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Introduction

Use of the polarized proton targets considerably extends possibilities of measuring polarization effects in various reactions. So, in the case of the meson-nucleon scattering the polarized target makes it possible to measure the parameters R and A which can not be determined from experiments with unpolarized target. As is shown in ref. [1] the problem of a **direct** reconstruction of the meson-nucleon scattering matrix from experimental data at a given angle may be solved if the cross section for scattering of mesons by an unpolarized proton target, the nucleon polarizaton produced in the scattering from an unpolarized target (asymmetry in the scattering from polarized target) as well as the parameters R and A are measured. This method of reconstruction of the scattering amplitude may turn out to be most effective in the region of high energies where unique phase shift analysis is difficult due to the large number of states involved and to the contribution of inelastic processes leading to the phase shifts being complex. The formulas for the reconstruction of the meson nucleon scattering matrix obtained in ref. [1] are valid only in the nonrelativistic region. As is known, in the relativistic case while obtaining formulas which connect experimentally observed quantities with the scattering amplitudes in c.m.s. it is necessary, in addition to the usual relativistic kinematics, to take into account also a specific spin rotation [2-5]. In Section 2 we get relativistic formulas for the direct reconstruction of the elastic meson-nucleon scattering matrix from data at a fixed angle and energy. In Section III the pion-nucleon scattering is considered. Owing to the isotopic invariance the pion-nucleon scattering matrix is characterized by four complex functions of the energy and the scattering angle. We shall show that to determine these functions (up to over-all phase) it is necessary for given value of the angle and the energy to measure nine quantities.

II. Meson-Nucleon Scattering

We consider the elastic scattering of mesons with spin zero on nucleons. Let $\vec{a}_\ell$ be unit vector in the lab. sys. The projection of the polarization vector of a recoil nucleon on the direction $\vec{a}_\ell$ measured in the lab. sys. is /2/

$$\vec{P}' \vec{a}_\ell = \frac{1}{\sigma} \, Sp \, \vec{\sigma} \, (\vec{a}_\ell)_R \, M(\vec{k}', \vec{k}) \, ½ \, (1 + \vec{\sigma} \, . \, \vec{P}) \, M^+ (\vec{k}', \vec{k}) \tag{1}$$

where $M(\vec{k}',\vec{k})$ is the scattering matrix in c.m.s. ($\vec{k}$ ($\vec{k}'$) is the unit vector in the direction of the momentum of the initial (final) mesons in c.m.s.), $\vec{P}$ is the initial polarization $\sigma = Sp \, M(\vec{k}', \vec{k}) \, ½ \, (1 + \vec{\sigma} \, \vec{P}) \, M^+ (\vec{k}', \vec{k})$ is the differential scattering cross section in c.m.s. and

$$(\vec{a}_\ell)_R = R_{\vec{n}} (\Omega) \, \vec{a}_\ell \tag{2}$$

Here $R_{\vec{n}} (\Omega)$ is the rotation operator at the angle Ω around the normal $\vec{n}$ ($\vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}$). The rotation (2) is considered in detail in refs /2-5/ where a general expression for the rotation angle is obtained. By means of this expression it can be shown that in the case of elastic meson-nucleon scattering the rotation angle Ω in (2) is

$$\Omega = 2\Phi_\ell - \Phi \tag{3}$$

where $\Phi = \pi - \theta$ is the recoil nucleon angle in c.m.s. (θ is the angle between $\vec{k}$ and $\vec{k}'$) and Φ_ℓ is the angle between the momenta of the initial meson and the recoil nucleon in lab. sys. It is obvious that in the nonrelativistic limit Ω vanishes. We note also that

$$tg \, \Phi_\ell = \frac{(M^2 + \mu^2 + 2ME)^{½}}{E + M} \, tg \, \frac{\Phi}{2} \tag{4}$$

where μ and M are the meson and nucleon masses, and E is the meson energy in lab. sys.

To determine experimentally measured quantities we introduce in the lab. sys. two orthonormalized sets of vectors

$$\vec{n}_\ell = \frac{\vec{k}_\ell \times \vec{k}'_\ell}{|\vec{k}_\ell \times \vec{k}'_\ell|} = \vec{n} \, , \qquad \vec{k}''_\ell \, , \quad \vec{s}''_\ell = \vec{n}_\ell \times \vec{k}''_\ell \tag{5}$$

$$\vec{n}_\ell \, , \quad \vec{k}_\ell \, , \quad \vec{s}_\ell = \vec{n}_\ell \times \vec{k}_\ell \tag{6}$$

Here $\vec{k}_\ell$ $(\vec{k}'_\ell)$, $\vec{k}''_\ell$ are the unit vectors in the directions of the momenta of the initial (final) meson and recoil nucleon respectively. We expand the final nucleon polarization $\vec{P}'$ in the set of vectors (5) and the initial one in the set (6).

