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DISPERSION SUM RULES AND SU(6) SYMMETRY II

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I. Introduction

Difficulties of the relativistic generalization of SU(6) symmetry have recently led to the development of an approach/1-6/ which may be characterized by two postulates:

1) one postulates some algebra of charges, i.e. one postulates the equal time commutators between the space integrals of the components of the densities of some currents (e.g., the commutation relations, which correspond to the generators of the group $U(6) \times U(6)$);

2) one supposes that the sum over intermediate states in these commutation relations can be restricted to some intermediate states chosen in a special way.

This approach allows to derive all the results of SU(6) and also some new relations (e.g., between the magnetic moment of the proton and its radius). However, as it is noted in/6/ the choice of the intermediate states in equal-time commutators has no dynamical foundation, since even the mass of an intermediate state by no means affects its role in the sum. A further step in this direction has been made in ref./4/ where the authors have proposed to consider the dispersion relations for the equal time commutators. In doing so, they express the corrections due to the disregarded states in terms of dispersion integrals off the mass shell. Therefore the estimation of the corrections from experiment is rather difficult.

In ref./7/ a simple approach has been proposed in which no algebra is postulated. It is based on the sum rules, following from the usual one-dimensional dispersion relations for a fixed momentum transfer under some assumptions about the high energy behaviour of physical amplitudes. As far as in these relations the law of conservation of energy is fulfilled the intermediate states with smaller masses correspond to the nearest singularities and give, generally speaking, the main contribution to the amplitudes at low energies. For each process one may choose such amplitudes for which the contribution of far singularities is relatively smallest. In ref./7/ it was shown, that if in the sum rules for such amplitudes we confine ourselves to the nearest intermediate states, we can obtain the relations of two types:

1) a set of relations, which usually follow from SU(6) symmetry,

2) new relations, which together with the results of $SU(6)$ allow to obtain the magnetic moments of the proton and the neutron.

In this approach we are dealing only with observables therefore our assumptions about high energy behaviour permit an experimental test.

This paper is devoted to the development of the approach outlined in [7] and to application to the interactions of the baryons N, Λ, Σ and Ξ with pions and photons.

In § 2 the sum rule for the spin-flip pion-baryon scattering amplitudes is used to obtain the relations between the pion-baryon coupling constants and the decay widths of the nearest p-wave resonances. The relation for N and $N^*(1236)$ is in good agreement with experiment. The relations for other baryons allow one to obtain the coupling constants through the resonance widths. In § 3 the sum rules for the longitudinal amplitudes for the virtual photoproduction of pions on baryons are considered. They lead to the following results. 1) The isoscalar Pauli form factor of the nucleon is equal to zero; together with $SU(6)_W$ this gives the proportionality between the electric and magnetic Sachs form-factors of the proton in agreement with experiment, and predicts the magnetic moments of the proton and the neutron [7]. 2) We obtain relations between the form factors of baryons and those of decays $B^* \to B + \gamma$. At $k^2 = 0$ (i.e. for magnetic moments) these relations for the nucleon are in good agreement with experiment. For all baryons they agree well with the predictions of $SU(6)$ symmetry. In Conclusion other possible applications of the considered approach and the limits of its applicability are discussed.

2. Pion-Baryon Scattering

Let us consider the elastic scattering of pions on the baryons N, Λ, Σ and Ξ. The amplitudes for these processes in the Breit coordinate system have the form[x]

$$\bar{u}_2 T u_1 = -\frac{\sqrt{\vec{p}^2 + m^2}}{m} A + \{ i\vec{\sigma}[\vec{e}, \vec{p}] \frac{\lambda}{m} - E \} B , \qquad (2.1)$$

[x] We set $\hbar = c = m_\pi = 1$

where A and B are invariant amplitudes, introduced in [8], $\vec{e}$ is the unit vector perpendicular to the baryon momentum $\vec{p}$, m is the mass of the baryon, E is the energy of the pion, and $\lambda = \sqrt{E^2 - \vec{p}^2 - 1}$.

