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DISPERSION SUM RULES AND SU(6) SYMMETRY

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Recently the new sum rules connecting magnetic moments and other coupling constants with many-particle amplitudes have been written down with the aid of dispersion relations for equal time commutators[1]. The aim of this note is to show that the analogous sum rules can be simply derived from usual dispersion relations at fixed momentum transfer under some assumptions about the high energy behaviour of physical amplitudes[2]. These sum rules are very useful when combined with unitary symmetry.

As in ref.[2] let us consider the amplitude $T\cdot\epsilon$ for virtual photoproduction (electroproduction) of pions on nucleons, where ϵ is the polarization four vector of the virtual photon with momentum k. In the Breit coordinate system

$$\vec{T} = i(\vec{\sigma}\vec{\lambda})\vec{\lambda}\, L +$$

$$+ i\vec{\sigma}\, L_1 + i(\vec{\sigma}\vec{p})\vec{p}\, L_2 + i(\vec{\sigma}\vec{\lambda})\vec{p}\, L_3 + [\vec{p},\vec{\lambda}]\, L_4 + i(\vec{\sigma}\vec{p})\vec{\lambda}\, L_5 , \qquad (1)$$

where $\vec{\lambda} = \lambda\vec{e}$, $\vec{e}$ is the unit vector perpendicular to the target momentum $\vec{p}$, $\lambda = (E^2 - f(k^2, \vec{p}^{\,2}))^{1/2}$ and E is the pion energy.

It has been proved[3] that the combination $T(\vec{e}) + T(-\vec{e})$ satisfies dispersion relation in E at fixed k^2 and $\vec{p}^{\,2}$. The same relation holds for $\lambda^2 L$. Furthermore, T does not depend on $\vec{e}$ when $\lambda = 0$ as it can be seen from the integral representation usually used in the dispersion relation theory. Hence $\lambda^2 L = 0$ at $\lambda^2 = 0$ and L has no kinematical singularity.

Let us now suppose that

$$T(E) \to \text{const} \qquad (2)$$

when $E \to \infty$ at fixed k^2 and $\vec{p}^{\,2}$. This assumption means that the laboratory differential cross section for the forward electroproduction at high energy does not depend on the longitudinal amplitudes L and L_5. In this case $L \to C/E^2$ and we can use dispersion relations both for L and EL without subtractions:

$$\mathrm{Re}\, L(E) = \frac{P}{\pi} \int_{-\infty}^{\infty} \mathrm{Im}\, L(E') \frac{dE'}{E'-E}, \qquad (3)$$

$$\mathrm{Re}\, L(E) = \frac{P}{\pi} \int_{-\infty}^{\infty} \mathrm{Im}\, L(E') \frac{E'\,dE'}{E(E'-E)}.$$

From these relations we immediately get the following sum rule:

$$\int_{-\infty}^{\infty} \mathrm{Im}\, L(E)\, dE = 0 . \qquad (4)$$

In the Lorentz invariant form

$$\Gamma = \sum_{i=1}^{6} \bar{u}(p') \gamma^5 R_i u(p) F_i , \qquad (5)$$

where

$$R_1 = R(p, p') \qquad R_5 = R(p-p', k)$$

$$R_{2,3} = R(p \pm p', \gamma) \qquad R_6 = (\gamma . k) R_4 \qquad (6)$$

$$R_4 = R(\gamma, \gamma) \qquad R(a,b) = a\,(b.k) - (a.k)\,b .$$

From eqs. (1), (5) and (6) it follows that $L = -2F_6$ and is Lorentz invariant. Taking into account isotopic structure /4/ and crossing symmetry of L and separating the one-nucleon term we finally get

$$-g F_{\mu}^{(V)}(k^2) + \frac{1}{\pi} \int_{(M+m)^2}^{\infty} \mathrm{Im}\, L^{(+)}(s, t, k^2)\, ds = 0, \qquad (7)$$

$$-g F_{\mu}^{(S)}(k^2) + \frac{1}{\pi} \int_{(M+m)^2}^{\infty} \mathrm{Im}\, L^{(0)}(s, t, k^2)\, ds = 0, \qquad (8)$$

where s and t are the usual Mandelstam variables, $M(m)$ is the nucleon (pion) mass, g is the pion-nucleon coupling constant, $F_{\mu}^{(V,S)}(k^2) = \mu'_{V,S} F_2^{(V,S)}(k^2)$, $\mu'_{V,S}$ are the anomalous isovector and isoscalar magnetic moments of the nucleon and $F_2^{(V,S)}(k^2)$ are the corresponding Pauli form factors.

Let us now neglect in eqs. (7) and (8) every intermediate state but those entering the 56-plet, the lowest baryon multiplet of SU(6). Then eq. (7) leads to a relation between nucleon form factors, form factors of the transition $N^* \to N\gamma$ and the width of the 33 resonance N^*. Eq. (8) gives

$$F_{\mu}^{(S)}(k^2) = 0 \qquad (9)$$

and, in particular,

$$\mu_p - e/2M + \mu_n = 0 , \qquad (10)$$

where $\mu_{p,n}$ are the total magnetic moments of the proton and neutron. Combining eq. (10) with the famous SU(6) relation

$$\mu_p / \mu_n = -3/2 \qquad (11)$$

we get the result

$$\mu_p = 3, \qquad \mu_n = -2 \qquad (12)$$

(in nuclear magnetons), obtained in ref. /5/ in a different way.

This approach can be applied to many other reactions to give relations between vertex functions of particles and resonances.

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