Using the invariance considerations, from (1) we get

$$\sigma(\vec{P}'\vec{n}_\ell) = \sigma_0(P_0 + D(\vec{P}\,\vec{n}_\ell))$$
$$\sigma(\vec{P}'\vec{k}''_\ell) = \sigma_0(A'\vec{k}_\ell + R'\vec{s}_\ell)\vec{P} \qquad (7)$$
$$\sigma(\vec{P}'\vec{s}''_\ell) = \sigma_0(A\vec{k}_\ell + R\vec{s}_\ell)\cdot\vec{P}$$

Here

$$\sigma_0 = \tfrac{1}{2}\,\mathrm{Sp}\,M\,M^+ \qquad (8)$$

is the differential cross section for scattering from an unpolarized target in c.m.s.,

$$P_0 = \frac{1}{2\sigma_0}\,\mathrm{Sp}\,\vec{\sigma}\,\vec{n}_\ell M M^+ = \frac{1}{2\sigma_0}\,\mathrm{Sp}\,M\,\vec{\sigma}\cdot\vec{n}_\ell M^+ \qquad (9)$$

is the polarization of the recoil nucleons arising in this case (the left-right asymmetry arising in the scattering from a target polarized orthogonally to the scattering plane) and

$$\sigma_0 D = \tfrac{1}{2}\,\mathrm{Sp}\,\vec{\sigma}\,\vec{n}_\ell M\,\vec{\sigma}\,\vec{n}_\ell M^+$$
$$\sigma_0 A' = \tfrac{1}{2}\,\mathrm{Sp}\,\vec{\sigma}\,(\vec{k}''_\ell)_R M\,\vec{\sigma}\,\vec{k}_\ell M^+$$
$$\sigma_0 R' = \tfrac{1}{2}\,\mathrm{Sp}\,\vec{\sigma}\,(\vec{k}''_\ell)_R M\,\vec{\sigma}\,\vec{s}_\ell M^+ \qquad (10)$$
$$\sigma_0 A = \tfrac{1}{2}\,\mathrm{Sp}\,\vec{\sigma}\,(\vec{s}''_\ell)_R M\,\vec{\sigma}\,\vec{k}_\ell M^+$$
$$\sigma_0 R = \tfrac{1}{2}\,\mathrm{Sp}\,\vec{\sigma}\cdot(\vec{s}''_\ell)_R M\,(\vec{\sigma}\,\vec{s}_\ell)\,M^+$$

Using the condition of invariance of the M matrix under the reflections in the reaction plane[6]

$$\vec{\sigma}\,\vec{n}\,M\,\vec{\sigma}\cdot\vec{n} = M \qquad (11)$$

it is easy to show that the parameters D, A', R', A and R in the relativistic case obey the same relations as in the nonrelativistic one:

$$D = 1 , \qquad A' = R , \qquad R' = - A \tag{12}$$

The matrix $M(\vec{k}', \vec{k})$ has, as is known, the following general form

$$M(\vec{k}', \vec{k}) = a + b \vec{\sigma}\, \vec{n} \tag{13}$$

where a and b are the complex functions of the energy and the scattering angle.

We introduce also the functions

$$g = \frac{a+b}{\sqrt{2}} , \qquad h = \frac{a-b}{\sqrt{2}} \tag{14}$$

By means of (2) and (3) we find

$$(\vec{k}''_{\ell})_R = \vec{k} \cos\alpha + \vec{n} \times \vec{k} \sin\alpha \tag{15}$$

$$(\vec{s}''_{\ell})_R = -\vec{k} \sin\alpha + \vec{n} \times \vec{k} \cos\alpha$$

where

$$\alpha = \Omega - \Phi_\ell = \Phi_\ell - \Phi \tag{16}$$

Using these relations, from eqs. (8)-(10), (13) and (14) we get the following expressions for the observables

$$\sigma_0 = |g|^2 + |h|^2$$

$$\sigma_0 P_0 = |g|^2 - |h|^2 \tag{17}$$

$$\sigma_0 R = 2\,\mathrm{Re}\, gh^* \cos\alpha - 2\,\mathrm{Im}\, gh^* \sin\alpha$$

$$\sigma_0 A = -2\,\mathrm{Re}\, gh^* \sin\alpha - 2\,\mathrm{Im}\, gh^* \cos\alpha$$