We see that for a given behaviour of the amplitude $\bar{u}_2 T u_1$ at $E \to \infty$ and fixed $\vec{p}^2$ the amplitude B decreases faster than A. At the same time it corresponds to a spin-flip process, i.e. to an inelastic process. Therefore there is no contradiction with the well-known (or supposed) high energy behaviour of the total scattering amplitude if we assume that at $E \to \infty$ and fixed $\vec{p}^2$

$$|B(E, \vec{p}^{\,2})| \le \frac{\text{const}}{E \ln^a E}, \quad a > 1 . \tag{2.2}$$

This assumption enables us to write dispersion relations both for B and EB without subtractions. Combining these relations in the same way as in [7], we get the following sum rule:

$$\int_{-\infty}^{\infty} \operatorname{Im} B(E, \vec{p}^{\,2})\, dE = 0 . \tag{2.3}$$

This equation is nontrivial only for the amplitude B which is an odd function under the crossing transformation (for which $\operatorname{Im} B(E) = \operatorname{Im} B(-E)$). Writing down the isotopic structure of B, we have for πN and $\pi \Xi$ scatterings

$$B = \delta_{i\ell}\, B_{odd} + \tfrac{1}{2}[\tau_\ell, \tau_i]\, B^{(-)} , \tag{2.4}$$

for $\pi\Sigma$ scattering

$$B = \delta_{i\ell}\, \delta_{jk}\, B_{odd} + \tfrac{1}{2}(\delta_{ij}\, \delta_{k\ell} + \delta_{ik}\, \delta_{j\ell})\, B'_{odd} + \\ + \tfrac{1}{2}(\delta_{ij}\, \delta_{k\ell} - \delta_{ik}\, \delta_{j\ell})\, B^{(-)} \tag{2.5}$$

and for $\pi\Lambda$ scattering

$$B = \delta_{i\ell}\, B_{odd} , \tag{2.6}$$

where i, ℓ and j, k are the isotopic indices of pions and Σ particles, and the subscript "odd" denotes the amplitudes we are interested in. Writting the sum rule (2.3) for these amplitudes in terms of the usual Mandelstam variables s and t and picking out the one-particle terms, we get for each reaction[x]

$$g^2 + \frac{1}{\pi} \int_{(m+1)^2}^{\infty} \operatorname{Im} B_{odd}(s,t)\, ds = 0 \tag{2.7}$$

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[x] For the reaction $\pi\Sigma \to \pi\Sigma$ $m = m_\Lambda$.

(sum rule for the amplitude B'_{odd} is discussed below), where g^2 stands for $g^2_{NN\pi}$, $g^2_{\Xi\Xi\pi}$, $g^2_{\Sigma\Sigma\pi}$ and $g^2_{\Sigma\Lambda\pi}$ in πN, $\pi\Xi$, $\pi\Sigma$ and $\pi\Lambda$ scatterings, respectively. The normalization of the constants corresponds to the Lagrangians

$$L_{BB\pi} = g_{BB\pi}\, i\bar{\psi}_B \gamma_5 \vec{\tau}\, \psi_B \vec{\phi}\ , B = N, \Xi\ .$$

$$L_{\Sigma\Sigma\pi} = g_{\Sigma\Sigma\pi} \bar{\psi}_k \gamma_5\, \psi_j\, \phi_i\, \epsilon_{ijk}\ . \qquad (2.8)$$

$$L_{\Sigma\Lambda\pi} = g_{\Sigma\Lambda\pi} \bar{\psi}_\Lambda \gamma_5 \vec{\psi}_\Sigma \vec{\phi} + h.c.\ .$$

In calculating $Im\, B_{odd}$ we shall take into account only the contributions of the nearest resonant states x/. Then this equality gives connections between the constants g^2 and the resonance decay widths. For calculating the contributions of the resonances in integrals (2.7) we restrict ourselves to the δ-approximation. Then we can use the effective Lagrangians of interaction of resonances with baryons and pions. It is easy to express the constants in these Lagrangians in terms of the decay widths. For the interaction of N^* with N and π we have

$$L_{N^*N\pi} = g_{N^*N\pi}\, \bar{\psi}\psi_\mu \partial_\mu \phi \chi^{a}\, \chi_{abc}\, \chi^{bc} + h.c.\ , \qquad (2.9)$$

where the particle fields are written in the form of the product of the coordinate and isotopic parts. $\psi\chi^a$, $\psi_\mu \chi^{abc}$ and $\phi\chi^{ab}$ are the fields of nucleon, resonance and pion respectively, χ^{ab} and χ^{abc} are the completely symmetric tensors of the second and third ranks and $a,b,c = 1,2$. The isotopic functions are determined as follows

