ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E-2557

D.Robaschik and A.Uhlmann

PLUS PARITY MESONS AND THE 189-PLET OF SU(6)

ЛАБОРАТОРИЯ ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

1966

E-2557

D.Robaschik and A.Uhlmann

PLUS PARITY MESONS AND THE 189-PLET OF SU(6)

Submitted to "High Energy Physics"

1. Introduction

The classification in the SU(6) 189-plet of the positive parity mesons was proposed by Dao Vong Duc, Pham Quy Tu[1] and on the basis of a relativistic extension of SU(6) by Delbourgo[2a] and by R.Delbourgo, M.A.Rashid, A.Salam, J.Strathdee[2b]. Using bootstrap ideas R.C.Hwa, S.H.Patil[3] have given arguments in favour of this particle assignment. A more elementary consideration is the following : Assume the low-lying plus parity mesons to be bound states of the 35-plet mesons. Then these bound states are composed of s-, d- and higher states. The p-state and higher odd angular momentum states are parity forbidden. If the s-state is the dominant state, one is allowed to hope that a SU(6) classification is a relevant one. Therefore the plus parity mesons should fit into the Kronecker product 35 x 35. On the other hand using U(6) x U(6) it is reasonable to value the representation 6 x 6 and 15 x 15 as minus and plus parity meson states, respectively. The SU(6) content of 15 x 15 is just 1 + 35 + 189 and this is contained in 35 x 35 indeed. In the following, we restrict ourselves to the 189-plet, having in mind, that the existence and the quantum numbers of some of the recently discovered plus parity mesons seem to be not well established.

2. Labelling of States

Following the ideas of F.Gürsey, A.Pais, L.A. Radicati[4] we use the p-chain (physical chain) and the u-chain (unphysical chain) to get two different systems of basis vectors of the 189-plet. The former will be used to fix the structure of the mass formula, the later one is assumed to give the physical particle states. The p-chain reads $SU(6) \supseteq SU(3) \times (\vec{J}) \supseteq (\vec{I}) \times Y \times (\vec{J})$, where $(\vec{T})$ denotes the group generated by the generators $\vec{T}$. Let us concentrate on the 189-plet. One knows the $SU(3) \times (\vec{J})$ content to be $189 = (1,1) + (8,1) + (27,1) + (8,3)+(10,3)+(10^x,3)+(1,5)+(8,5)$. Therefore we characterize a basis set of states by J^2, J_3, I^2, I_3, Y and by their $SU(3)$ representation. There remains a degeneracy for the spin one octets so that it is possible to rotate them in the linear space spanned by themselves. The u-chain reads $SU(6) \supseteq SU(4)_W \times (\vec{S}) \times (Y) \supseteq (\vec{N}) \times (\vec{S}) \times (\vec{I}) \times (Y)$ and we may fix a basis of the 189-plet by using J^2, J_3, I^2, I_3, N^2, S^2, Y and $C_2^{(4)}$. In the 189-plet $C_2^{(4)}$ is not a function of the other quantum numbers of the u-chain. We consider the u-chain states as particle states. This may be further confirmed by the possibility to give the $Y=0$ members of them a definite value of the G-parity. The G-parity is calculated as shown in ref. 5. In the p-chain decomposition there is a mixture of plus- and minus G-parity states, because the 10-plet is mapped onto the 10^*-plet by the G-parity operator. In table I the physical states and their quantum numbers are listed. In table II we have collected the mixing relations between the p- and the u-chain states for the 189-plet according to the matrix scheme. See also ref. 1.

$$\text{p-chain state}\,\rangle = (A)\ \text{u-chain state}\rangle .$$

3. Form of the Mass-Operator

It is rather an unconvenient task, to reduce out the 189 x 189 Kronecker product, to look for the $J=I=Y=0$ members of the occuring octets and last but not least, to discuss which of them should be neglected in order to get a useful mass operator. Therefore we do not give the mass operator in an explicit form but do the following: If M^2 is the operator for the squared masses, we require:

A. M^2 is diagonal in the particle states, e.g. in the basis vectors constructed with the aid of the u-chain.

B. There exist constants a_o, a_1, a_2, a_3 such that

$$a_o + a_1 C_2^{(3)} + a_2 J(J+1) + a_3 \left[I(I+1) - \frac{Y^2}{4} \right] = (\omega_p, M^2 \omega_p)$$

provided ω_p is a state of the basis that is constructed with the help of the p-chain.