From where we easily find

$$|g|^2 = \tfrac{1}{2}\sigma_0 (1 + P_0)$$

$$|h|^2 = \tfrac{1}{2}\sigma_0 (1 - P_0) \tag{18}$$

$$gh^* = \tfrac{1}{2}\sigma_0 (R - iA)\, e^{-i\alpha}$$

Thus, the measurement of the cross section σ_0 and of the polarization P_0 (the left-right asymmetry) for given value of the angle and the energy enable us to determine the moduli of the amplitudes g and h. Their relative phase is unambiguously determined by the parameters R and A.

In conclusion we note that the parameters R, A and P_0 are connected by the quadratic relation:

$$R^2 + A^2 + P_0^2 = 1 \tag{19}$$

which can be easily obtained by means of eq (18). From this relation it is seen that if P_0 and R are measured with a sufficient accuracy then the absolute value of the parameter A is determined by means of (19) and to reconstruct unambiguously the relative phase it is necessary to determine only the sign of A.

III. Pion-Nucleon Scattering

In this section we consider the pion-nucleon scattering. Let us denote by m and m' the projections of the isotopic spin of the initial and final nucleon and by μ and μ' those of the initial final meson. Due to the isotopic invariance transition amplitude is of the form

$$\langle \mu' m' | M | \mu m \rangle = \sum_J (1\mu' \tfrac{1}{2} m' | 1 \tfrac{1}{2} J M) M_J \cdot (1\mu \tfrac{1}{2} m | 1\tfrac{1}{2} J M) \tag{20}$$

Here J is the total isotopic spin of the system (J assumes the values 1/2 and 3/2) and $(1\mu \tfrac{1}{2} m | 1\tfrac{1}{2} J M)$ is the Clebsh-Gordan coefficient. The scattering matrix in the state with the total isotopic spin J is (c.m.s.)

$$M_J = a_J + b_J \vec{\sigma}\,\vec{n} \tag{21}$$

As in the above-considered case it is convenient to introduce

$$g_{2J} = \frac{1}{\sqrt{2}} (a_J + b_J) \tag{22}$$

$$h_{2J} = \frac{1}{\sqrt{2}} (a_J - b_J)$$

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x) The relation (19) was obtained earlier by N.P.Klepikov (private communication).

By means of (20)-(22) we get the following expressions for the observables

$$(\sigma_0)_{\mu'm';\mu m} = \sum_{J,J'} F(\mu'm';\mu m; JJ')(\mathrm{Re}\, g_{2J}\, g^*_{2J'} + \mathrm{Re}\, h_{2J}\, h^*_{2J'})$$

$$(\sigma_0 P_0)_{\mu'm';\mu m} = \sum_{J,J'} F(\mu'm';\mu m; JJ')(\mathrm{Re}\, g_{2J}\, g^*_{2J'} - \mathrm{Re}\, h_{2J}\, h^*_{2J'}) \qquad (23)$$

$$(\sigma_0 R)_{\mu'm';\mu m} = \sum_{J,J'} F(\mu'm';\mu m; JJ')[(\mathrm{Re}\, g_{2J}\, h^*_{2J'} + \mathrm{Re}\, g_{2J'}\, h^*_{2J})\cos\alpha -$$

$$-(\mathrm{Im}\, g_{2J}\, h^*_{2J'} + \mathrm{Im}\, g_{2J'}\, h^*_{2J})\sin\alpha\,].$$

$$(\sigma_0 A)_{\mu'm';\mu m} = -\sum_{J,J'} F(\mu'm';\mu m; JJ')[(\mathrm{Re}\, g_{2J}\, h^*_{2J'} + \mathrm{Re}\, g_{2J'}\, h^*_{2J})\sin\alpha$$

$$+(\mathrm{Im}\, g_{2J}\, h^*_{2J'} + \mathrm{Im}\, g_{2J'}\, h^*_{2J})\cos\alpha\,].$$

Here

$$F(\mu'm';\mu m; JJ') = (1\mu'\tfrac{1}{2}m' \mid 1\tfrac{1}{2}JM)\,(1\mu'\tfrac{1}{2}m' \mid 1\tfrac{1}{2}J'M)\,\cdot \qquad (24)$$

$$\cdot\,(1\mu\tfrac{1}{2}m \mid 1\tfrac{1}{2}JM)\,(1\mu\tfrac{1}{2}m \mid 1\tfrac{1}{2}J'M)$$