$$\chi^1 = p\ , \quad \chi^2 = n\ ;$$

$$\chi^{11} = -\sqrt{2}\,\pi^+\ , \quad \chi^{12} = \pi^0\ , \quad \chi^{22} = \sqrt{2}\,\pi^-\ ; \qquad (2.10)$$

$$\chi^{111} = N^{*++}\ , \quad \chi^{112} = \frac{1}{\sqrt{3}} N^{*+}\ , \quad \chi^{122} = \frac{1}{\sqrt{3}} N^{*0}\ , \chi^{222} = N^{*-}\ .$$

To the antiparticles there correspond the functions with lower indices the lowering of which is performed by means of the tensor ϵ_{ab}.

For other resonances the coordinate part of the Lagrangian is the same as in (2.9) (they all have spin 3/2) and the isotopic structure coincides with

x/ Note that from the view point of SU(6) symmetry this corresponds to the account of the states entering the 56-plet.

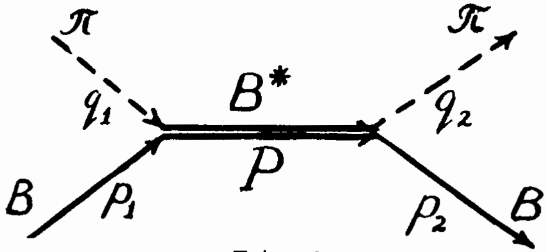
that of the Lagrangians (2.8) (the isotopic spins of Ξ^*, Ξ and Σ^*, Σ are equal to each other, respectively).

The decay widths of the resonances are equal to ($B = N, \Xi, \Sigma$)

$$\Gamma_{B^*B\pi} = \frac{g^2_{B^*B\pi}}{6\pi} \frac{q^3 (m+E)}{M}, \quad \Gamma_{\Sigma^*\Lambda\pi} = \frac{g^2_{\Sigma^*\Lambda\pi}}{12\pi} \frac{q^3 (m+E)}{M}, \tag{2.11}$$

where E is the baryon energy, q is the pion momentum in the rest system of the resonance and M is the resonance mass.

The contribution of the resonance to the amplitude B_{odd} entering (2.7) corresponds to the diagram Fig. 1,



F i g. 1

where the propagator of the resonance (without taking into account the isotopic functions) is equal to

$$P_{\mu\nu} = \frac{-i}{(2\pi)^4} \frac{M - i\gamma P}{P^2 + M^2} \{\delta_{\mu\nu} - \frac{1}{3}\gamma_\mu \gamma_\nu + \frac{i}{3M}(\gamma_\mu P_\nu - \gamma_\nu P_\mu) + \frac{2}{3M^2} P_\mu P_\nu\}. \tag{2.12}$$

Finding this contribution and using (2.11) we finally get from (2.7)

$$\frac{g^2_{BB\pi}}{4\pi} - a_{B^*B} \frac{M}{q} [\frac{1}{E-m} - \frac{3x}{E+m}] \Gamma_{B^*B\pi} = 0, \tag{2.13}$$

where

$$a_{N^*N} = \frac{2}{3}, \quad a_{\Xi^*\Xi} = \frac{1}{3}, \quad a_{\Sigma^*\Lambda} = 1, \quad a_{\Sigma^*\Sigma} = \frac{1}{2} \tag{2.14}$$

and x is the cosine of the scattering angle in the center-of-mass system. We see, that the dependence on x in (2.13) practically falls out x/.

Inserting $g^2_{NN\pi}/4\pi = 14{,}6$ into (2.13) we get for the width of the decay

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x/ For example, for πN scattering the square bracket in (2.13) has the form $[5 - 0.2x]$. In other cases the dependence on x is even weaker. In order not to do with the region of unobservable angles in the dispersion relations we can, from the very beginning, set $x = 1$ (the forward scattering); the figures written below correspond to this case.

$N^* \to N\pi$ a value $\Gamma_{N^*N\pi} = 117$ MeV, which is in good agreement with the experimental data 120.0 ± 1.5 MeV/9/. For other reactions we can derive the coupling constants knowing the decay widths. The results of the calculations are collected in the table. In the last column of the table there are data for the baryon coupling constants obtained in the scheme of SU(6) symmetry, broken in a special way/10/. The differences are considerable x/. In what follows we shall see, that for electromagnetic interactions the results of SU(6) symmetry and the considered approach are in good agreement with each other.

