the first and the second poles the second root and so on.

Using the normalization condition (8) the functions C_ρ and $D^{\lambda\mu i}_{\rho\nu\sigma}$ can be written in the form

$$C^{-2}_\rho = 1 + \tfrac{1}{4}\sum_{\lambda\mu i}\sum_\nu \frac{V^2_{\rho\nu}}{Y^i(\lambda\mu)}\frac{f^{\lambda\mu}(\rho\nu)^2 + \bar{f}^{\lambda\mu}(\rho\nu)^2}{(\epsilon(\nu) + \omega^{\lambda\mu}_i - \eta_i)^2}, \qquad (11)$$

x) By $Nn_z\Lambda$↑ we denote the $K=\Lambda+\Sigma$ state of the Nilsson potential, by $Nn_z\Lambda$↓ the $K=\Lambda-\Sigma$ one.

$$D^{\lambda\mu}_{\rho\nu\sigma} = \tfrac{1}{2}\frac{V_{\rho\nu}}{\sqrt{Y^i(\lambda\mu)}}\frac{f^{\lambda\mu}(\rho\nu) - \sigma\bar{f}^{\lambda\mu}(\rho\nu)}{\epsilon(\nu) + \omega^{\lambda\mu}_i - \eta_i} \qquad (12)$$

The quantity C^2_ρ determines the contribution of the one-quasi-particle state with a given ρ to the state under consideration. If $C^2_\rho \approx 1$ then the state is a one-quasi-particle state, if C^2_ρ is somewhat smaller than unity then the state possesses a complicated structure and if $C^2_\rho \ll 1$ then, as a rule the state is a collective one. The quantity $C^2_\rho(D^{\lambda\mu i}_{\rho\nu})^2$ determines the contribution of the component with a quasi-particle in the ν-state plus phonon $\lambda\mu i$ to the wave function $\Psi(K\pi)$. Here

$$(D^{\lambda\mu i}_{\rho\nu})^2 = \tfrac{1}{2}\sum_\sigma(D^{\lambda\mu i}_{\rho\nu\sigma})^2 = \tfrac{1}{4}\frac{V^2_{\rho\nu}}{Y^i(\lambda\mu)}\frac{f^{\lambda\mu}(\rho\nu)^2 + \bar{f}^{\lambda\mu}(\rho\nu)^2}{(\epsilon(\nu) + \omega^{\lambda\mu}_i - \eta_i)^2} \qquad (12)$$

It should be noted that the secular equation is derived under the assumption that $[a_{\nu\sigma}, Q_i(\lambda\mu)] = 0$ i.e. it is believed that the phonon is a boson and the fact that $Q_i(\lambda\mu)$ is a superposition of the operators $a_{\nu\sigma}a_{\nu'\sigma'}$, $a^+_{\nu\sigma}a^+_{\nu'\sigma'}$ is neglecte The phonon-phonon scattering and the Coriolis forces which are of importance in some cases are neglected, as well.

The secular equation (10) can be, in some respects, improved. So, the values of $\epsilon(\rho)$ should be calculated using the values of the correlation function $C(\rho)$ and of the chemical potential $\lambda(\rho)$ for a given ρ state of the system consisting of an odd number of particles, or replacing in eq.(10). $\epsilon(\rho)$ by $\mathcal{E}(\rho) - \mathcal{E}_o$ i.e. the difference of the energies (reckons from the corresponding λ, $\lambda(\rho)$) of the system consisting of N+1 particles with a quasi-particle in the ρ state and of the ground state of the N particle system. Further we can take into account the influence of the blocking effect on the phonons i.e. calculate the quantities $\omega^{\lambda\mu}_i$ and $Y^i(\lambda\mu)$ starting from the fact, that the wave function of the ground state is $a^+_{\nu\sigma}\Psi_o$ and of the excited one is $Q^+_i(\lambda\mu)a^+_{\nu\sigma}\Psi_o$. Due to the Pauli principle the phonon operator $Q_i(\lambda\mu)^+$ contains in this case, no component with a quasi-particle $a^+_{\nu\sigma}$ and the quantities $\omega^{\lambda\mu}_i$ and $Y^i(\lambda\mu)$ weakly depend on ν_1. Corrections of such a type are taken into consideration in ref./10

Let us use eq. (4) and exclude the spurious state, then the secular equatic reads

$$\epsilon(\rho) - \eta - \tfrac{1}{4}\sum_{\lambda\mu i}\sum_{\nu\neq\rho}\frac{V^2_{\rho\nu}}{Y^i(\lambda\mu)}\frac{f^{\lambda\mu}(\rho\nu)^2 + \bar{f}^{\lambda\mu}(\rho\nu)^2}{\epsilon(\nu) + \omega^{\lambda\mu}_i - \eta_i} -$$

$$-\tfrac{1}{4}\sum_i \frac{1}{Y^i(20)}\frac{1}{\epsilon(\rho) + \omega^{20}_i - \eta_i}\frac{1}{\epsilon(\rho)^2}\{f^{20}(\rho\rho)(E(\rho) - \lambda) -$$

$$-\frac{4C^2_\rho}{Y^i_\rho}\sum_{\nu\nu'}\frac{f^{20}(\nu\nu)}{\epsilon(\nu)(4\epsilon(\nu) - (\omega^{20}_i)^2)}\frac{E(\nu) - E(\nu')}{\epsilon(\nu')(4\epsilon(\nu') - (\omega^{20}_i)^2)}\}^2 = 0 \qquad (13)$$

all the notations are taken from ref./8,9/ . However, if the contribution in (13) of the terms with $\lambda=2$, $\mu=0$ is small, then eq. (10) can be used.

We make a further improvement of the secular equation (10). We bear in mind, that there are several one-particle states with a given $K\pi$ and take the wave function $\Psi(K\pi)$ in the form

$$\Psi(K\pi)=\Omega(K\pi\rho_1 \dots \rho_n)^+ \Psi_0 \tag{14}$$

where

$$\Omega(K\pi\rho_1\dots\rho_n)^+ = \frac{1}{\sqrt{2}} N(\rho_1\dots\rho_n)\sum_\sigma \{ C_{\rho_1} a^+_{\rho_1\sigma} + \dots + C_{\rho_n} a^+_{\rho_n\sigma} + \sum_{\lambda\mu i\nu} D^{\lambda\mu i}_{\nu\sigma}(\rho_1\dots\rho_n) a^+_{\nu\sigma} Q_i(\lambda\mu)^+ \} \tag{14'}$$

This general case is considered in ref./2/ . In the present paper we study a particular case of two- one-particle states ρ_1 and ρ_2 with a given $K\pi$. The wave function is written in the form:

$$\Psi(K\pi)=\Omega(K\pi\rho_1\rho_2)^+\Psi_0 \tag{15}$$

where
$$\Omega(K\pi\rho_1\rho_2)^+ = \frac{1}{\sqrt{2}} N(\rho_1\rho_2)\sum_\sigma \{ C'_{\rho_1} a^+_{\rho_1\sigma} + C'_{\rho_2} a^+_{\rho_2\sigma} + \sum_{\lambda\mu i\nu} D^{\lambda\mu i}_{\nu\sigma}(\rho_1\rho_2) a^+_{\nu\sigma} Q_i(\lambda\mu)^+ \} . \tag{15'}$$

Using the variational principle we obtain a secular equation in the form:

$$P(\eta) \equiv \begin{vmatrix} V_i(\rho_1\rho_1)-(\epsilon(\rho_1)-\eta_i) & V_i(\rho_1\rho_2) \\ V_i(\rho_1\rho_2) & V_i(\rho_2\rho_2)-(\epsilon(\rho_2)-\eta_i) \end{vmatrix} = 0 , \tag{16}$$

where

$$V_i(\rho_q,\rho_n) = \tfrac{1}{4}\sum_{\lambda\mu i\nu} \frac{V_{\rho_q\nu} V_{\rho_n\nu}}{Y^i(\lambda\mu)} \frac{f^{\lambda\mu}(\rho_q\nu)\, f^{\lambda\mu}(\rho_n\nu) + \bar{f}^{\lambda\mu}(\rho_q\nu)\,\bar{f}^{\lambda\mu}(\rho_n\nu)}{\epsilon(\nu)+\omega^{\lambda\mu}_i - \eta_i} \tag{17}.$$

Note that in eq. (16) there are poles of the first order only. C'_{ρ_1} and C'_{ρ_2} are taken in a symmetric form:

$$C'_{\rho_1} = 1 - \frac{V_i(\rho_1\rho_2)}{V_i(\rho_1\rho_1)-(\epsilon(\rho_1)-\eta_i)} \tag{18}$$

$$C'_{\rho_2} = 1 - \frac{V_i(\rho_1\rho_2)}{V_i(\rho_2\rho_2)-(\epsilon(\rho_2)-\eta_i)} \tag{18'}$$

then for $D^{\lambda\mu i}_{\nu\sigma}(\rho_1\rho_2)$ we have

$$D^{\lambda\mu i}_{\nu\sigma}(\rho_1\rho_2) = C'_{\rho_1} D^{\lambda\mu i}_{\rho_1\nu\sigma} + C'_{\rho_2} D^{\lambda\mu i}_{\rho_2\nu\sigma} \tag{20}$$

$$N(\rho_1\rho_2)^{-2} = C'_{\rho_1}\left(1+\frac{\partial V_1(\rho_1\rho_1)}{\partial \eta_1}\right) + C'_{\rho_2}\left(1+\frac{\partial V(\rho_2\rho_2)}{\partial \eta_1}\right)$$

$$+ 2C'_{\rho_1}C'_{\rho_2}\frac{\partial V(\rho_1\rho_2)}{\partial \eta_1} \qquad (20)$$

The contribution of the one-particle state ρ to the wave function (15) is $N(\rho_1\rho_2)^2 C'^2_{\rho_1}$.