The following processes are studied experimentally

$$\pi^+ + p \to \pi^+ + p \qquad (25.1)$$

$$\pi^- + p \to \pi^- + p \qquad (25.2)$$

$$\pi^- + p \to \pi^0 + n \qquad (25.3)$$

Let us look what experiments are to be done for an unambiguous (up to the over-all phase) reconstruction of the amplitudes g_{2J} and h_{2J}. It is obvious that $|g_3|$ and $|h_3|$ can be determined if the cross section and the polarization of the processes (25.1) with an unpolarized target are measured

$$|g_3|^2 = \tfrac{1}{2}[(\sigma_0)_{\pi^+;\pi^+} + (\sigma_0 P_0)_{\pi^+;\pi^+}].$$

$$|h_3|^2 = \tfrac{1}{2}[(\sigma_0)_{\pi^+;\pi^+} - (\sigma_0 P_0)_{\pi^+;\pi^+}]. \qquad (26)$$

From (23) we get also that

$$|g_1|^2 = 3/4\,[((\sigma_0)_{\pi^-;\,\pi^-} + (\sigma_0)_{\pi^0;\,\pi^-} - 1/3(\sigma_0)_{\pi^+;\,\pi^+})$$

$$+ ((\sigma_0 P_0)_{\pi^-;\,\pi^-} + (\sigma_0 P_0)_{\pi^0;\,\pi^-} - 1/3(\sigma_0 P_0)_{\pi^+;\,\pi^+})]$$

$$|h_1|^2 = 3/4[((\sigma_0)_{\pi^-;\,\pi^-} + (\sigma_0)_{\pi^0;\,\pi^-} - 1/3(\sigma_0)_{\pi^+;\,\pi^+}) -$$

$$-((\sigma_0 P_0)_{\pi^-;\,\pi^-} + (\sigma_0 P_0)_{\pi^0;\,\pi^-} - 1/3(\sigma_0 P_0)_{\pi^+;\,\pi^+})] \qquad (27)$$

$$\mathrm{Re}\, g_8 g_1^* = 3/8\,[((\sigma_0)_{\pi^-;\,\pi^-} - 2(\sigma_0)_{\pi^0;\,\pi^-} + 1/3(\sigma_0)_{\pi^+;\,\pi^+}) +$$

$$+ ((\sigma_0 P_0)_{\pi^-;\,\pi^-} - 2(\sigma_0 P_0)_{\pi^0;\,\pi^-} + 1/3(\sigma_0 P_0)_{\pi^+;\,\pi^+})]$$

$$\mathrm{Re}\, h_8 h_1^* = 3/8\,[((\sigma_0)_{\pi^-;\,\pi^-} - 2(\sigma_0)_{\pi^0;\,\pi^-} + 1/3(\sigma_0)_{\pi^+;\,\pi^+})$$

$$-((\sigma_0 P_0)_{\pi^-;\,\pi^-} - 2(\sigma_0 P_0)_{\pi^0;\,\pi^-} + 1/3(\sigma_0 P_0)_{\pi^+;\,\pi^+})]$$

Here $(\sigma_0)_{\pi^+;\,\pi^+}$, $(\sigma_0)_{\pi^-;\,\pi^-}$, $(\sigma_0)_{\pi^0;\,\pi^-}$ we denote, for the sake of brevity, the differential cross sections for the processes (25.1), (25.2), (25.3) in c.m.s. and by $(\sigma_0 P_0)_{\pi^+,\,\pi^+}$, $(\sigma_0 P_0)_{\pi^-;\,\pi^-}$, $(\sigma_0 P_0)_{\pi^0;\,\pi^-}$ the products of the cross section on the polarization of the recoil nucleons for the corresponding processes.

Thus, the measurement of the differential cross sections σ_0 and of the nucleon polarizations P_0 (the left-right asymmetry) in all the three processes (25) allows one to determine the moduli $|g_8|$, $|g_1|$, $|h_8|$, $|h_1|$ as well as the cosines of the differences of the phases

$$\arg g_8 - \arg g_1 = \phi \qquad (28)$$

$$\arg h_8 - \arg h_1 = \psi$$

i.e. the moduli of all the amplitudes and the absolute values of the two phase differences (we assume that the phases are determined in the interval from $-\pi$ to π). To determine the third independent phase difference and the signs of ϕ and ψ it is necessary to make use of other observables. As the third independent phase difference we choose

$$\arg g_8 - \arg h_8 = \eta \qquad (29)$$

From (23) we find

$$\begin{aligned} \mathrm{Re}\, g_3 h^*_3 &= \tfrac{1}{2}[(\sigma_0 R)_{\pi^+;\pi^+} \cos\alpha - (\sigma_0 A)_{\pi^+;\pi^+} \sin\alpha] \\ \mathrm{Im}\, g_3 h^*_3 &= -\tfrac{1}{2}[(\sigma_0 R)_{\pi^+;\pi^+} \sin\alpha + (\sigma_0 A)_{\pi^+;\pi^+} \cos\alpha] \end{aligned} \tag{30}$$