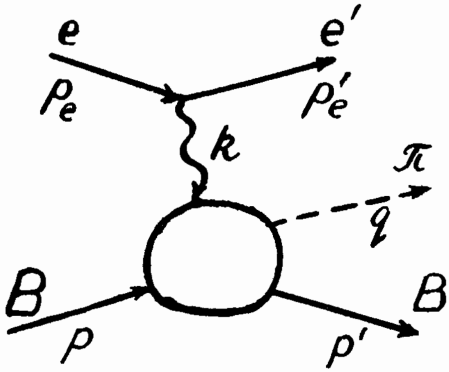
T a b l e

B B	$\frac{g^2_{BB\pi}}{4\pi}$	$\Gamma_{B^*B\pi}$ (MeV)	Error in %	$\frac{g^2_{BB\pi}}{4\pi}$ in broken SU(6) symmetry
$\Xi\Xi$	3,09	7,5	22	0,74
$\Sigma\Sigma$	4,3	4,4	26	0,75
$\Sigma\Lambda$	11,42	39,6	7	12,51

3. Electro- and Photoproduction of Pions on Baryons

It is possible to apply a similar approach to other processes. Let us consider the electroproduction of pions on baryons

$$e + B \to e + B + \pi$$



(Fig. 2)

It is described by the amplitude of the virtual photoproduction/11/

$$T_\mu = \langle qp' | j_\mu(0) | p \rangle = -i \sum_{i=1}^{6} \bar{u}(p')\gamma_5 R_{i\mu} u(p) F_i(s,t,k^2), \qquad (3.1)$$

where j_μ is the electromagnetic current, and

x/ Remind, that for $BB\pi$ interactions nonbroken SU(6) symmetry is not applicable. The introduction of breaking is generally speaking not unique. On the other hand, we do not consider here the higher resonances.

$$R_{1\mu} = p_\mu\, p'k - p'_\mu\, pk\ ,$$

$$R_{2,3\mu} = i[(p_\mu \pm p'_\mu)\gamma k - \gamma_\mu (p \pm p')k]\ , \tag{3.2}$$

$$R_{4\mu} = \gamma k \gamma_\mu - \gamma_\mu \gamma k\ ,$$

$$R_{5\mu} = (p_\mu - p'_\mu) k^2 - k_\mu (p - p')k\ ,$$

$$R_{6\mu} = - i\gamma k R_{4\mu}$$

At $k^2 = 0$ the components of T which are transversal with respect to $\vec{k}$ describe the usual photoproduction. Let us assume now, that at $s \to \infty$ and fixed t

$$|T_\mu| \leqslant \mathrm{const}\, s\ln^a s\ , \quad a < -1\ . \tag{3.3}$$

As is shown in /11/ this means that the differential cross section for the forward electroproduction in the lab.system at high energy is independent of the time- and the longitudinal polarization of the virtual photon. It is shown in /11,7/, that in this case the sum rule (2.3) holds for the amplitude F_6 at not very high k^2. The isotopic structure of F_6 has the form: for electroproduction on N and Ξ

$$F_6 = \delta_{\ell 3} F_{odd}^{(V)} + \tau_\ell F_{odd}^{(S)} + \tfrac{1}{2}[\tau_\ell , \tau_3] F^{(-)}\ , \tag{3.4}$$

for electroproduction on Σ

$$F_6 = \delta_{\ell 3}\, \delta_{jk}\, F_{odd}^{(V)} + \tfrac{1}{2}(\delta_{3j}\, \delta_{k\ell} + \delta_{3k}\,\delta_{j\ell})\, F'_{odd} + \tag{3.5}$$
$$+ \tfrac{1}{2}(\delta_{3j}\, \delta_{k\ell} - \delta_{3k}\delta_{j\ell})\, F^{(-)} + i\epsilon_{jk\ell}\, F_{odd}^{(S)}\ .$$

and for electroproduction on Λ

$$F_6 = \delta_{\ell 3}\, F_{odd}^{(V)}. \tag{3.6}$$

Picking out in the sum rules for F_{odd} the one-particle terms (the amplitude F'_{odd} is discussed in Conclusion) we have