This however does not fix the operator completely, because the p-chain gives rise to a degenerate system of diagonal operators: It remains to remove the degeneracy of the two $J=1$ octets. From this we get a new constant which expresses the mixing of these two octets. This parameter is essential for the $J=1$, $I=1/2$ mesons. Finally let us stress the fact, that in the p-chain representation only the diagonal terms of M^2 are determined by the requirement B. For the $J=1$, $I=1$ mesons this gives rise to a further constant. With the help of the mixing relations we get for the squared masses m_u^2 of the particles (states of the u-chain)

$$m_u^2 = b_o + b_1K_1^u + b_2K_2^u + b_3K_3^u + b_4K_4^u + b_5K_5^u$$

The numbers K_1^u, K_2^u ... are tabulated in table 1.

4. Particle Assignments

To check particle assignments mass relations have been considered, if they are satisfied within 5% in the squared masses.

	Y	I	N	S	J	G	m
f'	0	0	2	0	2	+	1500
f	0	0	1	1	2	+	1253
A_2	0	1	1	1	2	–	1324
K^*	1	1/2	3/2	1/2	2		1405

They have to satisfy the relation (squared masses)

$$4K^* = 2f' + f + A_2$$

which is well established (2%) . One and only one place is in the 189-plet for the mesons D and M_1

	Y	I	N	S	J	G	m
D	0	0	1	1	1	–	1286
M_1	2	1	0	0	0		1280

With these quantum numbers they should satisfy

$$A_2 + 3D = 2M_1 + 2/9\,f' + 16/9\,f$$

which is in good agreement with the experment. Now we are able to calculate (in $(\text{MeV})^2 10^3$)

$$b_o = 1239 \;,\; b_1 = 93,\; b_2 = 75,\; b_3 = -135.$$

For this calculation we have used f, f', A_2 and D.

For the 35-plet it is well-known, that the determination of the coefficients in the mass formula suffers from large uncertainties (e.g. in the 35-plet a_3

varies between -140 and -150 if determined from the pseudo-vector or pseudo-scalar mesons). The mass values calculated with the help of our b_k' s (look at table 1) are therefore to be considered as rough estimates only. Now let us look at the mesons A_1 and B. If these are actually particles with J=1 the 189-plet offers two possibilities

		Y	I	N	S	J	G	m
1.	A_1	0	1	1	1	1	-	1072
	B	0	1	1	0	1	+	1220
2.	A_1	0	1	1	0	1	-	1072
	B	0	1	0	1	1	+	1220

From both the relations

$$2f' + f + A_1 + 2B = 3D + 3A_2$$

follows, that is satisfied within 1% in the the squared masses![x)]. Assuming for the resonances K^{**} (1320) and C the quantum numbers

	Y	I	N	S	J	G	
K	1	1/2	1/2	1/2	0		1320
C	1	3/2	1/2	1/2	1		1215

one gets the relations

$$C + 1/6\, f' + 7/12 f = 1/3\,(A_1 + 2B) + 3/4\, A_2$$

$$9M_1 + 5C + 6D = 11/3(A_1 + 2B) + 9K^{**}$$

$$K^{**} + 1/4\, A_2 = M_1 + 19/54 f' + 11/108\, f.$$

The first is satisfied within 1%, the others within 4%. For the experimental data we have used the table of A.H. Rosenfeld et al[7]. It is a pleasure to thank dr. U. Kundt for interesting discussions on recently discovered resonances.

---

x) From the well-satisfied relation[6] $A_1 + A_2 = 2B$ we obtain $b_2 = b_5$ if the assignment I is true and $b_1 + b_3 \cdot 3 + b_5 = 0$ if the other one is correct.

After this paper was completed we received a preprint of Chia-Hun Chan and Nguyen-Hun Xuong containing the mixing matrices also.

References

1. Dao Vong. Duc, Pham Quy Tu. Ядерная физика 2, 748 (1965).

2a. R. Delbourgo. Phys. Lett. 15, 3 347 (1965).

2b. R. Delbourgo, M.A. Rashid, A. Salam and J. Strathdee. Preprint IC (65) 57.

3. R.C. Hwa, S.H. Patil. Phys. Rev. 139, B969 (1965).

4. F. Gürsey, A. Pais, L.A. Radicati. Phys. Rev. lett. 13, 299 (1965).

5. A. Uhlmann Preprint, Dubna E-2545.

6. T. Gatto, L. Maiani. Preprint 1965.

7. A.H. Rosenfeld et al. Preprint UCRL - 8030.

Received by Publishing Department
on February 1, 1966.

