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ELECTROMAGNETIC CURRENT AND FORM-FACTORS FOR THE MESON 35-PLET IN THE SL(6,C) SYMMETRY

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It is well known that the groups U(6,6) and SL(6,C) suggested by a number of authors[1-6 et al] as relativistic generalizations of the SU(6) symmetry, give predictions which disagree with the experimental data concerning four-particle processes. The situation is much better for three-particle vertices, where the agreement is usually very good. The reason for this could be that electromagnetic and weak vertices, similarly as forward and backward scattering, two-meson annihilation of a baryon-antibaryon pair at rest et al. can always be considered to be colinear processes in some coordinate system. In such a case the application of the inhomogeneous U(6,6) or SL(6,C) groups practically reduces to the application of certain "colinear" subgroups [7,8], such as $SU(6)_W$ or SU(3) x SU(3). Thus the assumptions made in calculations concerning colinear processes (e.g. vertices) are weaker than those for general processes and it is quite possible that the neglected symmetry breaking is not so important in this case.

For these reason it is interesting to consider all possible colinear processes. In this paper we shall consider the meson electromagnetic current in the framework of the nonhomogeneous SL(6,C) group. We shall use the formalism developped by Nguyen van Hieu[6], which has been applied to the baryon currents in[9].

The U(6,6) invariant meson current has been considered in[10] and experiments concerning the creation of meson pairs in colliding electron-positron beam experiments were suggested to test the corresponding predictions. An analogous method was used to study the baryon currents[11] and baryon-antibaryon annihilation into to mesons[12] in the inhomogeneous U(6,6) symmetry.

The mesons, transforming as an SU(6) 35-plet in the rest system, are described by four spinors with respest to SL(6,C) Φ^{A}_{B}, $\Phi^{A}_{\dot{B}}$, $\Phi^{\dot{A}}_{B}$, $\Phi^{\dot{A}}_{\dot{B}}$ satisfying invariant equations of the Pauli - Fierz type[13]. We use these spinors and the 36-dimensional initial and final state momenta $Q^{A}_{\dot{B}}$, $Q^{\dot{A}}_{B}$ and

$P^{A}_{\dot B}$, $P^{\dot A}_{B}$ to construct the most general matrix elements of the currents $I^{A}_{\dot B}$, $I^{\dot A}_{B}$ transforming like the corresponding SL(6,C) spinors. We shall consider the current to be hermitean

$$I^{+A}_{\dot B} = I^{B}_{\dot A} \qquad I^{+\dot A}_{B} = I^{\dot B}_{A} \tag{1}$$

i.e.

$$\langle Q \mid I^{A}_{\dot B} \mid P \rangle^{*} = \langle P \mid I^{B}_{\dot A} \mid Q \rangle \qquad \langle Q \mid I^{\dot A}_{B} \mid P \rangle^{*} = \langle P \mid I^{\dot B}_{A} \mid Q \rangle$$

and take P and C invariance and current conservation into account.

Setting $K = P - Q$, $L = P + Q$, taking the equations for the spinors and relations for the momenta (of the type $K^{A}_{\dot B} K^{\dot B}_{C} = K^{\dot A}_{B} K^{B}_{\dot C} = k^2 \delta^{A}_{C}$) into account we obtain :