$$\frac{g}{2} F_\mu^{(V,S)}(k^2) + \frac{1}{\pi}\int_{(m+1)^2}^{\infty} \mathrm{Im}\, F_{odd}^{(V,S)}(s,t,k^2)\, ds = 0\ , \tag{3.7}$$

where g are the same constants as in (2.7) and $F_\mu^{(V,S)}(k^2) = \mu'_{V,S} F_2^{(V,S)}(k^2)$, $F_2^{(V,S)}$ are the isovector and isoscalar Pauli form factors of baryons and $\mu'_{V,S}$ are the corresponding anomalous magnetic moments[x]. Let us take into account in integral (3.7) the same resonances, as in the scattering, with the aid of the effective Lagrangians introduced in § 2 and the following Lagrangians[xx]

$$L_{B^*B\gamma} = (\bar{\psi}\gamma_\mu \gamma_5 \psi_\nu + \frac{1}{M} \bar{\psi}\gamma_5 \partial_\mu \psi_\nu) F_{\mu\nu} I_{B^*B} + \text{h.c.} \quad , \tag{3.8}$$

where I_{B^*B} contains the form factors and the isotopic functions:

$$\begin{aligned} I_{N^*N} &= G^{(V)}_{N^*N} \chi^{a} \chi_{abc} (\tau_3)^b_d \epsilon^{dc} , \\ I_{\Xi^*\Xi} &= G^{(S)}_{\Xi^*\Xi} \Psi_a \psi^a + G^{(V)}_{\Xi^*\Xi} \Psi_a (\tau_3)^a_b \psi^b , \\ I_{\Sigma^*\Sigma} &= (G^{(S)}_{\Sigma^*\Sigma} \delta_{ij} + i\epsilon_{ij3} G^{(V)}_{\Sigma^*\Sigma}) \Psi_i \psi_j , \\ I_{\Sigma^*\Lambda} &= G^{(V)}_{\Sigma^*\Lambda} \Psi_3 , \end{aligned} \tag{3.10}$$

χ^a and χ^{abc} are determined in (2.10), Ψ^a and ψ^a are the isospinor functions of Ξ^* and Ξ, Ψ_i and ψ_j are the isovector functions of Σ^* and Σ.

Generally speaking, from the requirements of Lorentz and gauge invariance it is possible to construct three independent Lagrangians describing the interaction of the baryon, resonance $(\frac{3}{2}^+)$ and photon /12/. But if we require in accordance with the experimental data on photo- and electroproduction on nucleons, that the electric and longitudinal quadrupoles should not contribute to electroproduction of pions on baryons at energies close to the resonance ones, then these three Lagrangians are reduced to the only effective Lagrangian (3.8) which corresponds to the magnetic-dipole transition.

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x/ For nucleon $F^{(V,S)}_\mu = \frac{1}{2}(F^{(p)}_\mu \mp F^{(n)}_\mu)$, for Ξ $F_\mu = \frac{1}{2}(F^{(\Xi^0)}_\mu \mp F^{(\Xi^-)}_\mu)$, for Σ $F^{(V,S)}_\mu = \frac{1}{2}(F^{(\Sigma^+)}_\mu \mp F^{(\Sigma^-)}_\mu)$, $F^{(S)}_\mu = F^{(\Sigma^0)}_\mu$ and for Λ $F^{(V)}_\mu = F^{(\Sigma\Lambda)}_\mu$, where the last form factor is normalized by the condition $L_{\Sigma\Lambda\gamma} = \frac{i}{2} F^{(\Sigma\Lambda)}_\mu \bar{\psi}_\Sigma \gamma_\rho \gamma_\nu \psi_\Lambda F_{\rho\nu} + \text{h.c.}$

xx/ Note, that these Lagrangians can be transformed to

$$L_{B^*B\gamma} = -\frac{1}{M} \epsilon_{\mu\nu\sigma\rho} \bar{\psi}\partial_\mu \psi_\nu \partial_\sigma A_\rho I_{B^*B} + \text{h.c.} \tag{3.9}$$