Table 1. Particle states

J	Y	I	N	S	$C_2^{(4)}$	$C_2^{(3)}$	G	K_1^u	K_2^u	K_3^u	K_4^u	K_5^u	$m^2_{theor.}$	$m^2_{exp.}$	symbol
2	0	0	2	0	12	–	+	6	6	0	0	0	2250	2250	f'
2	0	0	1	1	8	–	+	–3	6	0	0	0	1570	1571	f
2	0	1	1	1	8	6	–	3	6	2	0	0	1750	1753	A_2
2	1	½	3/2	1/2	39/4	6		3	6	½	0	0	1950	1988	K
1	0	0	1	1	8	6	+	+3	2	0	0	0	1650	1650	D
1	0	0	1	0	8	6	–	3	2	0	0	0	1650		
1	0	1	1	1	8	–	–	3	–	2	0	–2		1150	A_1
1	0	1	0	1	8	–	+	–3	2	2	0	–1			
1	0	1	1	0	8	–	–	15	2	2	0	2			
1	0	1	1	0	12	–	+	3	2	2	0	1		1488	B
1	1	½	3/2	½	39/4	–		7	2	½	1	0			
1	1	½	½	½	15/4	–		7	2	½	–1	0			
1	1	½	½	½	39/4	–		–2	2	½	0	0	1210		
1	1	3/2	½	½	39/4	12		6	2	7/2	0	0	1410	1476	C
1	2	0	1	0	5	12		6	2	–1	0	0	2010		
0	0	0	1	1	8	–	+	–2	0	0	0	0	1090		
0	0	0	0	0	0	–	+	55/3	0	0	0	0	2610		
0	0	0	0	0	12	–	+	–16/3	0	0	0	0	840		
0	0	1	1	1	8	–	–	18	0	2	0	0	2320		
0	0	1	0	0	8	–	–	–7	0	2	0	0	1500		
0	0	2	0	0	8	16	+	8	0	6	0	0	1030		
0	1	½	½	½	15/4	–		29/3	0	½	0	0	1900	1742	
0	1	½	½	½	39/4	–		4/3	0	½	0	0	1270		
0	1	3/2	½	½	39/4	16		8	0	7/2	0	0	1370		
0	2	1	0	0	5	16		8	0	1	0	0	1700	1638	M_1

Table II

Transformation matrices

J=2 Y=0 I=0

$$\begin{pmatrix} \text{singlet} \\ \text{octet} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix} \qquad \begin{pmatrix} 2, & 0, & 12) \\ 1, & 1, & 8) \end{pmatrix}$$

J=1 Y=0 I=0

$$\begin{pmatrix} \text{octet a} \\ \text{octet b} \end{pmatrix} = - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \begin{pmatrix} 1, & 1, & 8) \\ 1, & 0, & 8) \end{pmatrix}$$

I=0

$$\begin{pmatrix} \text{octet a} \\ \text{octet b} \\ \text{decuplet} \\ \text{decuplet} \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 0 & \sqrt{2} & 0 & 1 \\ -1 & 0 & \sqrt{2} & 0 \\ 1 & -\sqrt{1/2} & -\sqrt{1/2} & 1 \\ 1 & \sqrt{1/2} & -\sqrt{1/2} & -1 \end{pmatrix} \qquad \begin{pmatrix} 1, & 1, & 8) \\ 0, & 1, & 8) \\ 1, & 0, & 12) \\ 1, & 0, & 8) \end{pmatrix}$$

Y=1 I=1/2

$$\begin{pmatrix} \text{octet a} \\ \text{octet b} \\ \text{decuplet} \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 2 & -2 & -1 \\ 1 & 2 & -2 \\ -2 & -1 & -2 \end{pmatrix} \qquad \begin{pmatrix} 3/2, & 1/2, & 39/4) \\ 1/2, & 1/2, & 39/4) \\ 1/2, & 1/2, & 15/4) \end{pmatrix}$$

J=0 Y=0 I=0

$$\begin{pmatrix} \text{singlet} \\ \text{octet} \\ \text{27-plet} \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{60}} & \frac{1}{\sqrt{6}} & -\frac{5}{\sqrt{60}} \\ -\sqrt{\frac{2}{15}} & -\frac{1}{\sqrt{3}} & -\sqrt{\frac{8}{15}} \\ -\sqrt{\frac{9}{20}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{20}} \end{pmatrix} \qquad \begin{pmatrix} 1, & 1, & 8) \\ 0, & 0, & 0) \\ 0, & 0, & 12) \end{pmatrix}$$

I=0

$$\begin{pmatrix} \text{octet} \\ \text{27-plet} \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \qquad \begin{pmatrix} 1, & 1, & 8) \\ 0, & 0, & 12) \end{pmatrix}$$

Y=1 I=1/2

$$\begin{pmatrix} \text{octet} \\ \text{27-plet} \end{pmatrix} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix} \qquad \begin{pmatrix} 1/2, & 1/2, & 15/4) \\ 1/2, & 1/2, & 39/4) \end{pmatrix}$$

Notation: SU(3) plet = (A) N, S, $C_2^{(4)}$