Hence it is seen that the measurement of the parameters R and A in the $\pi^+ - p$ scattering allows one to determine unambiguously the phase difference η x). Finally, the signs of the phases ϕ and ψ can be found if one of the remaining parameters is measured, for example $R_{\pi^-;\pi^-}$. From (23) we get

$$\begin{aligned} &(\sigma_0 R)_{\pi^-;\pi^-} - 1/9\,(\sigma_0 R)_{\pi^+;\pi^+} = \\ &= \frac{8}{9}\,(\mathrm{Re}\, g_1 h^*_1 \cos\alpha - \mathrm{Im}\, g_1 h^*_1 \sin\alpha) + \\ &+4/9\,[(\mathrm{Re}\, g_1 h^*_3 + \mathrm{Re}\, g_3 h^*_1)\cos\alpha - (\mathrm{Im}\, g_1 h^*_3 + \mathrm{Im}\, g_3 h^*_1)\sin\alpha] \end{aligned} \tag{31}$$

If we express the real and imaginary parts of this expression in terms of the real and imaginary parts $g_3 g^*_1$, $g_3 h^*_3$, $h_3 h^*_1$ and the amplitude moduli then it takes the form:

$$a = b\epsilon_\phi + c\epsilon_\psi + d\,\epsilon_\phi \epsilon_\psi \tag{32}$$

where $\epsilon_\phi = \frac{\phi}{|\phi|}$ and $\epsilon_\psi = \frac{\psi}{|\psi|}$ are the signs of the phases ϕ and ψ respectively, and

$$\begin{aligned} a &= [(\sigma_0 R)_{\pi^-;\pi^-} - 1/9\,(\sigma_0 R)_{\pi^+;\pi^+}]\,|g_3|\,|h_3| - \\ &- \frac{4}{9}\cos(\eta+\alpha)\,(|h_3|^2\,\mathrm{Re}\, g_3 g^*_1 + |g_3|^2\,\mathrm{Re}\, h_3 h^*_1 + 2\,\mathrm{Re}\, g_3 g^*_1\,\mathrm{Re}\, h_3 h^*_1) \\ b &= 4/9\sqrt{|g_3|^2\,|g_1|^2 - (\mathrm{Re}\, g_3 g^*_1)^2}\,\sin(\eta+\alpha)\,(2\,\mathrm{Re}\, h_3 h^*_1 + |h_3|^2) \\ c &= -4/9\sqrt{|h_3|^2|h_1|^2 - (\mathrm{Re}\, h_3 h^*_1)^2}\,\sin(\eta+\alpha)\,(2\,\mathrm{Re}\, g_3 g^*_1 + |g_3|^2) \\ d &= 8/9\sqrt{|g_3|^2|g_1|^2 - (\mathrm{Re}\, g_3 g^*_1)^2}\,\sqrt{|h_3|^2|h_1|^2 - (\mathrm{Re}\, h_3 h^*_1)^2}\,\cos(\eta+\alpha) \end{aligned} \tag{33}$$

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x) As was pointed out earlier, to determine uniquelly a relative phase it is sufficient, in principle, to determine only the sign of the parameter A.

All the quantities entering here can be expressed in terms of the observables ((27), (30)) and assumed to be known from previous measurements. By means of (32), (33) ϵ_{ϕ} and ϵ_{ψ} can be found.[x)]

In conclusion we note that the above-considered procedure of the reconstruction of the pion-nucleon scattering matrix implies that the experimental values are measured with a sufficient accuracy. If the expermental errors are large then to reconstruct the scattering matrix the information about the remaining observable may be needed.

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x) If a is equal (or is equal by its absolute value and opposite in sign) to one of the coefficient b, c, or d and the remaining two coefficients are equal by the absolute value and opposite in sign (or are equal) then to determine ϵ_{ϕ} and ϵ_{ψ} use should be made of one more relation.