Let us investigate the peculiarities of eq. (16) and compare them with the secular equation (10). Consider the function $V_1(\rho_q,\rho_n)$ for the values of η which are noticeably smaller than the first pole in eq. (16). The function $V_1(\rho_q,\rho_n)$ is a sum of a large number of terms taken over $\lambda\mu i$ and ν, however, the main contribution is given by terms the poles of which are not so far removed, the matrix elements of which are large, $V_{\rho\nu} \approx 1$ and $Y^i(\lambda\mu)$ are small. The diagonal functions $V_i(\rho_q,\rho_q)$ are the sum of only positive terms. The non-diagonal functions $V_1(\rho_1,\rho_2)$ are far smaller than the diagonal ones because, firstly $V_1(\rho_1,\rho_2)$ is a sum changing the sign, secondly, conditions under which the term in $V_1(\rho_1\rho_2)$ is large is not simultaneously fulfilled for ρ_1 and ρ_2. Therefore the determinant (16) turns actually into the product of diagonal terms only and we are led to the product of eqs. (10), one for ρ_1 and the other for ρ_2. If the value of η is rather close to that of the first pole then in the sum $V(\rho_q,\rho_n)$ the predominant role is played by the pole term. In this case the poles of the second order in eq. (16) are cancelled and the function $P(\eta)$ has, at the point $\epsilon(\nu)+\omega_1^{\lambda\mu}$ a pole of the first order. Then we are led again to an equation of the type (10). But eq. (16) does not turn into the product of eqs.(10), for large η. Besides, it is necessary to analyse the solutions of eqs. (16) in order to exclude false solutions, which appear in solving eqs. (10) for ρ_1 and ρ_2. Fig. 1 gives the function $P(\eta)$ for $\rho_1+\rho_2 = 521\uparrow + 512\downarrow$. From Fig. 1 it is seen that, in spite of the fact that the behaviour of the function $P(\eta)$ for $512\downarrow + 521\uparrow$ essentially differs from both curves $P(\eta)$ for $521\uparrow$ and $P(\eta)$ for $512\downarrow$ the roots of eq. (16) actually coincide with those of the (10). However, there are two false solutions for $512\downarrow$ when the values of η are close to those of the first and second poles. In the considered case $\epsilon(521\uparrow)$ is smaller than the first pole, and $\epsilon(512\downarrow)$ is larger than the second pole. We find the structure of the first three states; $\eta_1 = 0.275\, \hbar\overset{\circ}{\omega}_0$, the wave function contains 70 percent of the one-quasiparticle $521\uparrow$ state 25 percent of the $521\downarrow + Q_1(22)$ state, 2 percent of the $651\uparrow + Q_1(30)$ state and so on, $\eta_2 = 0.334\, \hbar\overset{\circ}{\omega}_0$ and the wave function contains 67 percent of the $512\downarrow$ state 20 percent of the $514\downarrow + Q_1(22)$ state, 5 percent of the $510\uparrow + Q_1(22)$ state and so on, $\eta_3 = 0.338\, \hbar\overset{\circ}{\omega}_0$ and the wave function contains 11 percent of the 521↑ state, .6 percent of the 512↓ state, 49 percent of the 521↓ + $Q_1(22)$ state, 30 percent of the 521↓ + $Q_2(22)$ state, 2 percent of the 514↓ + $Q_1(22)$ state and so on. From here it is seen that both one-quasi-particle states ρ_1 and ρ_2 contribute to the wave function corresponding to the third roots. The solutions of the equations such as (16) with the determinant of higher order have a similar structure.

The smallest roots of eqs. (10), and (16) as a rule, coincide. Some rules can be formulated for excluding false solutions of (10), not solving eq. (16). So, the solution of eq. (10) for ρ_1 is false if the root is very close to the pole $\epsilon(\nu) + \omega_1^{\lambda\mu}$ and the matrix element $f^{\lambda\mu}(\rho_1\nu)$ is small and at the same time there is a large matrix element $f^{\lambda\mu}(\rho_2\nu)$ which connects the given pole with another one-particle state ρ_2 having the same $K\pi$ as ρ_1. It should be noted that solutions of eqs. (10) somewhat removed from the poles are not usually false.

Thus, the main role of eq. (16) is the exclusion of false solutions of (10) and the determination of the structure of higher states with a given $K\pi$. The study of eq. (16) has led to the improvement of the conclusions about the position of the collective non-rotational states which were formulated in ref. /2/ basing on the analysis of eq. (10).

The secular equations (10) and (16) have not a single free parameter and the arbitrariness consists only in that how many values of $\lambda\mu$ and roots i should be taken into account. We obtained the lowest roots (as a rule, the first two or three roots) of eq. (10) and (16) for nuclei in the range $153 \lesssim A \lesssim 187$. As the quantity $\epsilon(\rho)$ we take the difference of the energies of an odd-mass nucleus in the ρ state (calculated taking into account the blocking effect) and of an even nucleus. The first and second roots ($i=1,2$) of the quadrupole $K\pi$ 0+ and 2+ and of the octupole $K\pi$ = 0-, 1- and 2- states, were taken into account in the calculations. The investigations showed that the lowering of the energies with respect to the one-quasi-particle values of $\epsilon(\rho)$ as well as with respect to the first poles $\epsilon(\nu)+\omega_i^{\lambda\mu}$ is mainly due to the terms of (10) and (16) with $\lambda=2, \mu=2, i=1$ and $\lambda=3, \mu=0, i=1$. The terms with $\lambda=2, \mu=2, i=1$ are of importance almost in all the nuclei, the terms $\lambda=3, \mu=0, i=1$ play an essential role in the beginning of the region of deformation and in the region of the isotopes of Yb-Hf. In some cases, e.g., for ½+, ρ = 660↑ states, 3/2+ , ρ = 651↑ state and 5/2- , ρ = 512↑ state in ^{161}Dy and in other ones the terms with $\lambda=3, \mu=0, i=1$ play a predominant role. In some other cases the terms in eqs. (10) and (16) with $\lambda=2, \mu=2, i=2$ and with $\lambda=2, \mu=0, i=1$ and others are important. So, the terms with $\lambda=2, \mu=2, i=2$ are of importance for the first $K\pi=5/2-$ and

$\frac{1}{2}-$ states in ^{175}Hf and the terms with $\lambda=2, \mu=0, i=1$ in $K\pi = \frac{1}{2}+$ states in ^{161}Dy and ^{155}Sm and so on.

The positive parity states in ^{157}Tb and ^{159}Tb up to 1.6 MeV are shown in Table 1. The energies, the structure and the lowering of the energies of these states with respect to the one-quasi-particle values $\epsilon(\rho)$ as well as with respect to the first poles $\epsilon(\nu) + \omega_i^{\lambda\mu}$ (if their energies are not higher than $\epsilon(\rho)$ or $\epsilon(\nu) + \omega^{\lambda\mu}$) are given. The experimental values of the energies are taken from refs. /11, 12, 13/. From this table it is seen that for a number of states the terms with $\lambda=2, \mu=0, i=1$ are very important.

It should be noted that for odd proton nuclei the states with positive parity are usually more strongly collectivized than those with negative parity. For odd neutron-nuclei, on the contrary, the states with negative parity are more strongly collectivized, on the average, than those with positive parity.

Thus, basing on the investigations performed we may prove the conclusion drawn earlier in ref. /3/ that those approximations which take into account (although more accurately /10/, that in our case) only phonons with $\lambda=2, \mu=2, i=1$ and neglect the remaining (first of all with $\lambda=3, \mu=0, i=1$) are rather rough for most nuclei in the range $153 \leq A \leq 187$. Nevertheless, even in the cases when the terms with $\lambda=2, \mu=2, i=1$ play a main role, the account of the terms with other $\lambda\mu i$ is necessary since in some cases the lowest pole corresponding to a phonon $\lambda\mu i$ which differ from $\lambda=2, \mu=2, i=1$ can noticeably change the energy of the calculated state. There is no necessity to take into account in eqs. (10) and (16) terms with $\lambda > 3$ and $i > 2$ since the total contribution of such terms is very small.