As it is shown in Appendix the magnetic moments (the form factors) of the transitions $B^* \to B + \gamma$ are connected with the form factors, entering (3.10), as follows:

$$\mu\,(N^{*+,0} \to N^{+,0} + \gamma) = \frac{2\sqrt{2}}{3}\, G^{(V)}_{N^*N} \,,$$

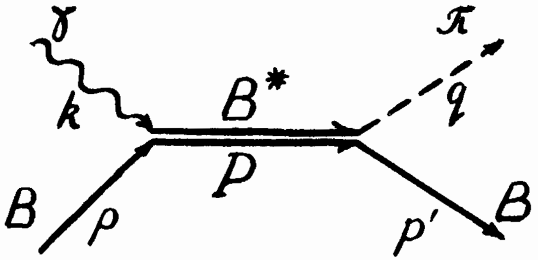
$$\mu(\Xi^{*0,-} \to \Xi^{0,-} + \gamma) = \sqrt{\frac{2}{3}}\,(G^{(S)}_{\Xi^*\Xi} \pm G^{(V)}_{\Xi^*\Xi}) \,,$$

$$\mu\,(\Sigma^{*+,-} \to \Sigma^{+,-} + \gamma) = \sqrt{\frac{2}{3}}\,(G^{(S)}_{\Sigma^*\Sigma} \pm G^{(V)}_{\Sigma^*\Sigma}) \,, \qquad (3.11)$$

$$\mu(\Sigma^{*0} \to \Sigma^{0} + \gamma) = \sqrt{\frac{2}{3}}\, G^{(S)}_{\Sigma^*\Sigma} \,,$$

$$\mu(\Sigma^{*0} \to \Lambda + \gamma) = \sqrt{\frac{2}{3}}\, G^{(V)}_{\Sigma^*\Lambda} \,.$$

The constant $G^{(V)}_{N^*N}$ and the magnetic moment of transition $N^* \to N + \gamma$ can be found from experiment in the following manner. With the aid of the diagram Fig. 3



F i g. 3

we can find the contribution of the magnetic dipole transition to the total cross section for photoproduction of neutral pions on protons near the resonance:

$$\sigma = 8\pi \frac{q}{k}\,|M^{(+)}_{1+}|^2 \,, \qquad (3.12)$$

$$\frac{M^{(+)}_{1+}}{kq} = \frac{G_{N^*N}\; g_{N^*N\pi}}{9\pi}\;\frac{\sqrt{(E+m)(E'+m)}}{M^2 - W^2 - iM\Gamma} \,, \qquad (3.13)$$

where E and E' are the initial and final nucleon energies in the centre-of-mass system, W is the total energy, Γ is the width of the resonance N^*. Then taking into account (3.11) and (2.11) we have at the resonance

$$\sigma = \frac{2}{3}\,\frac{k(E'+m)}{M\Gamma}\,\mu^2\,(N^* \to N + \gamma)\,. \qquad (3.14)$$

On the other hand it is known from experiment that near the resonance this process corresponds almost completely to the magnetic dipole transition. Therefore, it is possible to substitute in (3.14) the total experimental cross section

$\sigma = 0.25 \pm 0.01$ mb[13]. Using for the decay width $N^* \to N + \pi$ the experimental value $\Gamma = 120 \pm 1{,}5$ MeV[9] we get the experimental value of the magnetic moment of the transition $N^* \to N + \gamma$ [x]

$$\mu_{exp} (N^* \to N + \gamma) = \frac{2\sqrt{2}}{3} (1{,}25 \pm 0{,}02)\mu(p) . \qquad (3.15)$$

Let us return now to the dispersion sum rules (3.7). From the sum rule for the isoscalar amplitude for electroproduction on nucleons we immediately deduce the relation

$$F_\mu^{(p)}(k^2) + F_\mu^{(n)}(k^2) = 0 . \qquad (3.16)$$

If we combine this relation with the following relations for the Sachs form factors of proton and neutron, which are predicted by $SU(6)_W$ symmetry[14]

$$G_E^{(n)}(k^2) = 0 ;$$

$$\frac{G_M^{(p)}(k^2)}{G_M^{(n)}(k^2)} = -\frac{3}{2} , \qquad (3.17)$$

we get that the ratio between magnetic and electric form factors of proton is constant, independent of k^2, what is in good agreement with experiment, and this ratio is equal to

$$\frac{G_M^{(p)}(k^2)}{G_E^{(p)}(k^2)} = 3 . \qquad (3.18)$$

If we consider the equations (3.17) and (3.18) at $k^2 = 0$ we get the following values for magnetic moments of proton and neutron[4,7]

$$\mu_p = 3 , \quad \mu_n = -2 \qquad (3.19)$$

(in nuclear magnetons) which are in good agreement with experiment [xx].