$$\begin{aligned}
\langle P \mid I^{A}_{\dot B} \mid Q \rangle = {} & F_1 [\Phi^{+A}_{C} \Phi^{C}_{\dot B} + \Phi^{+A}_{\dot C} \Phi^{\dot C}_{\dot B} + \Phi^{+\dot C}_{\dot B} \Phi^{A}_{\dot C} + \Phi^{+C}_{\dot B} \Phi^{A}_{C}] + \\
& + F_2 [\Phi^{+D}_{\dot B} (\tfrac{iK}{m})^{A}_{\dot C} \Phi^{\dot C}_{D} - \Phi^{+\dot C}_{D} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{A}_{\dot C} + \Phi^{+\dot C}_{D} (\tfrac{iK}{m})^{A}_{\dot C} \Phi^{D}_{\dot B} - \Phi^{+A}_{\dot C} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{\dot C}_{D}] + \\
& + F_3 [\Phi^{+\dot C}_{D} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{A}_{\dot C} - \Phi^{+\dot C}_{\dot B} (\tfrac{iK}{m})^{A}_{\dot D} \Phi^{\dot D}_{\dot C} + \Phi^{+A}_{D} (\tfrac{iK}{m})^{C}_{\dot B} \Phi^{D}_{C} - \Phi^{+\dot C}_{\dot D} (\tfrac{iK}{m})^{A}_{\dot C} \Phi^{\dot D}_{\dot B}] + \\
& + F_4 [\Phi^{+\dot C}_{D} (\tfrac{iK}{m})^{A}_{\dot C} (\tfrac{iK}{m})^{E}_{\dot B} \Phi^{D}_{E} + \Phi^{+\dot C}_{\dot E} (\tfrac{iK}{m})^{A}_{\dot C} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{\dot E}_{D} + \\
& + \Phi^{+\dot C}_{D} (\tfrac{iK}{m})^{A}_{\dot E} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{\dot E}_{\dot C} + \Phi^{+C}_{D} (\tfrac{iK}{m})^{A}_{\dot E} (\tfrac{iK}{m})^{D}_{\dot B} \Phi^{E}_{C}] + \\
& + F_5 [\Phi^{+C}_{\dot D} (\tfrac{iK}{m})^{\dot D}_{C} (\tfrac{iK}{m})^{A}_{\dot E} \Phi^{\dot E}_{\dot B} + \Phi^{+A}_{C} (\tfrac{iK}{m})^{C}_{\dot B} (\tfrac{iK}{m})^{\dot D}_{E} \Phi^{E}_{\dot D} + \\
& + \Phi^{+C}_{\dot D} (\tfrac{iK}{m})^{\dot D}_{C} (\tfrac{iK}{m})^{E}_{\dot B} \Phi^{A}_{E} + \Phi^{+\dot C}_{\dot B} (\tfrac{iK}{m})^{A}_{\dot C} (\tfrac{iK}{m})^{\dot D}_{E} \Phi^{E}_{\dot D}] +
\end{aligned} \tag{2}$$

It follows from P-invariance that we obtain a completely analogous expression with the same formfactors for $<P|I^{\dot{A}}_{B}|Q>$. In (2) we have taken into account that $\Phi^{+\dot{E}}_{F} K^{F}_{\dot{E}} = \Phi^{+E}_{\dot{F}} K^{\dot{F}}_{E}$ and $K^{\dot{D}}_{E} \Phi^{E}_{\dot{D}} = K^{D}_{\dot{E}} \Phi^{\dot{E}}_{D}$. All the formfactors F_i are functions of the square k^2 of the momentum transfer.

We use this SL(6,C) invariant current to construct the usual vector (electromagnetic) current, invariant with respect to SU(3) x L, where L is the Lorentz group. To do this we project all momenta into the physical space

$$K^{\dot{A}}_{B} \to \hat{k}^{\dot{a}}_{b}\, \delta^{\alpha}_{\beta} \qquad K^{A}_{\dot{B}} \to \hat{k}^{a}_{\dot{b}}\, \delta^{\alpha}_{\beta} \tag{3}$$

and reduce the meson functions according to [6]

$$\Phi^{\dot{A}}_{B} = -\tfrac{1}{2}\left(\frac{i\hat{p}}{m}\right)^{\dot{a}}_{\dot{c}} \sigma_{\mu b}^{\dot{c}}\, \xi^{\alpha}_{\mu\beta} + \tfrac{1}{2}\delta^{\dot{a}}_{b}\, \phi^{\alpha}_{\beta}$$

$$\Phi^{\dot{A}}_{\dot{B}} = -\tfrac{1}{2}\left(\frac{i\hat{p}}{m}\right)^{\dot{a}}_{c} \sigma_{\mu\dot{b}}^{c}\, \xi^{\alpha}_{\mu\beta} - \tfrac{1}{2}\delta^{\dot{a}}_{\dot{b}}\, \phi^{\alpha}_{\beta} \tag{4}$$

$$\Phi^{\dot{A}}_{B} = \tfrac{1}{2}\sigma_{\mu b}^{\dot{a}}\, \xi^{\alpha}_{\mu\beta} - \tfrac{1}{2}\left(\frac{i\hat{p}}{m}\right)^{\dot{a}}_{b} \phi^{\alpha}_{\beta}$$

$$\Phi^{A}_{\dot{B}} = \tfrac{1}{2}\sigma_{\mu\dot{b}}^{a}\, \xi^{\alpha}_{\mu\beta} + \tfrac{1}{2}\left(\frac{i\hat{p}}{m}\right)^{a}_{\dot{b}} \phi^{\alpha}_{\beta}$$