3. Odd-Mass Nucleus States Close to the One-Quasi-Particle Ones

The analysis of the secular equations (10) and (16) shows that if η_1 is very close to $\epsilon(\rho)$ then the state will be actually of the one-quasi-particle type. If η_1 noticeably differs from $\epsilon(\rho)$ and from the first pole $\epsilon(\nu) + \omega_1^{\lambda\mu}$ the structure of such a state is very complicated since the contribution to the wave function is given not only by the one-quasi-particle states but also by many states with different quasi-particles and phonons. If η_1 is close to the first pole of the secular equation then the state is collective. So, if η_1 approaches the pole, i.e.

$$\Psi(K\pi)\,|_{\epsilon(\nu)+\omega_1^{\lambda\mu}-\eta_1 \to 0} = \sum_{\sigma} \frac{1}{\sqrt{2}}\, \alpha^{+}_{\nu\sigma}\, Q^{+}_{1}(\lambda\mu)\, \Psi_0 \,, \qquad (21)$$

then this state may be called a gamma-vibrational one, if $\lambda = 2$, $\mu = 2$ or an octupole one, if $\lambda = 3$, $\mu = 0$ and so on.

Let us consider the states in odd deformed nuclei which are close in their structure to the one-quasiparticle states. The contribution of the ρ state to the wave functions is predominant and the quantity C_ρ^2 is somewhat smaller than unity. The state possesses a structure similar to the one-quasi-particle one, if the following condition is fulfilled

$$\epsilon(\rho) \ll \min \{ \epsilon(\nu) + \omega_1^{\lambda\mu} \} , \tag{22}$$

i.e. when the quasi-particle energy is much lower than the value of the secular equation first pole. In some cases there are comparatively high-lying states close to the one-quasi-particle ones when η is close to $\epsilon(\rho)$ and the condition (22) is not fulfilled. In the case (22) $K = 11/2, 9/2$ states and partially 7/2 states are similar to the one-quasi-particle ones and the contribution of the ρ state to their wave function is, as a rule, (95-99) percent /3/. For example, the contribution of 523↑ one quasi-particle states to $K\pi = 7/2-$ state is 99 percent in ^{161}Tb, 98 percent in ^{159}Tb, and in ^{157}Tb and 97 percent in ^{155}Tb. $K = 1/2, 3/2, 5/2$ states are rather close to the one-quasi-particle states and the contribution of ρ state is (97-99) percent. The admixtures of the states quasi-particle plus phonon are more important and their energies are more strongly lowered with respect to $\epsilon(\rho)$ than for $K = 11/2, 9/2$ states. For example, as is seen from Table 1 the contribution to the ground $K\pi = 3/2+$ state of the one-quasi-particle 411↓ state in ^{157}Tb is 90 percent and $\epsilon(\rho) - \eta_1 = 200$ keV.

The calculations of the levels of odd-mass deformed nuclei in the range $153 \leq A \leq 187$ showed that the interactions of quasi-particles with phonons lead to a different decrease with respect to $\epsilon(\rho)$ of the energies of the states close to the one-quasi-particle states. Therefore in a number of nuclei the calculated succession of the excited states differs from the succession of the Nilsson scheme levels.

So, in the Lu and Ta isotopes the $K\pi = 9/2-$ -state is very close to the one-particle 514↑ state (the contribution of the ρ state is larger than 99 percent) and the lowering ϵ (514↑)- $\eta \approx$ (10 - 20) keV. At the same time in these nuclei the admixtures in $K\pi = 7/2+$ state close to 404↓ one are somewhat more important, since the contribution of ρ is of the order of 97 percent and the lowering is ϵ (404↓) $- \eta_1 \approx$ 50 - 100 keV. Therefore in all the Lu isotopes and in the Ta isotopes with A = 177, 179 and 181, 7/2+ 404↓ state is a ground one and 9/2 - 514↑ is an excited one.

Another example: in nuclei with N=91, according to the Nilsson scheme 11/2--505↑ state should be ground one, while according to our calculation, in ^{153}Sm 3/2 + 651↑ state is the ground one and in ^{155}Gd the 3/2 - 521↑ state, what agrees with experiment. Thus, the 11/2 - 505↑ state is not a ground state in nuclei with N = 91 - 93, although ϵ (505↑) $- \eta_1 \approx$ 100 keV.

Thus, the interaction of quasi-particles with phonons weakly affects K = 11/2, 9/2 states close to the one-quasi-particle ones and more strongly affects the states with smaller K. As a result K = 11/2, 9/2 states are not ground in odd-mass deformed nuclei if in the average field level scheme there are levels with smaller K, near these states.

We have calculated the energies of the levels close to the one-quasi-particle ones and their structure for a large number of odd-mass nuclei in the range $151 \leq A \leq 187$. The average field level scheme for δ = 0.3. was used[9] in which the following changes are introduced: in the neutron system 505↑ state is raised by 0.15 $\hbar \mathring{\omega}_0$, 651↑ state is lowered by 0.05 $\hbar \mathring{\omega}_0$ and 660↑ state by 0.10 $\hbar \mathring{\omega}_0$, in the proton system 404↑, 422↑ states and 404↓, 514↑ states interchanged their places and 541↓ state is lowered by 0.13 $\hbar \mathring{\omega}_0$. The values of $\omega_i^{\lambda\mu}$ and $Y^i(\lambda\mu)$ are recalculated according to the modified scheme, but in most cases they are close to those in ref.[9]. The Re isotopes are calculated for the deformation $\delta = 0.2$ according to the scheme and the values of $\omega_i^{\lambda\mu}$, $Y^i(\lambda\mu)$ obtained in ref.[14].

Some results of calculations are given in Table 2, namely the experimental and calculated values of the energies, $\epsilon(\rho) - \epsilon(K_0)$ ($\epsilon(K_0)$ is related to the ground state) excitation energies in the independent quasi-particle model (taking into account the blocking effect) and the structure of these states. The experimental data are taken from refs.[13, 15-22]. From the table it is seen that in some cases, even in comparatively strongly excited states the admixtures are not so important, e.g. in the $K\pi$ = 1/2+ with energy 612 keV in ^{181}Ta the contribution of 411↓state is 95 percent, in K = 7/2+ state with energy 995 in ^{175}Yb the contribution of 633↑ state is 98 percent and so on.

The calculated energies of the levels close to the one-quasi-particle ones somewhat better agree (especially high excited ones) with experimental data than those calculated according to the independent quasi-particle model taking into account the blocking effect[23,24]. However, this agreement is not sufficiently good since it depends on the position of the average field levels. In a number of cases the Coriolis interaction which is neglected by us could be very important.

The change in the energies of excited non-rotational states in odd neutron nuclei in the transition from one nucleus with a given N to another (or in different isotopes in odd proton nuclei) is due to many causes: change in the average field levels, change in the equilibrium deformation, interaction of quasi-particles with phonons (due to the change in the values of $\omega_i^{\lambda\mu}$, $Y^i(\lambda\mu)$) and others. In some cases the change of the energy in different isotopes or isobars may be due to the change of the equilibrium deformation of the nucleus in the excited state as compared to the ground one. So, the calculations made in ref./25/, showed that such a situation takes place for some states in a number of odd-odd nuclei in the transuranium region. Since the interaction of quasi-particles is one of many causes then using it one does not succeed in explaining the change of the energies of the levels for different isotopes and isobars, e.g. the behaviour of the states close to 541↓ in the Lu isotopes.

The fact that the states in odd-mass nuclei are not purely quasi-particle ones is displayed in the beta decay probabilities, in the magnitudes of the spectroscopic factors in direct nuclear reactions, in the values of the decoupling parameters a for $K = 1/2$ states and so on. Let us consider, as an example, the beta decay from 3/2 -411 state in ^{161}Tb to 1/2 -521 state in ^{161}Dy for which $\log ft = 8{,}2$ /26/. The correction due to pairing correlations $R = 0.06$/24/. (This transition is strictly forbidden in the independent particle model). The values of C_ρ^2 are equal to 0.93 for 411↑ state and to 0.53 for 521↓ state, therefore $R\, C^2_{\rho=411\uparrow} \cdot C^2_{\rho=521\downarrow} = 0.03$, thus this transition is hindered about 30 times.

Our investigations show that when a given average field level ρ is near the Fermi surface then the admixtures in a state close to that one-quasi-particle state are, as a rule, the smallest ones and C_ρ^2 is close to unity. As the level ρ moves away from the Fermi surface, i.e. as the excitation energy increases, the role of the admixtures in the state with a corresponding $K\pi$ becomes greater and the quantity C_ρ^2 decreases. This peculiarity can be seen from the change of the decoupling parameter and spectroscopic factors.

We investigate the influence of the interaction of quasi-particles with phonons on the decoupling parameter a for $K = 1/2$ state. Using the wave function of the state in the form (5) we get for the decoupling parameter the following expression:

$$a = C_\rho^2 \{ a^N_{\rho\rho} + \sum_{\nu\nu' i} a^{201}_{\nu\nu'} D^{201}_{\rho\nu} D^{201}_{\rho\nu'} - \\ - \sum_{\nu\nu' i} a^N_{\nu\nu'} D^{30i}_{\rho\nu} D^{30i}_{\rho\nu'} \} , \qquad (23)$$

where $a^N_{\rho\rho}$, $a^N_{\nu\nu'}$ are the decoupling parameters calculated with the Nilsson wave functions /15/. For the states (21) close to the pole with $\mu \neq 0$ the quantity a is equal to zero. The role of the second and the third term (23) is, in most cases, small.