We consider the other sum rules (3.7) at $k^2 = 0$. Calculating the contribution of resonances into the amplitudes $F_{odd}^{(V,S)}(s,t,k^2)$ in (3.7) using the diagram of Fig. 3 we get

$$\frac{g_{BB\pi}}{2} \mu'_{V,S}(B) - b_{BB} \frac{g_{B^*B\pi} G^{(V,S)}_{B^*B}}{6M} [m^2 + \frac{Mm}{2} + \frac{m^3}{2M} - \frac{3}{2} - \frac{m}{2M} - 3(kq)] = 0 \qquad (3.20)$$

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x/ The experimental estimate of this moment given in paper[15] with reference to paper[16] seems to be erroneous.

xx/ Note that in paper[4] the equations (3.19) are obtained from the sum rule for the photoproduction amplitude A (in notations of[17]) off the mass shell, while on the mass shell the one-nucleon term in this amplitude contains no magnetic moments (see formula (8.4) of ref.[17]).

$$b_{\Xi\Xi} = b_{\Sigma\Sigma} = b_{\Sigma\Lambda} = 1 , \quad b_{NN} = \frac{4}{3} .$$

Taking into account (2.11) and (3.11) we deduce from this equation

$$\mu(N^{*+,0} \to N^{+,0} + \gamma) = 1,28 \cdot \frac{2\sqrt{2}}{3} \mu(p) ,$$

$$\mu(\Xi^{*0,-} \to \Xi^{0,-} + \gamma) = 1.09 \cdot \sqrt{2}\,\mu'(\Xi^{0,-}) ,$$

$$\mu(\Sigma^{*+,-,0} \to \Sigma^{+,-,0} + \gamma) = 1.15 \cdot \sqrt{2}\,\mu'(\Sigma^{+,-,0}) , \qquad (3.21)$$

$$\mu(\Sigma^{*0} \to \Lambda + \gamma) = 1.17 \cdot \sqrt{2}\,\mu(\Sigma^{0} \to \Lambda + \gamma) .$$

The first relation is in good agreement with experiment (3.15). All relations are in good agreement with the predictions of SU(6) symmetry, which gives I instead of the first factor on the right-hand sides of (3.21) for all particles, except Ξ^- and Σ^-. For Ξ^- and Σ^- SU(6) symmetry gives 0 on the left-hand sides of these relations and $\mu'(\Sigma^-) = \mu'(\Xi^-) = 0.026\,\mu(p)$ for the right-hand sides, Therefore, the relations for Ξ^- and Σ^- also do not contradict the results of SU(6) symmetry.

Conclusion

We see that the simple dynamical approach considered above makes it possible to obtain many relations which are usually derived from SU(6) symmetry.

We have also applied this method to the reactions $\pi\Sigma \to \pi\Lambda$ and $e\Sigma \to e\Lambda\pi$ and have got the relations between the constants $g_{\Sigma\Sigma\pi}$ and $g_{\Sigma\Lambda\pi}$ and between the magnetic moments $\mu(\Sigma^* \to \Sigma\gamma)$ and μ'_Σ which are in full agreement with the table and (3.21).

This approach can be applied to other reactions, first of all, to the Compton scattering on baryons. This will give directly the relations between electromagnetic constants without using the results for scattering.

Let us discuss the limits of applicability of the present approach. First of all, it works only for the amplitudes which are odd under the crossing transformation. Such amplitudes are absent in scattering and photoproduction of mesons on mesons. At the present time the predictions for mesons can be made only on the basis of symmetries. Therefore, the experimental test of these predictions seems to be most interesting. It should be noted that up to now the predictions of $SU(6)$ for mesons are supported experimentally much weaker than for baryons.

Further, the present method does not permit to connect the properties of baryons with different strangenesses.