Defining the vector current matrix elements as

$$<p|j_\mu|q> = \tfrac{1}{2}\sigma_{\mu a}^{\dot{b}} \Lambda^{\beta}_{\alpha} <P|I^{A}_{\dot{B}}|Q> + \tfrac{1}{2}\sigma_{\mu\dot{a}}^{b} \Lambda^{\beta}_{\alpha} <P|I^{\dot{A}}_{B}|Q> \tag{5}$$

where $\Lambda = \tfrac{1}{2}(\Lambda_3 + \frac{1}{\sqrt{3}}\Lambda_8)$ is the charge matrix, we have

$$<p|j_\mu|q> = (f_1 + \frac{k^2}{2m^2} f_2)\, i\, \ell_\mu (\phi^+ \Lambda \phi)_F +$$

$$+ \{(-f_1 + f_2 + f_3)[\xi^+_\mu (i\ell\xi) + \xi_\mu (i\ell\xi^+)] + (f_1 + \frac{k^2}{2m^2} f_2)\, i\, \ell_\mu (\xi^+ \xi) + \tag{6}$$

$$+ \frac{f_3}{m^2}\, i\ell_\mu (\ell\xi^+)(\ell\xi)\}(V^+ \Lambda V)_F +$$

$$+ \frac{1}{2m}(f_1 - f_2 - f_3)\epsilon_{\rho\tau\nu\mu}\, k_\rho \ell_\tau [\xi^+_\nu (V^+ \Lambda \phi)_D - \xi_\nu (\phi^+ \Lambda V)_D]$$

where e.g.

$$(\phi^{+}\Lambda\phi)_{F} = \phi^{+\alpha}_{\cdot\gamma}\,\Lambda^{\beta}_{\alpha}\,\phi^{\gamma}_{\beta} - \phi^{+\alpha}_{\cdot\gamma}\,\phi^{\beta}_{\alpha}\,\Lambda^{\gamma}_{\beta}$$

$$(\phi^{+}\Lambda\, V)_{D} = \phi^{+\alpha}_{\cdot\gamma}\,\Lambda^{\beta}_{\alpha}\,V^{\gamma}_{\beta} + \phi^{+\alpha}_{\cdot\gamma}\,V^{\beta}_{\alpha}\,\Lambda^{\gamma}_{\beta}$$

and

$$f_1 = -\frac{1}{2m}\left(F_1 + \frac{k^2}{m^2}F_4\right), \quad f_2 = \frac{F_2}{m}, \qquad f_3 = -\frac{F_3}{m}\ .$$

We see that only three combinations of the five SL(6,C) formfactors contribute to the vector current (all five would contribute to the axial vector current) and that the five formfactors corresponding to SU(3) x L can be expressed with the help of three functions.

It follows from (6) that the formfactor $f_m = f_1 - f_2 - f_3$ determines both the radiation decay probability for vector mesons and their magnetic moments. Unfortunately no data on the vector meson magnetic momenta are available so that it is so far not possible to make a comparison with experiments. The current (6) can be used to calculate the cross section of the processes

$$e^{+} + e^{-} \to M + M$$

i.e. the creation of meson pairs in colliding electron-positron beam experiments, however, no simple relatons between the cross sections of various processes can be obtained (they will depend significantly on the energy) .

There is a simple connection between the meson current in U(6,6)[10] and in SL (6,C) : for $f_2 = f_3$ the relation (6) goes over into the corresponding U (6,6) current. Thus we have an additional relation in U(6,6) : the formfactor for pseudoscalar mesons is equal to the charge formfactor for vector mesons.

It is interesting that our results in certain points contradict those of Rühl[14] for the meson electromagnetic vertex. The reason is that we use different assumptions concerning the transformatio properties of photons with respeat to SL (6,C) . In[14] the three-meson vertex (with no irregular couplings) is considered and a photon is substituted for one of the vector mesons. The absence of irregular couplings in this case corresponds to the assumption that the photon has a finite mass, obeys the same invariant equations as the mesons and that its mass should be limited to zero in the final expression. We only assume that the electromagnetic current is given by (5) where $I^{A}_{\dot{B}}$, $I^{\dot{A}}_{\ B}$ transform like the corresponding SL (6.C) spinors. The absence of scalar and longitudinal photons is ensured by current conservation.

Both our current and that in [14] depend on three formfactors only, however the five SU(3) x L formfactors are connected by different relations. E.g. there is no relation between the $V \to PS + \gamma$ decay probability and vector mesons magnetic momenta in [14]. Thus the equality of the corresponding formfactors could be a criterion of the two alternative hypotheses.

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