It should be noted that if in the commutator $[Q_i(\lambda\mu), a_{\nu\sigma}]$ we take into account that the phonon operator is a superposition of the quasi-particle operators, then in the expression for a there appear terms linear in the admixture, which will lead to an additional change of a .

Let us consider the changes of a for $K\pi = 1/2-$ state close to the one-quasi-particle state 521↓ in the transition from nuclei, where this state is a particle one, to nuclei, where it is ground and further hole excited one. Table 3 gives the experimental and calculated values of the energies of these states, the experimental /27/ and calculated values of a as well as the quantity C^2_ρ describing the contribution of the one-quasi-particle state. The table gives also the difference of the energies $\epsilon(\rho)-\epsilon(K_o)$ calculated in the independent quasi-particle model, taking into account the blocking effect ($\epsilon(K_o)$ is the value for the ground state). If the considered state is believed to be a pure one-quasi-particle 521↓ state, the values of a calculated with the Nilsson wave functions for $\delta = 0{,}3$ are $a^N = 0.89$. From the table it is seen that when 521↓ state lies on the Fermi surface then the admixture are small $C^2_\rho = 0.96 - 0.99$ and a are close to a^N . In the cases, when 521↓ state is a particle excited one the admixture to it of states of the quasi-particle plus phonon type is large and a is far smaller than a^N. From Table 3 it is seen that the account of the interaction of quasi-particles with phonons allowed to explain changes in the behaviour of a for states close to 521↓ ones in different nuclei.

However, in some cases the account of the interactions of quasi-particles with phonons does not lead to the elimination of discrepancies between the calculated and experimental values of the decoupling parameters a . For instance, for 510↑ state $a^N = -0.2$ while in ^{183}W, $a = 0.19$ and the account of the interactions of quasi-particles with phonons does not eliminate this disagreement. As is shown in ref. /28/ only a noticeable change of the Nilsson potential parameters leads to the elimination of this disagreement. In some other cases, e.g., for 411↓ state in odd proton nuclei the experimentally determined values of a little differ from a^N and the interaction of quasi-particle with phonons is not displayed so effectively as, e.g., for states close to 521↓ ones.

We consider the effect of the admixtures on the spectroscopic factors in direct nuclear reactions. So, when the one-quasi-particle state ρ is excited in the (dp) reaction on an even A target the spectroscopic factor is u^2_ρ. If we take into account the admixtures, i.e. the wave function is assumed to have the form of eq. (5), then the spectroscopic factor is $C^2_\rho u^2_\rho$. Table 4 gives the calculated values of the spectroscopic factors for excited $K\pi = 1/2-$ states close to $510\uparrow$ ones in the Yb and Hf isotopes, the values of C^2_ρ as well as the energies of these states. The calculated values of the spectroscopic factor $C^2_\rho u^2_\rho$ correctly reproduce the behaviour of the cross sections for the (d, p) reactions/19/, the decrease of $C^2_\rho u^2_\rho$ in the light Yb isotopes being due to the decrease of C^2_ρ.

Thus, the interaction of quasi-particles with phonons in some cases essentially affects states close to the one-quasi-particle ones in odd deformed nuclei and it should be taken into account in investigating excited states.

4. Collective Non-Rotational States

Let us consider another limiting case, when the energy corresponding to the first pole (10) or (16) is much lower than the one-quasi-particle one, i.e.

$$\min\{(\epsilon(\nu)+\omega_1^{\lambda\mu}\} \ll \epsilon(\rho) . \qquad (24)$$

In this case the term (10), (16) corresponding to the first pole plays a predominant role in $K=11/2$, $9/2$ states, the latter have a structure: quasi-particle plus phonon. It is only in this case that we may use the words gamma-vibrational, octupole and so on states in odd deformed nuclei. The contribution of ρ to these states is $C^2_\rho = 0.001 - 0.050$. So, $K\pi = 11/2$-state in ^{165}Ho with an energy 687 keV is gamma-vibrational one. The calculated energy is 850 keV, the lowering with respect to $\epsilon(523\uparrow) + \omega_1^{22} = 1700$ keV is 3 keV, $\epsilon(\rho = 505\uparrow) = 4500$ keV, $C^2_\rho = 0.001$.

In the case (24) $K = 1/2$, $3/2$ states have a somewhat more complicated nature, since in eq. (10) (16) several terms with different λ and μ play often an important role. The contribution of the one-quasi-particle state ρ is $C^2_\rho = 0.01 - 0.10$. $K = 5/2$, $7/2$ states occupy an intermediate position. The state is collective if its energy is very close to the pole, for high states this can occur when eq. (24) is not fulfilled.

The most frequent is the intermediate case

$$\epsilon(\rho) = (0.5 - 2.0)\min\{\epsilon(\nu)+\omega_1^{\lambda\mu}\} \qquad (25)$$

Here the interaction between quasi-particles and phonons is most effective, it causes the strongest lowering of the roots of eqs. (10), (16) both with respect

to $\epsilon(\rho)$ and to the first pole of the secular equation. The contribution of the one-quasi-particle state is $C^2_\rho = 0.3 - 0.8$. The secular equation contains many terms with different $\lambda\mu i$ and ν which are important. The energies of $K = 1/2$, $3/2$ states are lowered with respect to $\epsilon(\rho)$ and to the first pole more strongly than the $K = 9/2$, $11/2$ state energies. Thus, in the case (25) the interaction of quasi-particles with phonons leads to the formation in odd deformed nuclei of collective non-rotational states having a complex structure.

The important quantity which characterizes the structure of an excited state is the reduced probability of the electromagnetic transition. So, the increase of the reduced probability $B(E2)$ for the electric E2 transition as compared to the one-particle value points out that the wave function of $K=K_0-2$ or $K=K_0+2$ states (K_0 is related to the ground nucleus state) has an appreciable admixture of the component quasi-particle plus phonon $\lambda=2$, $\mu=2$. An increase of $B(E3)$ as compared to the one-particle value points to a noticeable admixture of the component quasi-particle plus octupole phonon and so on.

For the reduced probability of the electrical E2 transition between the two states (7) we get:

$$B(E2, IK \to I'K') = |(I2\,K\,K'-K\,|\,I'K')\langle K'|\mathfrak{M}(2,K'-K)|K\rangle + (I2K\,-K'-K\,|\,I'-K')(-1)^{I'-K'}\langle K'R_i^{-1}|\mathfrak{M}(2,-K'-K)|K\rangle|^2 \qquad (26)$$

where $\langle K'|\mathfrak{M}(2\nu)|K\rangle$ is the matrix element of the ν component of E2-transition operator, R_i is the operator of rotation at 180° around the axis 2. Though actually eq. (26) contains two terms, in all the cases we are interested in, only one of them operates and the other either is exactly zero or negligibly small. In Tables 5-8 one gives the values calculated with the Clebsh-Gordon coefficients equal to unity. Such values are usually obtained by experimenters in Coulomb excitation experiments (see, e.g./12/). For the transition from K to $K+2$ this corresponds to the usual $B(E2)$ one, for the transition from K to $K-2$ this corresponds to the $B(E2)_T$ one i.e. to the total probability of excitation of the whole rotational band.

The matrix element $\langle K'|\mathfrak{M}(2\nu)|K\rangle$ consists of four parts

$$\langle K'|\mathfrak{M}(2\nu)|K\rangle = C_\rho C_{\rho'}\{\sum_i D^{22i}_{\rho'\rho+} M(2i) + e_{eff} f^{22}(\rho'\rho) V + e_{eff}\sum_{\lambda\mu i}\sum_{\nu\nu'} D^{\lambda\mu i}_{\rho'\nu+} D^{\lambda\mu i}_{\rho'\nu+} f^{22}(\nu'\nu) V_{\nu'\nu} + \sum_i D^{22i}_{\rho\rho'+} M(2i) \qquad (27)$$

Here $M(2i)$ is the matrix element of the collective E2 transition in even nucleus between the ground state Ψ_0 and the phonon state $Q_i(22)^+\Psi_0$. All the terms in eq. (27) have a direct physical meaning. The first one corresponds to absorption of the phonon in the $\Psi(K\pi)$ state, the second one to the one-particle transition between quasi-particles in both states, the third term to the one-particle transition accompanied by absorption and creation of a phonon and the fourth one is connected with the phonon admixture in the $\Psi(K\pi)$ state. In purely one-quasi-particle states only the second term will operate, in the transition from a purely collective to purely one-quasi-particle state only the first will operate (and we get $B(E2)_{odd} = 1/2\ B(E2)_{even}$).