In this paper we have considered the amplitudes for which high energy contributions are relatively unimportant and have taken into account the nearest resonances with masses not bigger than that of the corresponding baryons plus $2m_\pi$. But these amplitudes have kinematical factors which strongly reduce the S-wave contributions at low energies. Therefore, we can not use the present method for the amplitudes B'_{odd} and F'_{odd} in which the nearest resonance $\Lambda(1405)$ appears in the S-wave. For these amplitudes as well as for determination of the ΛNK coupling constant one should consider other intermediate states along with the nearest resonances.

The authors are sincerely grateful to N.N. Bogolubov for the attention to this work and to D.V.Shirkov for a valuable discussion.

APPENDIX

The wave function of a particle with spin 3/2 satisfies the equations [18]

$$\begin{aligned} &\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + M\right)\psi_\nu = 0, \\ &\gamma_\mu \psi_\mu = 0. \end{aligned} \qquad (A.1)$$

We go over to the momentum representation

$$\psi_\mu(x) = \frac{1}{(2\pi)^{3/2}} \int U_\mu(p) e^{ipx} dp. \qquad (A.2)$$

Then in accordance with the equations (A.1) $U_\mu(p)$ may be written in the form

$$U_\mu(p) = \sqrt{\frac{p_0+M}{2M}} \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\,\vec{p}}{p_0+M} \end{pmatrix} \phi_\mu , \qquad (A.3)$$

where four two-dimensional spinors satisfy the conditions

$$\phi_4 = i\frac{\vec{p}\vec{\phi}}{p_0}, \quad \vec{\sigma}\vec{\phi} = \frac{(\vec{\sigma}\vec{p})(\vec{p}\vec{\phi})}{p_0(p_0+M)} \qquad (A.4)$$

and the factor $\frac{p_0+M}{2M}$ corresponds to the normalization

$$\sum_{\mu} \bar{U}_{\mu}(p) U_{\mu}(p) = 1, \quad \sum_{\mu} \phi_{\mu}^{+} \phi_{\mu} = 1. \tag{A.5}$$

The conditions (A.4) exclude the particle with spin 1/2. In the particle rest system they have the form

$$\phi_4 = 0, \quad \vec{\sigma}\vec{\phi} = 0. \tag{A.6}$$

In the same system ϕ_μ are connected with the functions describing the states with fixed spin projections as follows

$$-\frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) = \begin{pmatrix} \phi_{3/2} \\ \frac{1}{\sqrt{3}}\phi_{1/2} \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) = \begin{pmatrix} \frac{1}{\sqrt{3}}\phi_{-1/2} \\ \phi_{-3/2} \end{pmatrix} \tag{A.7}$$

$$\phi_3 = \sqrt{\frac{2}{3}} \begin{pmatrix} \phi_{1/2} \\ \phi_{-1/2} \end{pmatrix}$$

Let us consider now the Lagrangian (3.9) describing the interaction of the resonance $(\frac{3}{2}^{+})$ with baryon and photon. We go over in this Lagrangian to the momentum representation and write it in the resonance rest system taking into account (A.6) and (A.7) and assuming that the magnetic field $\vec{H}$ is directed along the axis z. We get:

$$L_{B^*B\gamma} = i\sqrt{\frac{2}{3}}\sqrt{\frac{E+m}{2m}}\left(\phi^{B^*}_{1/2}\phi^{B}_{1/2} + \phi^{B^*}_{-1/2}\phi^{B}_{-1/2}\right) I_{B^*B} H, \tag{A.8}$$

where E is the baryon energy in the decay $B^* \to B + \gamma$ in the resonance rest system. A similar expression for baryons has the form

$$L_{BB\gamma}(p) = i\mu_B\left(\phi^{B}_{1/2}\phi^{B}_{1/2} - \phi^{B}_{-1/2}\phi^{B}_{-1/2}\right) H. \tag{A.9}$$

From comparison of (A.8) and (A.9), by analogy with baryons we determine the magnetic moment of the transition $B^* \to B + \gamma$ as follows

$$\mu(B^* \to B + \gamma) = \sqrt{\frac{2}{3}}\, I_{B^*B} \cdot \qquad (A.10)$$

From here taking into account the isotopic structure I_{B^*B} we get the relations (3.11). Determining the magnetic moment (A.10), we do not introduce into it the factor $\sqrt{\frac{E+m}{2m}}$ which is close to unity.

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