For many solutions the contribution is given by all the terms in eq.(27). If η is lower than the first pole then the first, second and fourth terms are summed up coherently. In this case enhanced E2 transitions can occur, although the phonon admixture is comparatively small. This takes place e.g., for the first $K\pi=1/2+$ states in the Tb isotopes. If η is higher than the first pole different terms in eq. (27) have different signs and an additional enhancement does not take place.

The quantities $M(2i)$ in eq. (27) are taken from earlier calculated $B(E2)$ /29/ for E2 transitions between the ground and gamma-vibrational states of even nuclei. The values of the effective charge are taken from the same paper. In calculating on the electronic computer an error was made /29/ which did not affect the values of $B(E2)$ but led to a wrong determination of e_{eff}. In the present paper corrected values $e_{eff} = 1,2e$ for protons and $e_{eff} = 0,2e$ for neutrons were used. It should be also noted that in ref./29/. $B(E2)$ for Yb^{172} is rather small and therefore in ^{173}Lu and ^{173}Yb underestimated values for $B(E2)$ were obtained, while for E2 transition for the even Gd and Dy isotopes in ref./29/ the values were slightly overestimated, what should affect the values of $B(E2)$ in the corresponding odd nuclei.

A part of the results which are concerned with the states of the complex and collective structure is given in Tables 5-9. Tables 5 and 6 on odd-proton nuclei and Tables 7 and 8 on odd-neutron nuclei give the results for $K=K_o-2$ and $K=K_o+2$ states where K_o is related to the ground states of odd-mass nuclei. The experimental and calculated values of the energies, of the decoupling parameters a and of the reduced probabilities $B(E2)$ as well as the contribution of the one-quasi-particle state C_ρ^2 and the contribution $C_\rho^2 (D_{\rho\nu}^{221})^2$ of the term in eqs. (10) or (16) corresponding to the first pole with $\lambda=2$, $\mu=2$ are given. In some cases there are several states with given $K\pi$ having different structure therefore Tables 5-8 give not only the first but sometimes the second and the third states with those $K\pi$. Some complex structure states which are most interesting are given in Table 9. The experimental data are taken from the reviews/13,20/ and also from refs./15-22, 26-37/. It should be noted that the contribution of the states corresponding to the nearest poles of the secular equations given in Tables 5-9 is obtained from the normalization condition of the wave function. The role of the terms corresponding to the nearest poles is considerably increased as compared to the role of these terms in eqs. (10) and (16).

Let us consider the peculiarities of some nuclei having an odd number of protons. In ^{153}Eu, ^{155}Eu the admixture of the $413\downarrow + Q_1(22)$ state to the first $K = 1/2+$ states is 11 and 10 percent, and to the second ones 80 percent. The calculated values of the decoupling parameter a disagree with the experimental values which are larger than $a^N = -0.79$ for $\delta = 0.3$ and $\delta = -0.89$ for $\delta = 0.2$. The contribution of the gamma-vibrational state to the first $K\pi = 9/2+$ states is negligibly small while the second $K\pi = 9/2+$ states are pure gamma-vibrational ones. In ^{155}Eu $K\pi = 3/2-$ state of energy 1095 keV is interesting, its structure is given in Table 9.

In the Tb isotopes the first $K\pi = 1/2+$ states contain 60-65 percent of the one-quasi-particle $411\downarrow$ state and, in spite of this, $B(E2)$ is large and equals 1.6-2.8. Due to the large contribution of $411\downarrow$ state the disagreement with experiment in the value of a is large. In ^{159}Tb the calculated decoupling parameter for the first $K\pi = 1/2+$ state is closer to the experimental one for the second $K\pi = 1/2+$ state and on the other hand the calculated a for the second state is closer to the experimental a for the first one. As to $K\pi = 7/2+$ states the first ones have a complex structure, the third states are, to a large extent, octupole ones.

As is seen from Table 1, comparatively high-lying $K\pi = 3/2+$, $5/2+$ states in ^{159}Tb strongly differ from those in ^{157}Tb. According to the calculations, in ^{157}Tb the first excited $K\pi = 3/2+$ state with an energy 1050 keV is beta vibrational one and the second state with an energy 1600 keV is close to $422\downarrow$ one, in ^{159}Tb the first $K\pi = 3/2+$ state with an energy 1600 keV is close to $422\downarrow$ state and the second one is a beta-vibrational state. The second excited $K\pi = 5/2+$ state in ^{157}Tb with an energy 1100 keV is a beta-vibrational one and in ^{159}Tb with an energy 1250 keV octupole one. This is due to the increase of ω_1^{20} in Gd^{158} as compared to Gd^{156}.

In the Ho isotopes the calculated energy of $K\pi = 3/2-$ state is much higher than the experimental one. The solutions of (16) with $\rho_1 = 541\uparrow$ $\rho_2 = 532\downarrow$ do not improve the situation. $K\pi = 11/2-$ states are gamma-vibrational ones. $K\pi = 5/2+$ states in ^{165}Ho with an energy 995 keV for which $\log ft = 5,7$ in the beta decay of ^{165}Dy/32/ is possibly a three-quasi-particle state with the configuration $n\ 633\uparrow + n\ 523\downarrow - p\ 523\uparrow$.

In the Tm isotopes the first $K\pi = 3/2+$ states contain a large contribution of the one-quasi-particle 411↑ states and the second ones are mainly gamma-vibrational states. In the region 0.8 - 1.4 MeV there are three $K\pi = 5/2+$ states, two of which contain a large contribution of 411↓ + $Q_{11}(22)$ and 413↓ states and the third one is close to 402↑ state.

In the Lu isotopes the first $K\pi = 3/2+$ states contain a large contribution of one-quasi-particle 411↑ state, and the second ones are mainly gamma-vibrational states, $K\pi = 11/2+$ states are gamma-vibrational as well, the condition (24) being well fulfilled. In the ^{173}Lu there is a $K\pi = 3/2-$ state with energy 888keV. According to our calculations $K\pi = 3/2-$ state has an energy 1.3 MeV and it is rather close to 532↓ state, since $C^2_\rho = 0.9$. This is due to the fact that in our level scheme ϵ (541↑) = 1.6 MeV and $Y^1(22) = 7.10^3$ because the first $K\pi = 2+$ state in ^{172}Yb is close to the two-quasiparticle one, according to ref./9/.

The calculations of the Re isotopes have a tentative character because of some defects of the average field level scheme. So, $\epsilon(400\uparrow) = 0.210\hbar\mathring{\omega}_0$ is very small, this leads to underestimated values of the energies of the first $K\pi = 1/2+$ states and to underestimated values of B(E2) due to a large contribution of 400↑ state. $K\pi = 9/2+$ states of the Re isotopes are mainly gamma-vibrational states what well agrees with experiment/33,34/.

Let us consider the peculiarities of some odd neutron nuclei. In ^{153}Sm and ^{155}Gd there is a $K\pi = 1/2+$ state which is rather close to 400↑ state since $C^2_\rho = 0.65 - 0.69$ this state is lowered with respect to $\epsilon(\rho)$ by 600 keV and with respect to the first pole by 400 keV, and $K\pi = 1/2-$ state with an energy 600keV has a complex structure. In the nuclei with N=93 $K\pi = 1/2-$, $7/2-$ states have a complex structure and it would be interesting to measure for them the quantities B(E2).

In the nuclei with N=95 the first $K\pi = 9/2-$ state is gamma-vibrational one. In ^{161}Dy $K\pi = 1/2-$ state with an energy 365 keV has a complex structure according to our calculations, the contribution of 521↓ state is 53 percent. In ^{163}Dy $K\pi = 3/2+$ state with a calculated energy 370 keV should have a complex structure. In the first $K\pi = 1/2$ state of ^{163}Dy the calculations give an overestimated contribution of 521↓ state.

In the nuclei with N=99 the first $K\pi = 3/2+$ states in ^{165}Dy and ^{167}Er are gamma-vibrational ones, as is seen from Table 7, the calculated energies and B(E2) are in a rather good agreement with experiment, however, in ^{169}Yb, according to the calculations, the gamma-vibrational state energy increases by 600 keV as compared to ^{165}Dy and ^{167}Er, which is due to the increase of ω^{22}_1 in ^{168}Yb as compared to ^{164}Dy and ^{166}Er. Table 9 gives a complicated structure of $K\pi = 3/2-$ and $1/2-$ states in ^{165}Dy and ^{167}Er.

In ^{171}Yb and ^{173}Yb the calculations give overestimated energies of K_0-2 and K_0+2 states due to the increase of the values of ω^{22}_1 and $Y^1(22)$ in ^{170}Yb and especially in ^{172}Yb as compared to ^{168}Yb and ^{176}Yb. $K\pi = 3/2-$ and $1/2-$ states in ^{173}Yb have a complex structure. In ^{173}Yb there are two comparatively high-lying $K\pi = 3/2-$ states, one of them has an energy 1340 keV and is close to 512↓ state, for the other with an energy 1224 keV we get two possible solutions, one with a complex structure and the second close to 521↑ state. In ^{175}Yb the first $K\pi = 3/2-$ state with an energy 809 keV is relatively close to 512↓ state, $C^2_\rho = 0.77$ the second $K\pi = 3/2-$ state with an energy ~~1616~~ 1616 keV is mainly gamma-vibrational one and the contribution of 521↑ state is 18 percent. The first $K\pi = 3/2-$ state in ^{177}Yb with an energy 709 keV is close to 512↓ one, $C^2_\rho = 0.77$, and the second $K\pi = 3/2-$ state with an energy 1365keV has the contribution of one-quasi-particle 501↑ state of the order 67 percent.

It should be noted that in calculating the levels of ^{175}Yb and ^{177}Yb and of the Hf and W isotopes we should take into account the decrease of the equilibrium deformation as compared to $\delta = 0.3$ what was not done.

In refs./2,3/ it was noted that the energies of the states close to $K=K_0-2$ gamma-vibrational ones are lower than those for $K=K_0+2$ states:

$$\mathcal{E}(K_0-2) < \mathcal{E}(K_0+2) \tag{28}$$

The fulfillment of this relation is caused by the two following facts: first, for small K in eqs. (10) and (16) there are more terms in summing up over ν than for larger K. Second, in the scheme of the average field levels there are less K=11/2, 9/2 states as compared to K=1/2, 3/2 states and therefore for large K there are rarely cases when $\epsilon(\rho)$ is only somewhat higher than the energy of the first pole and when the first non-rotational state energy is lowered very strongly.

Eq. (28) must be fulfilled for states for which the energy of the first gamma-vibrational pole is lower than $\epsilon(\rho)$. Otherwise, these relations have a complex structure and eq. (28) may not be fulfilled. The calculations prove the validity of eq. (28) in the cases when both states are close to the gamma-vibrational ones, what is seen from Tables 5-8. It is difficult to compare the results of calculations with the experimental data on $\mathcal{E}(K_0+2)-\mathcal{E}(K_0-2)$ since there are at present not many experimental data on the splitting energies where both states are close to the gamma-vibrational ones.

The comparison of the results of calculations of the characteristics of the collective states and the complex structure states with the corresponding experimental data allows us to conclude that the interaction of quasi-particles with phonons gives a rather good description of the energies of these states and of the quantities such as B(E2) the decoupling parameter a and others. It is necessary to stress that in many cases states which were earlier interpreted e.g. in refs. /13, 30, 37/ as gamma vibrational ones, according to our calculation have a complex structure.

Conclusions

We have made calculations of the properties of the ground and excited states for 62 nuclei in the region $151 \geq A \geq 187$. 20-30 states were calculated for each nucleus. Thus, the obtained material is sufficiently large. Tables 1-9 give only a small part of the results concerning the most interesting cases and the cases for which there are experimental data. The remaining material can be used when additional experimental data will be available.

The aim of the present paper is to give a general picture of the excited states for many odd-mass nuclei. Therefore we do not analyse here each nucleus taken separately. In developing further this topic a detailed and careful calculations of the properties of the most interesting nuclei with an improvement of the Nilsson potential parameters, taking into account the Coriolis interaction and so on is needed. Using such an approach better agreement between the results of calculations and experiment can be obtained and the predictions for the considered states can be improved. So, e.g. a small displacement of the average field one-particle levels 411↓ and 411↑ can noticeably improve the description of a number of states of odd-proton nuclei.

It should be noted that in investigating the interaction of quasi-particles with phonons there is not a single free parameter. The quantities $\omega_i^{\lambda\mu}$ and $Y^i(\lambda\mu)$ are obtained in calculating the collective states of even nuclei. Therefore in the cases when the agreement between theory and experiment was not sufficiently good in even nuclei, this discrepancy should take place also in odd-mass nuclei. A general picture of the excited states of odd-mass nuclei is more complicated, and the descriptions are somewhat more rough as compared with even nuclei. By the way, in the present paper we do not consider pure three-quasi-particle states.

The investigation performed showed that the structure of the excited non-rotational states of deformed odd-mass nuclei is very different. Most low-lying states are one-quasi-particles one, but when the energy increases the number of states close to the collective ones and to the states with a complex structure increases. The account of the interaction of quasi-particles with phonons has led to the improvement of the description of the states close to the one-quasi-particle ones as compared to the independent quasi-particle model and to a rather correct description of the collective states and the complex structure states. For a further investigation of the structure of the states of odd-mass deformed nuclei it is necessary to increase the amount of the experimental data on the state energies, on the beta and gamma transition probabilities, on the spectroscopic factors in direct nuclear reactions and so on.

The position of the levels of deformed odd-mass nuclei is to a large extent determined by the behaviour of the average field one-particle levels. Therefore the accuracy of calculation of different characteristics of odd-mass nuclei is essentially restricted by a rough description of the energies and the wave functions with the Nilsson potential.

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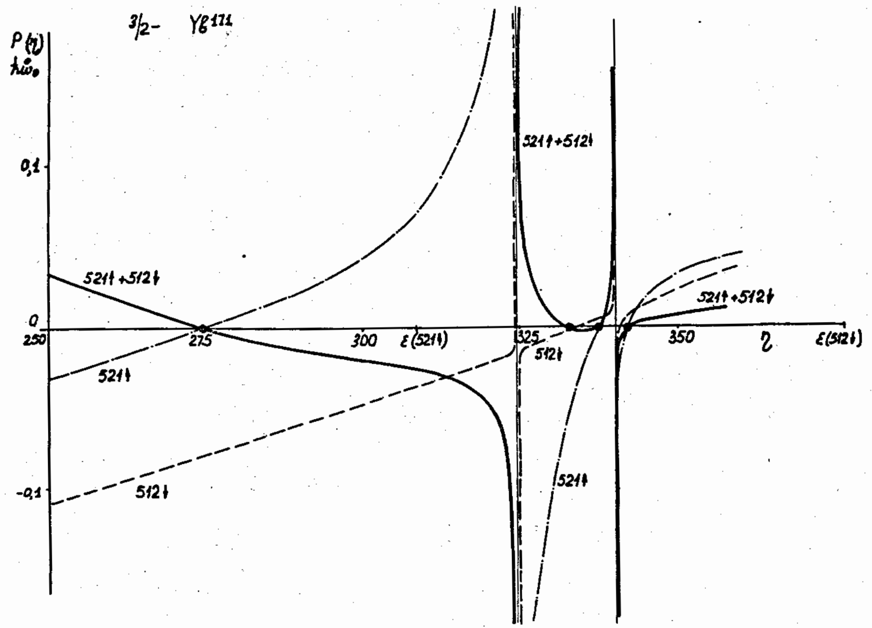

Fig. 1. The behaviour of P(η) (in units $\hbar\mathring{\omega}_0$) in the left-hand side of eq. (10) for ρ =521↑ the dashed and dotted line, ρ =512↓ is the dashed line and the behaviour of P(η) (in units $(\hbar\mathring{\omega}_0)^2$, the scale is increased ten times for clarity) in the left-hand side of eq. (16) for $\rho_1+\rho_2$ = 521↑ + 512↓ is the continuous curve. The first pole corresponds to ϵ(521↑) + ω_1^{22} = 0.324 $\hbar\mathring{\omega}_0$ the second one to ϵ(521↑) + ω_2^{22} = 0.340 $\hbar\mathring{\omega}_0$.

Table 1

Positive parity states in ^{157}Tb and ^{159}Tb

Nuclei	Kπ	Energy(keV) exp.	Energy(keV) Cacl.	Lowering(keV) respect to the ε(ς)	Lowering(keV) respect to the L pole	Structure
Tb^{157}	3/2+	0	0	200	1100	411↑ 90%; 411↓ +Q_1(22) 6%
	5/2+		300	80	770	413↓ 96%
	1/2+	597	530	440	770	411↓ 65%; 411↑ + Q_1(22) 25%
	3/2+	990	1050	420	–	411↑+Q_1(20) 100%
	7/2+		1100	420	260	404↓ 81%; 404↓+ Q_1(20) 10%
	5/2+		1100		0	413↓+ Q_1(20) 100%
	9/2+		1100	270	200	404↑ 70%, 404↑+ Q_1(20) 25%
	1/2+		1300			413↓+ Q_1(22) 75%, 411↑ +Q_1(22) 20%
	9/2+		1300			413↓+ Q_1(22) 95, 404↑ 1,4%
	7/2+		1400			411↑+ Q_1(22) 100%
	1/2+		1550	520		420↑ 70%, 422↓+Q_1(22) 15%
	7/2+		1550			523↑+Q_1(30) 90%, 404↓ 3%
	3/2+		1660	400		422↓ 75%
Tb^{159}	3/2+	0	0	260	1600	411↑ 90%, 411↓+Q_1(22) 8%
	5/2+	348	330	120	900	413↓+ 94%, 411↓+Q(22) 4%
	1/2+	580	430	570	700	411↓ 60%, 411↑+Q_1(22) 30%
	7/2+		1100	420	60	404↓ 66%, 523↑+Q_1(30) 30%
	9/2+		1100	290	10	413↓+Q_1(22) 95%; 404↑ 4%
	5/2+		1250		0	532↑+Q_1(30) 100%
	1/2+	971	1150		0	413↓+Q_1(22) 80%; 411↑ +Q_1(22) 15%
	7/2+	1270	1150		0	411↑+Q_1(22) 100%
	7/2+		1350			523↑+Q_1(30) 70%, 404↓ 25%
	9/2+		1400			404↑ 90%, 404↑+Q_1(20) 4%
	1/2+		1500	650		420↑ 65%, 422↓+Q_1(22) 20%
	3/2+		1600	450		422↓ 69%
	3/2+		1650		0	411↑+Q_1(20) 100%

Table 2.

States close to the one-quasi-particle ones, in odd mass nuclei

Nucleus	Kπ	Energy(keV) exp.	Energy(keV) calc.	Energy(keV) ε(ς) - ε(K₀)	Structure
^{153}Eu	3/2+	1o3	1oo	18o	411↑ 94%
^{161}Tb	5/2+	316	280	2oo	413↓ 96%; 411↓+Q_1(22) 2%
^{161}Tb	5/2–	482	580	450	532↑ 96%
^{161}Ho	1/2+	211	220	240	411↓ 96%
^{161}Ho	5/2–	826	900	920	532↑ 91,3%
^{165}Ho	7/2+	715	650	850	404↓ 93%; 402↓+Q_1(22) 5%
^{169}Tm	7/2–	379	350	330	523↑ 99%
^{173}Lu	3/2–	888	1300	1500	522↓ 92%
^{181}Ta	1/2+	612	540	600	411↓ 95%, 411↑+Q_1(22) 3%; 413↓+ Q_1(22) 2%
^{181}Re	3/2+	851	550	440	404↓ 98%
^{161}Dy	3/2–	75	60	110	521↑ 95%; 651↑+Q_1(30) 2%; 521↓+ + Q_1(22) 2%
^{163}Dy	3/2–	251	280	400	521↑ 91%
^{165}Dy	5/2–	535	530	560	523↓ 94%
^{165}Dy	5/2–	184	240	410	512↑ 89%
^{167}Er	5/2–	348	300	410	512↑ 91%
^{169}Yb	5/2–	191	400	470	512↑ 95%
^{169}Yb	5/2–	570	540	560	523↓ 97%
^{169}Yb	5/2+	584	600	740	642↑ 89%; 523↓+Q_1(30) 4%; 642↑+ Q_1(20) 4%
^{169}Yb	3/2–	657	820	1000	521↑ 94%; 651↑+Q_1(30) 4%
^{171}Yb	7/2–	835	1200	1300	514↓+ 88%
^{171}Yb	7/2+	95	160	120	633↑ 99%
^{173}Yb	3/2–	1340	1560	1600	512↓ 90%
^{173}Yb	7/2–	636	450	430	514↓ 99%
^{173}Yb	7/2+	351	530	520	633↑ 98%
^{173}Yb	3/2–	(1224)	1900	2000	521↑ 80%
^{175}Yb	7/2+	995	1140	1100	633↑ 98%
^{177}Hf	9/2+	321	400	350	624↑ 100%
^{177}Hf	7/2–	1o60	1200	1400	503↑ 89%

Table 3

Energy, decoupling parameter and C_s^2 quantity for a state close 1/2– 521↓ state (Caculation for δ = 0.3, where for $C_s^2 = 1$, α = 0.89)

Nucleus	Energy(keV)			Decoupling parameter α		C_s^2
	exp.	calc.	$\varepsilon(\varrho)-\varepsilon(K_0)$	exp.	calc.	·100
$^{153}Sm_{91}$	698	900	1500	0.33 ± 0.36	0.50	56
$^{155}Sm_{93}$	824	950	1350	0.32±0.28	0.58	66
$^{161}Dy_{95}$	365	450	920	0.44	0.47	53
$^{163}Er_{95}$	346	480	920	0.47	0.49	55
$^{163}Dy_{97}$	–	300	530	–	0.60	69
$^{165}Er_{97}$	297	340	530	0.56	0.65	73
$^{165}Dy_{99}$	108	130	180	0.58	0.86	97
$^{167}Er_{99}$	208	150	180	0.71	0.87	98
$^{169}Yb_{99}$	24	150	180	0.79	0.87	98
$^{169}Er_{101}$	0	0	0	0.83	0.85	96
$^{171}Yb_{101}$	0	0	0	0.85	0.86	98
$^{173}Hf_{101}$	0	0	0	0.82	0.87	98
$^{173}Yb_{103}$	400	280	290	0.74	0.88	99
$^{175}Hf_{103}$	126	280	290	0.75	0.85	97
$^{175}Yb_{105}$	913	800	850	0.71	0.84	95
$^{177}Hf_{105}$		800	850	–	0.84	95
$^{177}Yb_{107}$		1000	1230	–	0.81	91
$^{179}Hf_{107}$		1050	1230	–	0.82	92
$^{181}W_{107}$	746	1000	1230	0.59	0.85	81

Table 4

Spectroscopical factor and the one-particle amplitude for states close to 1/2 – 510↑

Nuclei	Energy (keV)			C_s^2	Spectroscopic factor $C_s^2 a_s^2$
	exp.	calc.	$\varepsilon(\varrho)-\varepsilon(K_0)$		
$^{169}Yb_{99}$	805	1300	2300	0.39	0.38
$^{171}Yb_{101}$	950	1200	1800	0.51	0.49
$^{173}Yb_{103}$	1040	1160	1340	0.65	0.63
$^{175}Yb_{105}$	500	660	800	0.90	0.85
$^{177}Yb_{107}$	320	103	300	0.89	0.77
$^{175}Hf_{103}$		1070	1340	0.75	0.73
$^{177}Hf_{105}$		680	800	0.90	0.85
$^{179}Hf_{107}$	378	110	300	0.90	0.79
$^{181}Hf_{109}$	0	0	0	0.96	0.70

Table 5

$K=K_0-2$ states in odd proton nuclei (K_0 is related to the ground state)

Nucleus	$K\pi$	ϱ	ν for ground state	Energy(keV) exp.	Energy(keV) calc.	$B(E2)/B(E2)_{s.p.}$ exp.	$B(E2)/B(E2)_{s.p.}$ calc.	Decoupling parameter α exp.	Decoupling parameter α calc.	$C_\varsigma^2 \cdot 100$	$C_\varsigma^2 (D^{221}_{5\nu})^2 \cdot 100$
$^{153}_{63}Eu$	1/2+	411↓	413↓	635	650	–	0.5	-0.95	-0.61	72	11
					1800	–	1.1		-0.10	10	80
$^{155}_{69}Eu$	1/2+	411↓	413↓	765	650	–	0.6	-1.0	-0.63	74	10
					1800	–	1.2		-0.10	10	80
$^{155}_{65}Tb$	1/2+	411↓	411↑	761	500	–	2.2	0.1	-0.50	64	28
					1200	–	0.2	–	0	0.3	10
$^{157}_{65}Tb$	1/2+	411↓	411↑	597	530	–	2.1	0.04	-0.50	65	25
					1300	–	0.5	–	0	0.2	20
$^{159}_{65}Tb$	1/2+	411↓	411↑	580	430	1.5	2.8	0.05	-0.47	60	30
				971	1150		0.3	-0.81	0	0.4	15
					1600					38	40
$^{161}_{65}Tb$	1/2+	411↓	411↑		550		1.6	–	-0.50	64	25
					1200		0.2		0	0.5	15
$^{161}_{67}Ho$	3/2–	541↑	523↑	593	1000		1.9			12	65
$^{163}_{67}Ho$	3/2–	541↑	523↑		940		2.4	–	–	9	78
$^{165}_{67}Ho$	3/2–	541↑	523↑	515	820	1.9	2.8	–	–	4	90
$^{167}_{69}Tm$	3/2+	411↑	411↓	–	570	–	0.7			81	16
					1050	–	2.3	–	–	15	80
$^{169}_{69}Tm$	3/2 +	411↑	411↓	570	620	1.5	0.3			91	6
				900	1200	0.3	2.0			6	90
$^{171}_{69}Tm$	3/2+	411↑	411↓	675	650	–	0.1			95	2
					1350		1.6			3	90
$^{173}_{71}Lu$	3/2+	411↑	404↓		1050					96	1
					1400					1	90
$^{175}_{71}Lu$	3/2+	411↑	404↓		1000		0.2			82	16
$^{177}_{71}Lu$	3/2+	411↑	404↓		900		0.5			61	36
$^{179}_{73}Ta$	3/2+	411↑	404↓		1000					53	2
		402↓			1200		1.1			5	90
$^{181}_{73}Ta$	3/2+	411↑	404↓		1200					62	
$^{183}_{75}Re$	1/2+	400↑	402↑		460		0.5		0.34	87	5
		411↓		1103	900					94	0
		400↑			1400		2.5		0	2	95
$^{185}_{75}Re$	1/2+	400↑	402↑	647	400	3.6	0.5	0.38	0.32	79	7
		411↓		872	850					92	0
		400↑			1100		1.8			4	90
$^{187}_{75}Re$	1/2+	400↑	402↑	511	400	3.1	0.6	0.38	0.31	78	9
		411↓		618	850			-1.1		90	0
		400↑			1150		2.4		0	3	90

Table 6

$K=K_0+2$ states in odd-proton nuclei (K_0 corresponds to the ground state)

Nuclei	$K\pi$	ρ	ν for ground state	Energy (keV) exp.	Energy (keV) calc.	$B(E2)/B(E2)_{s.p.}$ exp.	$B(E2)/B(E2)_{s.p.}$ calc.	C_ρ^2 100	$C_\rho^2(D_{\rho\nu}^{221})^2$ 100
$^{153}_{63}Eu$	9/2+	404↑	413↓	–	700	–	–	84	0
					1800		1.5	0	100
$^{155}_{63}Eu$	9/2+	404↑	413↓		750			86	0
					1800		1.6	0	100
$^{155}_{65}Tb$	7/2+	404↓	411↑		1060			77	0
					1300		2.7	0	100
$^{157}_{65}Tb$	7/2+	404↓	411↑		1100			81	0
$^{159}_{65}Tb$	7/2+	404↓	411↑	1270	1100			66	0
					1150	2.0	3.0	0.5	99
$^{161}_{65}Tb$	7/2 +	404↓	411↑		1100			81	100
					1200		2.5	0	100
$^{161}_{67}Ho$	11/2-	505↑	523↑		1050		2.8	0.1	99
$^{163}_{67}Ho$	11/2-	505↑	523↑		1000		3.1	0.1	99
$^{165}_{67}Ho$	11/2-	505↑	523↑	687	850	1.7	3.2	0.1	99
$^{167}_{69}Tm$	5/2+	413↓	411↓		820		2.1	30	70
		402↑			900			77	0
		413↓			1300		0.7	66	30
$^{169}_{69}Tm$	5/2+	402↑	411↓		900			92	0
		413↓		1170	950	1.5	1.2	56	40
		413↓			1350		1.2	42	57
$^{171}_{69}Tm$	5/2+	402↑	411↓		950			95	0
		413↓		912	1050		0.6	80	20
		413↓			1040		1.3	18	80

Nuclei	$K\pi$	ρ	ν for ground state	Energy (keV) exp.	Energy (keV) calc.	$B(E2)/B(E2)_{s.p.}$ exp.	$B(E2)/B(E2)_{s.p.}$ calc.	C_ρ^2 100	$C_\rho^2(D_{\rho\nu}^{221})^2$ 100
$^{175}_{71}Lu$	11/2+		404↓		1600		1.0	0	100
$^{177}_{71}Lu$	11/2+		404↓		1300		1.3	0	100
$^{181}_{73}Ta$	11/2+		404↓		1400		1.0	0	100
$^{183}_{73}Re$	9/2+	404↑	402↑		1350		2.5	4	95
$^{185}_{73}Re$	9/2+	404↑	402↑	966	1020	2.6	1.9	3	95
$^{187}_{73}Re$	9/2+	404↑	402↑	840	1080	3.8	2.6	2	95

Table 7

K=K$_0$-2 states in odd neutron nuclei

Nucleus	Kπ	ρ	ν for ground state	Energy (keV) exp.	Energy (keV) calc.	B(E2)/B(E2) s.p. exp.	B(E2)/B(E2) s.p. calc.	Decoupling parameter α exp.	Decoupling parameter α calc.	C_ρ^2 ·100	$C_\rho^2 (D^{221}_{\rho\nu})^2$ ·100
^{155}Gd	1/2-	521↓	521↑	-	600	-	1.1	-	0.39	44	37
^{155}Sm	1/2-	521↓	521↑	844	950	-	0.4	0.3± 0.3	0.59	66	17
^{157}Gd	1/2-	521↓	52 1↑	-	700	-	0.9	-	0.45	51	27
^{159}Dy	1/2-	521↓	521↑	-	750	-	1.0	-	0.42	47	33
^{159}Gd	1/2+	660↑	642↑		250		0.1		4.3	70	5
^{161}Dy	1/2+	660↑	642↑		420	-	0.2	-	4.3	70	8
^{163}Dy	1/2-	521↓	523↓	351	300	2.7	1.0	0.25	0.60	68	27
					900		1.5	0.65	0.07	3	49
^{165}Er	1/2-	521↓	523↓	298	340		0.7	0.56	0.55	73	22
					950		1.6		0.99	10	62
^{165}Dy	3/2+	651↑	633↓	539	700	2.6	3.0			9	90
^{167}Er	3/2+	651↑	633↓	532	750	3.2	2.5			7	88
^{169}Yb	3/2+	651↑	633↑		1000		0.1			72	4
					1350		0.1			3	87
^{171}Yb	3/2-	521↑	521↓	902	1200		0.4			70	25
^{171}Er	1/2-	510↑	512↑	701	800		0.9		-0.17	48	48
^{173}Yb	1/2-	510↑	512↑	1040	1200				-0.22	65	20
^{175}Yb	3/2-	512↓	514↓	809	800		0.5			77	14
^{177}Hf	3/2-	512↓	514↓		800		0. 5			87	14
^{177}Yb	5/2+	642↑	624↑		1200		1.3			0.4	99
^{183}W	3/2-	501↑	510↑	458	650	0.3				86	0
		512↓		209	270		0.2			93	4

Table 8

K=K$_0$+2 states in odd neutron nuclei

N	Nucleus	Kπ	ρ	ν for ground state	Energy (keV) Calc.	B(E2)/B(E2)s.p. calc.	C_ρ^2 ·100	$C_\rho^2 (D^{221}_{\rho\nu})^2$ ·100
93	^{159}Dy	7/2-	514↓	521↑	1200	2.7	0.1	85
95	^{159}Gd	9/2+	624↑	642↑	1000	2.8	2	94
95	^{161}Dy	9/2+	624↑	642↑	950	2.7	1	95
97	^{163}Dy	9/2-	514↑	523↓	750	3.1	0.1	87
97	^{165}Er	9/2-	514↑	523↓	800	2.6	0.1	99
99	^{165}Dy	11/2+	615↑	633↑	730	3.3	0.7	95
99	^{167}Er	11/2+	615↑	633↑	830	2.8	0.6	98
99	^{169}Yb	11/2+	615↑	633↑	1350	2.1	0.7	99
101	^{169}Er	5/2-	523↓	521↓	850	1.3	46	47
					1250	0.8	49	30
101	^{171}Yb	5/2-	523↓	521↓	1000	0.1	90	6
					1600	1.0	5	90
101	^{173}Hf	5/2-	523↓	521↓	960		81	
103	^{173}Yb	9/2-	505↓	512↑	1160		0	99
105	^{175}Yb	11/2-	505↑	514↓	1700	1.0	0	99
109	^{183}W	5/2-	512↑	510↑	1050	0.7	77	20
					1400	1.2	20	80

Table 9. Complex structure states

Nucleus	K_π	Energy (keV) exp.	calculation	Structure
^{155}Eu	3/2-	1095	1060	541↑ 70%; 541↑+Q_1(20) 19%; 411↑+Q_1(30) 5,5%
^{155}Gd	1/2+		850	400↑ 69%
^{159}Gd	3/2+		340	651↑ 77%; 532↑+Q_1(30) 6%; 521↑+Q_1(30) 6%
^{161}Dy	1/2-	365	450	521↓ 53%; 523↓+Q_1(22) 28%; 521↑+ Q_1(22) 18%
^{163}Dy	3/2+		370	651↑ 71%; 521↑+Q_1(30) 14%; 532↓+Q_1(30) 6%
^{165}Dy	3/2-	574	800	521↑ 68%; 521↓+Q_1(22) 25%; 651↑+ Q_1(30) 4%
^{165}Dy	1/2-	570	640	510↑ 32%; 512↑+Q_1(22) 63%; 512↓+ Q_1(22) 4%
^{165}Er	1/2+	508	630	660↑ 42%; 642↑+Q_1(22) 43%; 651↑+Q_1(22) 7%
^{167}Er	3/2-	545	750	521↑ 79%; 521↓+Q_1(22) 15%; 651↑+Q_1(30) 4%
^{167}Er	1/2-		800	510↑ 32%; 512↑+Q_1(22) 65%; 512↓+Q_1(22) 3%
^{169}Er	5/2-	915	850	523↓ 46%; 521↓+Q_1(22) 47%; 642↑+Q_1(30) 2%
^{169}Yb	1/2-	805	1300	510↑ 39%; 512↓+Q_1(22) 56%; 512↑+Q_1(22) 3%
^{171}Yb	1/2-	945	1200	510↑ 51%; 512↓+Q_1(22) 41%; 521↓+Q_1(20) 3.5%
^{173}Yb	3/2-	1224	1350	521↑ 7%; 521↓+Q_1(22) 90%
^{175}Yb	3/2-	1616	1700	521↑ 18%; 521↓+Q_1(22) 80%
^{177}Yb	3/2-	1365	900	501↑ 67%; 503↑+ Q_1(22) 26%